Endogenous Mortality, Human Capital and Economic Growth*

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Abstract

We consider growth and welfare effects of lifetime-uncertainty in an economy with human capital-led endogenous growth. We argue that lifetime uncertainty reduces private incentives to invest in both physical and human capital. Using an overlapping generations framework with finite-lived households we analyze the relevance of government expenditure on health and education to counter such growth-reducing forces. We focus on three different models that differ with respect to the mode of financing of education: (i) both private and public spending, (ii) only public spending, and (iii) only private spending. Results show that models (i) and (iii) outperform model (ii) with respect to long-term growth rates of per capita income, welfare levels and other important macroeconomic indicators. Theoretical predictions of model rankings for these macroeconomic indicators are also supported by observed stylized facts.

***JEL Classification*:** I1; I2; O1; H5
***Keywords*:** Health; Life Expectancy; Human Capital; Public Spending; Endogenous Growth.

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1. Introduction

Countries differ dramatically in the way they finance their educational systems. Education can be provided through public funds, through private funds, or a combination of the two. The Education at a Glance (OECD, 2003) reports private and public shares in overall education outlays for a total of 36 countries including most OECD and a small number of non-OECD countries for the year 2000. While no country uses either funding system exclusively, there are a number of countries with public education shares in 2000 that are close to 100% such as Norway (98.7%), Turkey and Portugal (98.6% each), Finland (98%) and Sweden (97%). Of the 36 countries, 19 countries (53%) finance at least 90% of their overall educational expenditures through public spending. In contrast, there are a number of countries that put a larger responsibility on the private provision of education. Among those, Chile has gone the furthest with a private education share of more than 46%. Other countries with large private education shares include South Korea (41%), Indonesia (36%), Jamaica (35%), and the United States (32%).

Overall investment in education, through its effect on the stock of human capital, has long been identified as a source for long-run growth in an economy (see, for example, Rebelo, 1991). In this broad category, a number of studies have specifically stressed the importance of public investment in education in further enhancing the growth performance of the economy (Glomm and Ravikumar, 1992, 1997, Boldrin, 1992, and Benabou, 1996). Given the trade-off between public and private sources in financing education, the question arises how different degrees of government involvement in the production of human capital described above affect welfare, long-term growth, and other indicators of economic performance.

To further complicate matters, budget components like public education spending must compete with other budget items such as internal and external security, infrastructure expenditures, and debt servicing, to name a few. One of the fastest growing budget components of many countries is public health expenditure. IMF data show that between 1972 and 1999 the share of public health expenditure in total public spending has increased two-fold or more in many countries. Over the 1972-99 period, the average annual growth rate of the public health expenditure share was 1.7%.[1] Just like the market for education, the provision of health services can also be linked to a number of positive externalities, which in turn explain the large public involvement in the health sector. One such externality that has not been sufficiently recognized in the literature is the impact of public health spending on longevity: Increased levels of health expenditures are most likely to be positively associated with higher life expectancy. In addition to the individual benefits of living longer lives, increased life expectancy may confer important growth effects. These effects arise since increased longevity produces stronger incentives to invest in physical and human capital as these long-term assets yield high returns only later in life (Chakraborty, 2004). Importantly, since public spending on education competes with public spending on health, a second trade-off exists that, like the first one between public and private education shares, may matter for the long-term growth rate of the economy as well as its level of welfare.

In this paper, we study the trade-off between public and private spending on education in a model with uncertain lifetimes and endogenous growth. To this end, we construct a two-period overlapping generations model in which survival of an individual in the second period is uncertain. The probability of survival depends on her health, which, in turn, is determined by both public health spending and her health when young. To the extent that good (poor) health and consequent higher (lower) longevity generates (dis)incentive for private accumulation of human capital, public health expenditure plays an important role in generation of human capital, thereby affecting long-run growth. However, the more the government spends on health, the less

[1]This average growth rate is based on data from 26 countries. 10 countries have been excluded due to missing or unreliable time series data on public health care expenditures. Countries with the fastest growing public health expenditure share are Uruguay (from 1% to 6%), Israel (from 4% to 13.7%), the U.S. (from 8.6% to 20.5%), Paraguay (from 3.5% to 7.3%), Australia (from 7% to 14.8%), and Thailand (from 3.7% to 7.4%).

it can spend on public education, which, together with private spending on education, determines future human capital. Since human capital accumulation is the engine of growth in this model, differences between public spending on health and education on one side and public and private spending on education on the other constitute the two fundamental trade-offs in our model that generate important growth and welfare consequences.

This paper connects two different strands of the growth literature. One of them focuses on 'productive' government spending assuming certainty with regard to the length of life for each individual (Glomm and Ravikumar, 1992, 1997; Boldrin, 1992; Benabou, 1996 and de la Croix and Doepke, 2004). Glomm and Ravikumar (1992) and de la Croix and Doepke (2004) focus on the effect of private and public provision of education on long-term economic growth and income inequality, while Glomm and Ravikumar (1997) discuss different forms of productive public expenditures and their effects on long-run growth.[2] Boldrin (1992) and Benabou (1996), on the other hand, focus on endogenous determination of public policies in a political economy setting. The other, more recent strand of the literature deals with uncertain lifetimes without exploring the potential endogenous growth implications of this assumption (Kalemli-Ozcan, 2002 and Chakraborty, 2004). While the former analyzes the effect of child mortality on 'precautionary demand for children' and human capital formation, the latter focuses on the 'poverty trap' phenomenon that endogenous mortality generates in an otherwise standard overlapping generations framework. Blackburn and Cipriani (2002) uses a discontinuous (step-function) endogenous survival function to explain the existence of multiple development regimes, in which the survival of an agent into old age depends upon her inherited level of human capital.

In this paper, we analyze three distinct model scenarios. We begin with the analysis of a model with both public and private education expenditures (the benchmark model) followed by

[2] Interestingly, public expenditure on health is *not* part of their analysis.

two alternative specifications: One with only public investment in education (the public education model) and one with exclusive private provision of education (the private education model). Except for the differences in funding education, all models share identical taste, technology, and policy (tax) parameters.

Our main results are as follows. First, longevity is highest in the private education model, followed by the benchmark model. Second, with regard to long-run growth, interest rate, and human-to-physical capital ratio, the benchmark and private education model generate similar values, all of which are higher than the corresponding values in the public education model. Third, with respect to welfare, the private education model ranks highest, followed by the benchmark and the public education models. Fourth, we compare the benchmark to the public education model for an optimally chosen tax rate (in a second-best sense). We show that the welfare ranking of two regimes depends on the (exogenous) relative size of government spending on education and health. For high levels of public spending on education relative to health, the public education model is welfare superior, while the reverse is true for low public education spending. Finally, we show that the observed stylized facts support the theoretical model rankings for several macroeconomic indicators.

The conclusions drawn from our analysis add to our understanding of the link between longevity, growth, and welfare. The growth, longevity, and welfare superiority of the private education vis-à-vis the public education model is based on the fact that in a world with limited government resources, a country is better off if the government concentrates its scarce resources on fewer budget items (here: health) instead of spreading itself thin on too many budgetary needs (here: health and public education). However, we do find that the welfare and growth rankings of the two models can be reversed, but only for very high levels of taxation. When high rates of taxation are not politically feasible, an economy with a “leaner government” with fewer tasks at hand performs better than one with a government that is fiscally starved relative to the size of its duties.

Our results extend some of the findings of the existing literature. Glomm and Ravikumar (1992) show that in countries with low-income inequality, a fiscal externality created by public schooling leads to lower growth under public schooling than under private schooling. Highlighting the interaction between fertility and education decision of parents, de la Croix and Doepke (2004) also reach similar conclusion. As in our model, Glomm and Ravikumar (1997) find that balanced growth is achievable through an intergenerational externality produced by productive government spending. As in Chakraborty (2004), our model predicts that large gains in human capital and per-capita income are possible by improving life expectancy. Unlike Chakraborty but similar to Glomm and Ravikumar (1997), we obtain a trade-off between long-run growth and taxation in the two models with private involvement in education. Boldrin (1992) shows that human capital shrinks to zero when tax rate falls to zero, while in our model, human capital stock (and hence the growth of income) does not vanish even in the absence of taxation due to private spending on education.

The paper unfolds as follows: Section 2 lays down the basic model framework and the characterization of the competitive equilibrium. The three models are analyzed in sections 3, 4 and 5, respectively. Section 6 provides a numerical comparison of the three models including welfare analysis when tax policy is endogenous and transitional dynamics. Section 7 concludes.

2. The Common Framework

Consider an overlapping generations economy with productive capital and life uncertainty of the individuals. Individuals potentially live for two periods. Survival in the first period of life is a certain event, but whether or not the individuals will survive in the next period depends on a positive probability ϕ, called the probability of survival. The Individuals are identical within each cohort. At time $t = 0$, there is an initial generation of old agents with health stock, x_0, human capital stock h_0 and physical capital stock k_0. In any period t, young agents of measure one are born, each with a strictly positive amount of health stock x_t^t, human capital h_t^t and a time endowment of one unit, where the superscripts refer to generation and the subscripts refer to time period. A generation-t individual combines her human capital with the labor time and supplies this effective labor inelastically, earning a wage income $w_t h_t^t$. The probability of survival in the second period, ϕ_{t+1} depends on the state of their health (or, their health stock) in the second period, x_{t+1}^t. In particular, it is given by a non-decreasing concave function:

$$\phi_{t+1} = \phi(x_{t+1}^t),\ \phi' > 0,\ \phi'' < 0,\ \phi(0) = 0 \tag{1}$$

Health stock of an individual is assumed to affect only her survival into the old age, and not her labor productivity or survival beyond her old age. It is assumed that health stock depends not on her direct private spending, but on the overall income level of the economy at the time of her birth. Specifically, health stock of a generation-t individual in the second period, x_{t+1}^t, depends positively on her own health stock in the first period, x_t^t, and the government investment in health in the first period, g_t^x, as a proportion of the total capital stock k_t. It is assumed that government investment in health does not have any contemporaneous effect. It improves health of individuals only with a lag of one period. Nevertheless, it is a public good and therefore equally available to the two generations present at any point in time and therefore has no generation superscript. Health is also assumed to depreciate at a constant rate $\delta_x \in (0,1)$. Health stock of an individual of generation-t in period t+1 is given by

$$x_{t+1}^t = M(x_t^t)^{1-\psi} \left(g_x^t / k_t\right)^{\psi} - \delta_x x_t^t \tag{2}$$

where M is the productivity parameter in the health production, δ_x is the fixed rate of depreciation of health stock between successive periods of life and $1-\psi$ measures the degree to which an individual's own intrinsic state of health x_t^t matters in determining her final health status.[3] The evolution of health stock also depends on g_t^x, normalized by capital stock k_t, which is necessary for existence of a balanced growth path equilibrium. The reason for this normalization is to make the long-run health stock compatible with balanced growth. In an endogenous growth model like this, along a balanced growth path, an ever-increasing level of income (output) and government tax revenue would implausibly lead to ever-increasing stock of health. This normalization prevents such a scenario.[4] Public provision of health gives rise to a positive externality to the private health production and the parameter ψ measures the strength of such externality (for example, for an young agent, the higher the value of ψ, the stronger (or, more effective) the externality of public health system on her future health).

The assumption that health stock does not depend on direct private spending on health is supported by available empirical evidence. In a study of sources of U.S. longevity during 1960-97, Lichtenberg (2002) concludes, "estimates provide strong evidence against the null hypothesis that public health expenditure has no effect on longevity, but not against the null hypothesis that private health expenditure has no effect on longevity". Blackburn and Cipriani (2002) argues that probability of life-extension could be driven by public spending on health. Our own empirical study also supports this conclusion. Table 1 reports the summary statistics for Life Expectancy Index (*LEI*) as constructed by United Nations Human Development Report (2003) and shares of

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[3] We could have used an alternative way of incorporating depreciation in the health accumulation function, namely. $x_{t+1}^t = M\,(1-\delta_x)(x_t^t)^{1-\psi}\left(g_x^t/k_t\right)^{\psi}$ without altering the qualitative results of the paper.

[4] According to Blackburn and Cipriani (1998), who use a similar specification; argue that such normalization captures the idea that greater public spending on health is an essential means of mitigating the potential adverse effects on longevity of greater economic activity as proxied by capital stock. In our case, economic activity could have been proxied by income, ensuring constancy of health stock in the long-run. However, this would have rendered the evolution of health completely autonomous. In this paper, we intend to highlight the interaction among health, human capital and physical capital stocks and normalization by k_t helps us achieve that.

private and public expenditures on health as a percentage of GDP. Figures 1 and 2 show the relationship between *LEI* and private and public spending on health as a percentage of *GDP*, respectively for 173 countries. Share of public spending on health has a significant positive effect, whereas share of private spending on health has no significant effect on *LEI*. The results are very similar if, instead of *LEI*, we use “Probability of survival beyond age 40” as the dependent variable. Also, if health of an economy is roughly measured by number of doctors per 100,000 population, empirical evidence shows that such a measure of health is positively (and significantly) associated with share of public health expenditure, while its association with the share of private health expenditure is statistically not different from zero.[5]

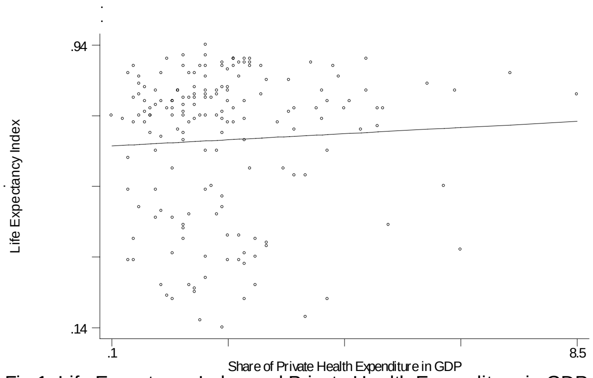


Fig.1: Life Expectancy Index and Private Health Expenditure in GDP

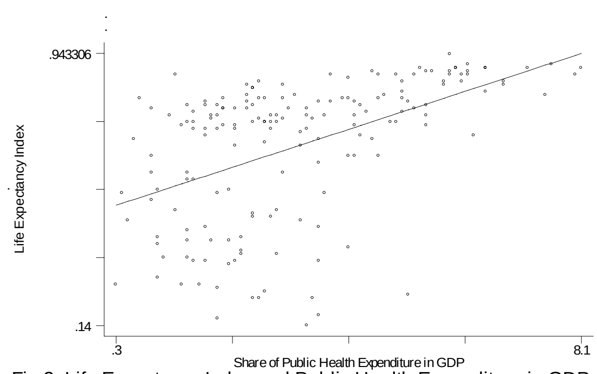


Fig.2: Life Expectancy Index and Public Health Expenditure in GDP

Human capital is another stock in the economy. It is different from health stock of an individual in three respects: First, mortality of an agent is solely determined by her health stock and does not directly depend on her human capital. Second, her labor productivity depends on her human capital, and not on her health. Third, individuals receive direct utility from the level of human capital (not health) of their offsprings. Therefore, parents consciously spend on their children’s schooling whenever it is effective in raising the level of future human capital. The

[5] Regression results using the HDR dataset (UNDP, 2003) show that, while controlling for population size and an index of education, a one percent increase in share of public health expenditure (in GDP) raises the number of physicians (per 100,000 population) by 0.73%, while the share of private health expenditure (in GDP) does not seem to have any effect on the number of physicians. The coefficient of the share of public health expenditure is statistically significant (*t*-stat = 3.86), while that of the share of private health expenditure is negative and statistically insignificant (*t*-stat = -1.33). Detailed results are available upon request.

stock of human capital of an individual of generation t+1, h_{t+1}^{t+1}, depends on three factors. First, it depends on the stock of human capital of her parents, h_t^t. This assumption captures the intergenerational transmission of knowledge (e.g. education or skill) between parents and their children. Next, h_{t+1}^{t+1} depends on the level of educational expenditure undertaken by her parents when she is young, e_t.[6] Finally, h_{t+1}^{t+1} depends on the level of government investment in public education in the previous period, g_t^h, (e.g. education infrastructure, or the quality of the schools) as a proportion of total capital stock k_t. The evolution process of human capital is given by

$$h_{t+1}^{t+1} = B(h_t^t)^{1-\theta}(\mu e_t + \nu g_t^h)^{\theta}, \qquad B > 0,\ \theta \in (0,1),\ \mu, \nu > 0 \tag{3}$$

where B is the productivity parameter. The human capital accumulation function implies that education of an agent of generation t+1 depends both on private and public investment in education in the previous generation (e_t and g_t^h respectively) as well as on her parent's stock of human capital h_t^t, which is assumed to be strictly positive to avoid non-triviality. It is assumed that private and public spending carries a weight μ and ν, respectively, in human capital accumulation. The parameter ν can be interpreted as the strength of the externality generated by public investment in education. The greater the value of ν the stronger is the effectiveness of the public education system. The parameter θ denotes the elasticity of real investment in education (private and public), 1-θ captures the degree of intergenerational transmission of human capital. Since human capital is not needed in the old age (the old agents do not work), agents do not care about their own human capital and instead invest in their child's education. Thus like physical

[6] Since in this model individuals' lifetimes are confined to two periods, childhood is not modeled explicitly, and it is assumed that the effect of parental investment in education has an instantaneous or 'immediate' effect on child's human capital, on which her labor productivity and wage income depends. Interpreted another way, a young worker receives a salary based on her 'projected' level of human capital that she would achieve sometime during her productive life when her education gets completed

capital, human capital is accumulated over every generation rather than over every period.[7] Again, public investment in education does not have a generation superscript because of its 'public good' nature.

Table 1: Summary Statistics

	Obs.	Mean	SD	Min.	Max.
Life Expectancy Index	175	0.67	0.21	0.14	0.94
Share of Pub. Exp. on health (% of GDP)	173	3.37	1.78	0.30	8.10

Source: Human Development report (2003); UNDP.

As mentioned earlier, individual agents are not able to internalize the effect of uncertain lifetime on their lifetime utility, and hence they do not undertake personal investment in health to augment their survival probability. Thus, in this model, health is always a public good provided by the government. On the other hand, education is financed both by private individuals as well as the government, and the two are substitutes (Buiter and Keltzer, 1995). Income is taxed at a uniform rate in order to finance the public investment in health and education. Balanced budget condition is assumed to be fulfilled in each period, so that the budget constraint of the government is given by

$$\text{Total Revenue} = \tau w_t h_t^t = \underbrace{\gamma_h \tau w_t h_t^t}_{g_t^h} + \underbrace{(1-\gamma_h)\tau w_t h_t^t}_{g_t^x} = \text{Total Expenditure} \quad (4)$$

where, γ_h and (1-γ_h) are the shares of tax revenue spent on human capital and health respectively. Public investment in health (g_t^x) and education (g_t^h) are perfect substitutes in this model from a budgetary point of view.

[7] More concretely, in this model there is no use of an explicit h_{t+1}^t. Thus an agent's old-age human capital, h_{t+1}^t, can be thought of as the same as her young-age human capital h_t^t ignoring any depreciation. This old age human capital does not matter for the agent since she does not derive any income or utility out of it.

Each generation-t person gives birth to a new individual at the end of period-t, before she realizes her 'mortality shock'. This new individual inherits her parent's health and human capital stocks and becomes economically active only at the beginning of period t+1. Every decision is made in the young age and it is assumed that an individual realizes her mortality shock after the decisions regarding saving, expenditure on child's education etc. have been made.

To abstract from the risk associated with uncertain lifetime and concentrate on the length-of-life effect, we follow Yaari (1965) and Blanchard (1985) in assuming a perfect annuities market whereby all savings are intermediated through mutual funds.[8] At the end of her youth, each agent deposits her savings with a mutual fund. The mutual fund invests these savings in physical capital and guarantees a gross return of $\hat{R}_{t+1}$ to the surviving old.[9] If a fund earns a gross return R_{t+1} on its investment, then perfect competition would ensure that $\hat{R}_{t+1} = R_{t+1}/\phi_{t+1}$.

As in the standard neoclassical model, the production process utilizes a constant returns-to-scale, time invariant technology utilizing physical capital and effective labor units. The output produced at time t, y_t, is governed by

$$y_t = Ak_t^{\alpha}(h_t^t)^{1-\alpha}, \quad \alpha \in (0,1) \tag{5}$$

Perfect competition in the factor markets ensures that each factor is paid its marginal product so that at time t (assuming full depreciation of physical capital during the length of one period), the wage rate and the rental rate, respectively, are given by

$$w_t h_t^t = A(1-\alpha)k_t^{\alpha}(h_t^t)^{1-\alpha} \tag{6a}$$

[8] Imperfection in the annuities market can be introduced in this model without much complication. If annuities market was imperfect, a lower mortality rate would raise realized returns on each type of investment an individual undertakes encouraging such investments. In that case, the results that follow understate the true magnitudes. But a perfect annuities market assumption may not be far-fetched even for developing countries. When life-contingent annuities market is not well-developed, the family can self-insure against mortality risks through interfamily transfers, substituting by more than 70% for perfect market annuities (Kotlikoff and Spivak, 1981).

[9] An alternative assumption is one where the government takes over the assets of generation-t agents who do not survive and transfers them lump-sum to those alive. This gives qualitatively similar results as long as the transfers are made to the surviving members of the same cohort. If, however, these assets are transferred to the progeny of the deceased (accidental bequests), asymptotic growth may result (Fuster, 1999).

$$R_t k_t = A\alpha k_t^{\alpha}(h_t^t)^{1-\alpha} \tag{6b}$$

Individuals are identical within as well as across time. During the first periods of their lifetimes individuals supply their unit endowments of labor inelastically, and in the second period they retire. An individual with h_t^t level of human capital, receives $(1-\tau)w_t h_t^t$ as post-tax wages, which is spent on consumption in the first period, c_t^t, saving, s_t and child's education, e_t. Since no income is earned in the old age, all individuals save in order to finance consumption, c_{t+1}^t, in the old age. This saving is inealstically supplied to mutual funds, which pay a gross return $\hat{R}_{t+1}$ to the individual when she is old. Therefore, for an individual of generation-*t*, the budget constraints are given by:

$$c_t^t \leq (1-\tau)w_t h_t^t - s_t - e_t \tag{7a}$$

$$c_{t+1}^t = \hat{R}_{t+1}s_t, \text{ where, } \hat{R}_{t+1} = R_{t+1}/\phi_{t+1} \tag{7b}$$

In our model economy, the successive generations are altruistically linked. Altruism is 'impure' in the sense that in their old age, parents derive utility from the level of human capital of successive generations. Denoting the degree of altruism between current and future generations by $\beta > 0$, and assuming logarithmic felicity functions for simplicity, a generation-*t* agent's expected lifetime utility is given by

$$U_t = \log(c_t^t) + \phi[\log(c_{t+1}^t) + \beta\log(h_{t+1}^{t+1})] \tag{8}$$

where, the parameter $\beta > 0$ measures of the degree of altruism between the current and future generation. Thus, a generation-*t* individual maximizes Eq.(8) subject to the above constraints and equations of motions (2) - (3) and the non-negativity constraints $s_t > 0$, and $e_t \geq 0$, taking as given the vector of prices (w_t, $\hat{R}_{t+1}$) and the survival probability ϕ.

2.1 Market Equilibrium

Given the initial k_0, x_0 and h_0, a competitive equilibrium for the model described in the previous section is a sequence of allocations for aggregate production $\left\{k_t, h_t^t, x_t^t\right\}_{t=0}^{\infty}$, a set of prices $\left\{w_t, \hat{R}_{t+1}\right\}_{t=0}^{\infty}$ and a set of choices for the representative agent $\left\{c_t^t, c_{t+1}^t, e_t\right\}_{t=0}^{\infty}$ that solve the generation-t young agent's problem (8), the representative firm's profits such that conditions (1) – (5) and (7) are always satisfied, and product market clears, i.e. $s_t = k_{t+1}$.

A balanced growth path $\left\{k_t, h_t^t, x_t^t, c_t^t, c_t^{t+1}\right\}_{t=0}^{\infty}$ is a competitive equilibrium such that $\left\{k_t, h_t^t, c_t^t, c_t^{t+1}\right\}_{t=0}^{\infty}$ grow at a constant rate n, and $\left\{x_t^t\right\}_{t=0}^{\infty}$ stay constant. If $n > 0$, then the balanced growth path is said to be interior. Throughout our analysis, we assume that the balanced growth path is interior, which, we will see, is a rather mild requirement.

In the subsequent sections, we investigate the properties of balanced growth paths as well as the transitional dynamics in versions of the above model, which vary in their human capital accumulation processes as given by Eq.(3).

3. The Benchmark Model

3.1 Model Specifications ($\mu > 0,\ \nu > 0$)

Young individuals (parents) in the benchmark model optimize over own consumption levels in the two periods and expenditure on her child's education. The problem facing an individual of generation t at time t is one of maximizing Eq.(8) by choosing s_t and e_t subject to (7a) and (7b). Assuming interior solutions and omitting the generation superscripts, optimization on the part of the young agent of generation-t implies

$$s_t = \left[\frac{\phi[(1-\tau)\mu + \nu\gamma_h\tau]}{\mu[1+(1+\theta\beta)\phi]}\right] w_t h_t \tag{9}$$

$$\text{and,}\qquad e_t = \left[\frac{\mu\theta\beta\phi(1-\tau)-\nu\gamma_h\tau(1+\phi)}{\mu[1+(1+\theta\beta)\phi]}\right]w_t h_t \tag{10}$$

where $\phi = \phi(x_{t+1}^t)$. The equilibrium saving function (Eq. 9) has the following properties: $s_\phi > 0,\ s_w > 0,\ s_h > 0,\ s_\mu < 0,\ s_\theta < 0,\ s_\beta < 0,\ s_\tau \lesseqgtr 0$. Equilibrium saving is increasing in the probability of survival (ϕ) since the higher expected length of life raises the return from savings and hence increases the incentive to save. Saving is, as usual, an increasing function of wage income ($w_t h_t$) of an agent. It is decreasing in θ, the elasticity of educational investment in (child's) human capital. Also, the stronger motive for altruism, i.e. the higher the magnitude of β, the higher the investment in child's education and consequently, the lower the equilibrium saving. The weight of private spending on education, μ, has a negative effect on equilibrium saving. Interestingly, the impact of tax rate on saving is ambiguous. The ambiguity arises from two opposing forces that an increase in tax rate generates. A rise in τ raises (i) health and longevity of parents as well as (ii) public spending on education, while lowering the disposable income for saving and private educational spending. If the positive effects (i) and (ii) dominate the negative income effect, an increase in tax rate would raise private saving.

Eq. (10) denotes the equilibrium private expenditure on education (for the next generation-t+1) and is characterized by: $e_\phi > 0,\ e_w > 0,\ e_h > 0,\ e_\mu > 0,\ e_\theta > 0,\ e_\beta > 0,\ e_\tau \lesseqgtr 0$. As expected, a higher survival probability, wage income, productivity of educational spending, degree of altruism, all raise parent's investment in child's education, and the tax rate on income has an ambiguous effect on e_t. The last result stems, once again, from the dual impact that a change in tax rate has on parental decision on educating her child. First, an increase in tax rate reduces disposable income, hence e_t. On the other hand, as a result of the increased tax rate parental longevity will be higher inducing them to spend more on child's education. The final effect will depend on whether the income effect is stronger than the incentive effect.

It is evident from Eq.(10) that the model gives rise to possibility of a corner solution ($e_t = 0$), where low survival probability eliminates any incentive for the parents to invest in child's education. The return from investment in child's education is typically uncertain for poor parents because the fruits of that investment are to be reaped only later in their lives when they might not be alive. Among other things, uncertainty regarding the length of life makes such investments in child's education risky. Such a situation arises when $\phi \leq \frac{\nu\gamma_h\tau}{\theta\beta(1-\tau)-\nu\gamma_h\tau}$, i.e. when tax rate or share of public revenue spent on education is too low. On the other hand, given ϕ, if tax rate is too high, the resulting negative income effect would drive down the optimal education spending by the parents toward zero. Thus, if $\tau \geq \frac{\theta\beta\phi}{\nu\gamma_h(1+\phi)+\theta\beta\phi} \equiv \hat{\tau}$, we have $e_t = 0$. Thus for an interior solution to private optimal education choice, the tax rate should be bounded above by $\hat{\tau}$. In the subsequent analysis we assume that this condition is satisfied.[10] Let us define a new variable z_t as a ratio of two stocks h_t and k_t:

$$z_t \equiv \frac{h_t}{k_t} \quad \forall t > 0 \tag{11}$$

The ratio z_t can be interpreted as the inverse of the capital-labor ratio with labor being measured in units of human capital. A steady state of the model is achieved when the ratio z_t becomes stationary, i.e. when $z_t = z^*$. Since there is no perpetual growth in health, the steady-state health stock x^* is given by:

$$x^* = \left(\frac{M}{1+\delta_x}\right)^{1/\psi} (1-\gamma_h)\tau A(1-\alpha)(z^*)^{(1-\alpha)} \tag{12}$$

Using the factor price Eqs. (7a,b), the market clearing condition, $s_t = k_{t+1}$, the allocation of government tax revenue (Eq.4), the saving function (Eq.6) and the steady state health stock (Eq.12), the BGP growth rates for physical and human capital stocks are given by, respectively,

[10] In the numerical simulations, the parameters are chosen so as to satisfy $e_t > 0$

$$g_k = \frac{k_{t+1}}{k_t} = \left[\frac{\phi(x^*)[(1-\tau)\mu - \nu\gamma_h\tau]}{\mu[1+(1+\theta\beta)\phi(x^*)]}\right] A(1-\alpha)(z^*)^{1-\alpha} \tag{13}$$

and $$g_h = \frac{h_{t+1}}{h_t} = B\left[\frac{\phi(x^*)[\mu(1-\tau)+\nu\gamma_h\tau]}{1+(1+\theta\beta)\phi(x^*)}\right]^{\theta} (\theta\beta A(1-\alpha))^{\theta}(z^*)^{-\alpha\theta} \tag{14}$$

In turn, Eqs. (13) and (14) implicitly determine z^*:

$$[B\mu(\theta\beta)^{\theta}]^{\frac{1}{1-\theta}} - \left[\frac{\phi(x^*)[(1-\tau)\mu+\nu\gamma_h\tau]}{1+(1+\theta\beta)\phi(x^*)}\right] A(1-\alpha)(z^*)^{\frac{1}{1-\theta}-\alpha} = 0 \tag{15}$$

where x^* is given by Eq.(12). Using Eq.(15), the balanced growth rate of physical and human capital can be written as

$$g^* = Q(z^*)^{\frac{-\theta}{1-\theta}}, \text{ where } Q = B^{\frac{1}{1-\theta}}(\mu\theta\beta)^{\frac{\theta}{1-\theta}} \tag{16}$$

3.2 Analysis of the Balanced Growth Path

Assuming a solution to Eq.(15) exists, it is straightforward to show uniqueness of the BGP equilibrium.[11] The difference equation governing the evolution of z_t is given by:

$$z_{t+1} - z_t = f(z_t) = [B\mu(\theta\beta)^{\theta}]^{\frac{1}{1-\theta}} - \left[\frac{\phi(x_{t+1})[(1-\tau)\mu+\nu\gamma_h\tau]}{1+(1+\theta\beta)\phi(x_{t+1})}\right] A(1-\alpha)z_t^{\frac{1}{1-\theta}-\alpha} \tag{17}$$

Differentiation of Eq.(17) with respect to z_t shows that the slope of this function is negative for all values of z, indicating that the solution to $f(z_t) = 0$ is unique.

Local stability of BGP equilibrium requires $\left|df(z_t)/dz_t\right|_{z_t=z^*} < 1$. While it is not possible to show that this condition holds in general, numerical simulation based on a parameterized version of the model (to be discussed below) reveals that the BGP equilibrium is indeed locally stable.

Next, we investigate the BGP relationship between the balanced growth rate g^* (as given by Eq.(16)) and z^*. Differentiating Eq.(16) with respect to z^* yields:

[11] It is straightforward to show that g_k is increasing and g_h is decreasing in z^*. Therefore, the resulting equilibrium (g^*, z^*) must be unique.

$$\frac{\partial g^*}{\partial z^*}=Q\left(\frac{-\theta}{1-\theta}\right)(z^*)^{\frac{-1}{1-\theta}}<0$$

The negative sign of the above partial implies that z^* unambiguously lowers balanced growth, a result that reflects the stronger diminishing returns to human capital accumulation relative to physical capital accumulation in our model.

Having analyzed some of the properties of the BGP equilibrium, we now address the issue of comparative statics along the BGP. While most of the comparative statics result are analytically ascertained, some are derived numerically (presented as shaded areas in Table 2).[12] We start with changes in z^*, from which we derive changes in g^* and x^*. In particular, we find that $dz/dj>0$, for $j=\psi,\beta,B,\delta_x,\theta,\mu,\tau$ and $dz/dj<0$ for $j=M,A,\nu,\gamma_h$. $dz/d\alpha>0$ for $z^*>$ 1 and ambiguous otherwise.[13]

Eq (12) shows that certain parameters such as γ_h, A, α, ψ, δ_x and M have a direct impact on x^*, in addition to having an indirect effect through z^*. Given that, along the BGP, health and human capital formation are 'competing' with each other at the margin, changes in these parameters will cause opposing changes in x^* and z^*. In contrast, changes in the remaining parameters (B, β, θ, τ, μ, ν) cause z^* and x^* to change in the same direction. Eq. (16) reveals an inverse equilibrium relationship between z^* and g^*. Therefore, we would expect that most parameter changes that change z^* in one direction would change g^* in the other. This is indeed the case, except for changes in B, θ, and β which, in addition to their indirect impact through changes in z^*, affect g^* directly (see Eq. (16)).

More specifically, the signs of certain partial derivatives, such as changes in x^* and g^* caused by changes in the productivity parameters A, B, and M, the deprecation rate δ_x, and the

[12] In our numerical exercise we use the functional specification $\phi(x)=x/(1+x)$, This specific form has been chosen so that the 'survival probability' satisfies $\phi(0)=0$ and lim $x\to\infty$ [$\phi(x)$] =1. For a discussion of the parameter values used to derive the numerical results, see section 7.1 below.

[13] The analytical derivatives have been omitted to conserve space and are available upon request.

altruism parameter β, are as expected and need no further discussion. On the other hand, the relationship between the endogenous variables and θ is rather complicated and the signs of the partial derivatives do not lend themselves easily to intuitive explanations.

An increase in the parameter ψ lowers the 'autoregressive coefficient' of the health stock (also see Eq. 12), reducing the steady state magnitude of x^*. An increase in α reduces x^* by reducing labor income, income tax revenue, and ultimately public health spending. Given everything else, a rise in the weight of private spending, μ, raises the human capital stock (and z^*) since increases in these parameters make private spending on education, e_t, more productive. A rise in z^*, in turn, positively affects x^* since the latter is positively associated with the former. A rise in ν, on the other hand, lowers the relative attractiveness of e_t, lowering z^* and consequently x^* falls.

Table 2: Comparative Static for the Benchmark Model

Parameters related to	**Health**			**Human capital**					**Output**		**Utility**	**Taxation**
Parameters	M	ψ	δ_x	B	γ_h	θ	μ	ν	A	α	β	τ
Effects on x^* (or ϕ)	+	–	–	+	–	–	+	–	+	–	+	+
Effects on z^*	–	+	+	+	+	–	+	–	–	+	+	+
Effects on g^*	+	–	–	+	–	–	+	+	+	–	+	–

Notes: Results in the shaded boxes are based on numerical simulations (see section 7.1 for calibration details).

Of special interest are the comparative static signs of two parameters: the tax rate τ and the education share of government expenditures γ_h. These are the policy instruments with which the government can influence the level of the health, human and physical capital stocks. As discussed in section 7.3 below, τ has a non-monotonic effect on g^* (see Fig. 3 below). Initially, g^* is increasing in τ because the tax rate is too low to internalize the externalities present in the model. The opposite relation holds for high values of the tax rate: with internalization accomplished, the distortionary effects of high taxes dominate. Since the chosen (calibrated) level of τ (16 %) is higher than the growth maximizing level ($\approx$15%, see Fig. 3 below), g^* declines with a rise in τ. Also, higher tax revenues imply higher public spending on health which subsequently causes an increase in x^*. Not surprisingly, γ_h has a positive effect on human capital formation (z^* rises) and a negative impact on government spending on health (g_t^x falls) which in turn lowers x^*.

Table 3: Non-linear effects of γ_h on z^* and g^*

Given τ = 0.16	γ_h **= 0.10**	γ_h **= 0.20**	γ_h **= 0.30**	γ_h **= 0.40**	γ_h **= 0.50**	γ_h **= 0.55**
z^*	3.0768	3.0679	3.0657	3.0732	3.0955	3.1149
g^*	2.6681	2.6701	2.6705	2.6689	2.6641	2.6599

As can be seen from Table 3, γ_h, like τ, has a non-linear relationship with g^*. This result is driven by the fact that growth rate of human capital, g_h, is negatively related to z_t, while that of physical capital, g_k, is positively related. For low values of γ_h, both g_h^t and z^* take on small values. In this situation an increase in z_t (due to a rise in γ_h) will decrease the growth rate of human capital, and raise the growth rate of capital via a high x^*. At low levels of γ_h, the negative effect on g_h dominates the positive effect on g_k. Since along the BGP, the two growth rates are equal, the equilibrium value of z_t should fall bringing g_h closer to g_k. As a result, the balanced growth rate rises, and x^* falls. At high levels of γ_h, however, the sensitivities of g_h and g_k are just opposite and a rise in γ_h would lead to an increase in z^* and a consequent fall in the balanced growth rate. Since the numerical benchmark value of the education spending share (γ_h = 0.46) is higher than its growth maximizing level ($\gamma_h \approx 0.3$), the impact of γ_h on g^* must be negative, as shown in Table 2.

4. Public Education Model

4.1 Model Specification ($\mu = 0,\ \nu > 0$)

In this model the human capital accumulation is driven only by public spending on education and therefore there is no role for private spending on education in this model. Young individuals (parents) maximize their intertemporal utility only by choosing the optimal levels of consumption. In particular, a representative young agent maximizes (5) with respect to s_t, subject to the following budget constraints:

$$c_t^t \leq (1-\tau)w_t h_t^t - s_t \tag{7a′}$$

$$c_{t+1}^t = \hat{R}_{t+1} s_t, \text{ where, } \hat{R}_{t+1} = R_{t+1} / \phi_{t+1} \tag{7b′}$$

Given these budget constraints, a young agent of generation-*t* maximizes her lifetime utility given by Eq.(8). In this model, the only choice variable is saving, which, given the survival probability, ϕ, and interiority of solution, is chosen optimally as:

$$s_t = \left(\frac{\phi}{1+\phi}\right)(1-\tau)w_t h_t \text{, where } \phi = \phi(x_{t+1}^t) \tag{18}$$

The equilibrium saving function given by Eq.(18) has the following properties: $s_\phi > 0, s_w > 0, s_h > 0, s_\tau < 0$. Thus saving increases as individuals' perceived probability of survival rises. As expected, saving increases with wages and decreases with income tax. Note that in this model, the saving function is identical to the one used in Chakrabarty (2004). As in the previous model, the steady state health stock is given by Eq.(12).

Using the factor prices, the market clearing condition, the saving function (Eq.18) the steady state health stock (Eq.12), and noting that $g_h^t = \gamma_h \tau w_t h_t = \gamma_h \tau A(1-\alpha) k_t^{\alpha} h_t^{1-\alpha}$, the BGP growth rates for physical and human capital stocks are given by, respectively,

$$g_k = \frac{k_{t+1}}{k_t} = \left[\frac{\phi(x^*)}{1+\phi(x^*)}\right](1-\tau)A(1-\alpha)z_t^{1-\alpha} \tag{19}$$

$$g_h = \frac{h_{t+1}}{h_t} = B[\nu\gamma_h\tau(1-\alpha)A]^{\theta} z_t^{-\alpha\theta} \tag{20}$$

Along the BGP stationarity of z_t leads to equality of Eq.(19) and Eq.(20), yielding the following BGP condition:

$$B(\nu\gamma_h\tau)^{\theta} = \left[\frac{\phi(x^*)}{1+\phi(x^*)}\right](1-\tau)[A(1-\alpha)]^{1-\theta}(z^*)^{1-\alpha(1-\theta)} \tag{21}$$

where x^* is given by Eq.(12). The balanced growth rate of physical and human capital can be written as:

$$g^* = B[\nu\gamma_h\tau A(1-\alpha)]^{\theta}(z^*)^{-\alpha\theta} \tag{22}$$

4.2 Comparative Statics for Public Education Model

Following the same method as in the benchmark model, we derive comparative static results for the public education model (see Table 4). Following a change in the parameter value, we first obtain the responses of z^*, from which, as in the case of benchmark model, we derive changes in g^* and x^*. we can find that for parameters $j = \psi, \gamma_h, B, \delta_x, \nu, \tau$ and θ $dz/dj > 0$, and for $j = M, A$, $dz/dj < 0$. For $z^* > 1$, $dz/d\alpha > 0$ (and ambiguous otherwise).[14]

The comparative static results for this model are presented in Table 4. A comparison with the corresponding results from the benchmark model (see Table 2) reveals that most signs are identical. The notable exception is the sign of ν, the parameter representing the weight of public education spending in total human capital formation. Not surprisingly, in the case where there is

[14] The derivations of the comparative static results are available upon request.

no private involvement in funding education, it has a positive impact on human capital formation (therefore on z^*) and x^*. For the same reason, the effect of τ on z^* is now positive, compared to a negative impact in the benchmark model (see Table 2). Since the balanced growth rate g^* and the capital stock ratio z^* are inversely related, one would expect parameter changes to affect them in opposite directions. Exceptions are changes in B, ν, γ_h, and τ, all of which affect g^* and z^* in the same direction. These parameters have a direct positive impact on g^* (see Eq. (26)) that dominates the negative indirect effect via z^*. As in the benchmark model, no straightforward intuitive explanation exists for the (inverse) relationship between θ and g^*.

Table 4: Comparative Static for the public education model

Parameters related to	**Health**			**Human capital**				**Output**		**Taxation**
Parameters	M	ψ	δ_x	B	γ_h	θ	ν	α	A	τ
Effects on x^* (or ϕ)	+	–	–	+	–	–	+	–	+	+
Effects on z^*	–	+	+	+	+	–	+	+	–	+
Effects on g^*	+	–	–	+	+	–	+	–	+	+

Notes: Results in the shaded boxes are based on numerical simulations (see section 7.1 for calibration details).

5. Private Education Model

5.1 Model Specification ($\mu > 0,\ \nu = 0$)

In this model, we consider the scenario when there is no public expenditure on education. Thus education is entirely financed privately while the entire tax revenue is spent on health. This case is the opposite extreme of the public education model where education is financed entirely by the government. Another, and perhaps a better interpretation of this model is that it is a special case of the benchmark model in which the all public revenue is spent on health, 1-γ_h = 1 (or, γ_h = 0). With this interpretation we can use all the result from the benchmark model with the modification γ_h = 0. Thus, we have the following expressions for private saving and educational spending that are obtained from individual optimization:

$$s_t = \left(\frac{\phi(1-\tau)}{1+(1+\theta\beta)\phi}\right) w_t h_t \tag{23}$$

and
$$e_t = \left(\frac{\theta\beta\phi(1-\tau)}{1+(1+\theta\beta)\phi}\right) w_t h_t \tag{24}$$

Steady state health stock is the same as in the previous models, and is given by

$$x^* = \left(\frac{M}{1+\delta_x}\right)^{\frac{1}{\psi}} \tau A(1-\alpha)(z^*)^{(1-\alpha)} \tag{25}$$

Note that for a given z^*, the steady state health stock in this model is higher than that in the benchmark model. Given Eq.(25), the growth rates of physical and human capital are, respectively, given by:

$$g_k = \frac{k_{t+1}}{k_t} = \left[\frac{\phi(x^*)(1-\tau)}{1+(1+\theta\beta)\phi(x^*)}\right] A(1-\alpha)(z^*)^{1-\alpha} \tag{26}$$

and
$$g_h = \frac{h_{t+1}}{h_t} = B\left[\frac{\phi(x^*)(1-\tau)}{1+(1+\theta\beta)\phi(x^*)}\right]^{\theta} (\mu\theta\beta A(1-\alpha))^{\theta} (z^*)^{1-\alpha\theta} \tag{27}$$

As before, the BGP condition is obtained by equating the above two growth rates:

$$[B(\mu\theta\beta)^{\theta}]^{\frac{1}{1-\theta}} = \left[\frac{\phi(x^*)}{1+(1+\theta\beta)\phi(x^*)}\right](1-\tau)A(1-\alpha)(z^*)^{\frac{1}{1-\theta}-\alpha} \tag{28}$$

Apparently the BGP condition in Eq.(28) is same as the BGP condition in Eq.(15) in the benchmark model, but the solution, z^* in the two models are different due to differences in x^*. Here the balanced growth rate (g^*) is given by:

$$g^* = Q(z^*)^{\frac{-\theta}{1-\theta}}, \text{ where, } Q = B^{\frac{1}{1-\theta}}(\mu\theta\beta)^{\frac{\theta}{1-\theta}} \tag{29}$$

For the parameter values chosen for the benchmark model, the BGP in this model is stable and unique.

In this model, the only utility of raising tax rate comes from higher health stock that raises longevity. However, marginal increase in longevity falls very quickly as health stock increases. This is the reason why the peak growth rate is reached at a relatively low level of tax rate after which growth rate starts to decline as shown in Fig.4 below.

To avoid repetition, we refrain from re-deriving the self-sustained growth and the analytics of comparative statics. The analysis is qualitatively similar to that of benchmark model and the comparative static results remain unchanged from those presented in Table 2.

6. Welfare and Macroeonomic Indicators

Government provision of education and health has important welfare effects through changes in consumption levels, survival probability and human capital. To compare welfare levels between the three models, we employ the standard social welfare index (W) that includes the utilities of individuals across generations and over time. More specifically, W is expressed as the discounted sum of utilities of young and old over a finite time horizon, T:

$$W = \sum_{t=-1}^{T} \rho^t \left\{ \log(c_t^t) + \phi(x_{t+1}^t)[\log(c_{t+1}^t) + \beta \log(h_{t+1}^{t+1})] \right\} \quad \rho \in (0,1) \tag{30}$$

where ρ is the social discount factor and t denotes time periods.

We impose $\rho < 1$ to ensure that the welfare index converges to a finite value even when $t \rightarrow \infty$. In addition, since the rate of convergence of the welfare indicator is slow, we need to choose a long enough time horizon, such as $T = 25$, to guarantee convergence in our numerical simulations. We report the long-run values of the welfare indicators for the three models in Table 5 below.

6.1 Calibration

Our numerical analysis will be based on the parameters summarized in Table 5. With the exception of the education spending weights in the human accumulation function (μ and ν), parameter values are identical across models.

Table 5: Benchmark model calibration

Parameters	M	ψ	δ_x	B	γ_h	θ	μ	ν	β	α	A	τ	ρ
Values	*4.5*	*0.55*	*0.5*	*4.5*	*0.46*	*0.20*	*0.65*	*0.35*	*0.65*	*0.33*	*5.0*	*0.16*	*0.40*

The share of physical capital in output production, α, is set to 0.33, a value within the range of its empirical estimates. We chose the total productivity parameter A to be equal to five so that the steady state net interest rate (R - 1) yielded by the benchmark model comes close to the benchmark countries' average (1970-2001) real interest rate on savings deposits of 5.94% (World Bank, 2003). As in Soares (2005) we pick a value of θ = 0.2 for the elasticity of total educational expenditure with regard to wage earnings (Eq. 3). Our choice of B = *4.5* together with the values for the weights of private and public funding in the human capital production function guarantees that the share of private education in total education spending matches that of the benchmark countries of about 27% in 2000 (OECD, 2003). For the benchmark model, this requires setting μ = 0.65 and ν = 0.35. In contrast, μ (ν) is set to zero in the public (private) education model.

No dependable empirical estimate for ψ, the elasticity of health spending in the survival function, exists. We thus choose an arbitrary value of ψ = 0.55. We assume that the health stock of an individual depreciates by about 50% from childhood to adulthood (δ_x = 0.5). Our choice of τ = 0.16 implies that the share of government spending on education and health in GDP approximates 10.6% - the corresponding value for the benchmark countries in 1995-2000

(UNDP, 2004). The parameter γ_h determines the allocation of government tax revenues between public health and education budget. The share of government spending on education as a percentage of total government spending on health and education equals 46% for the group of benchmark countries in 1995-2000 (UNDP, 2004). We thus choose γ_h = 0.46.

For the scaling parameter, M, in the health accumulation function we choose a value of 4.5 because it produces a probability of survival into the old age that comes close to the observed average probability at birth of surviving to age 65 in the benchmark countries of roughly 0.82 (UNDP, 2004). Finally, we choose the altruism parameter, β, to be equal to 0.65 which is similar to values found in the literature (for example, Rout, 2003). A social discount factor of ρ = 0.4 for the 30-year period is equivalent to a quarterly discount factor of 0.99, a widely used number of in the real-business-cycle literature

Table 6 summarizes certain economic indicators for the benchmark model as well as actual data for a group of countries with levels of private spending on education that make up a sizable share of overall expenditures on education (hereafter called benchmark countries).

Table 6: Comparison of the benchmark model with the benchmark countries (figures in %)

	Probability of survival to age 65	*Real Per Capita Growth rate**	*Real Rate of interest**	*Private share in total educational spending*	*Share of government health-spending in GDP*	*Share of government educational spending in GDP*
Benchmark model	81.94	3.32	4.27	27.43	5.79	4.93
Benchmark countries	82.31	2.96	5.94	26.59	5.82	4.79

*: Numbers reflect annualized rates.

6.2 Comparison of the Three Models

Table 7 summarizes select macroeconomic indicators along the BGP as well as welfare levels for each of the three models. In terms of longevity (ϕ), the private education model has by far the highest survival probability compared to both the benchmark and public education model. This result is a direct consequence of the differences in the level of health expenditures in the three models (see g_x/Y column). In the private education model, the government concentrates its entire revenues on health, while in the benchmark and public education models the government has a dual responsibility and diverts some its revenues toward education.

Table 7: Main economic indicators: A comparison between the three models

	ϕ	g_x/Y	$g^{*\dagger}$	h/K	$r^{*\dagger}$	*Welfare*
Benchmark model	81.94%	5.79	3.32	3.08	4.27	14.21
Pub.Edu model	81.29%	5.79	3.20	2.89	4.12	14.05
Pvt.Edu model	89.38%	10.72	3.32	3.09	4.28	14.49

† Numbers reflect annualized rates.

With regard to long-run growth (g^*), the net interest rate (r^*), and the human-to-physical capital ratio (*h/K*) the benchmark and private education model generate similar values. In addition, these values are higher than the corresponding values in the public education model. There are two forces affecting the human-to-capital ratio. In the public education model, the government is the sole provider of education spending leaving human capital at a comparatively lower level. At the same time, given the lower level of income in the public education model, the actual amount saved is less than what is being saved in the other models causing a lower stock of physical capital.[15] With both human and physical capital below the levels in the other models, the relative magnitude of the human-to-physical capital ratio in the public education model is ambiguous a priori. However, based on our numerical simulations, we find that the ratio is lower in the public education model than in the other models. Since the production function exhibits constant returns to scale and diminishing returns to each factor, the rate of return to physical

[15] The reduced longevity in the public education model does not have a strong adverse impact on private saving since returns are annuitized.

capital is an increasing function of the human-to-physical-capital ratio. Given the positive impact of the rate of interest on long-run growth, the public education model must exhibit lower balanced growth than both the benchmark and the private education model.

Differences in health expenditures and thus longevity among the three models have direct implications for the model rankings with regard to welfare.[16] The private education model ranks highest, followed by the benchmark model, while the public education model generates the lowest level of welfare. While many factors influence the level of welfare, the most important single determinant of welfare is the survival probability. A high level of ϕ translates into a low discount rate which explains the superior welfare level of the private education regime. Similarly, the low level of welfare in the public education models stems to a large degree from its low survival probability and thus its high discount rate.

6.3 Comparing Models: Empirical Evidence

In this section, we investigate whether the ordering of models by key economic indicators implicit in Table 6 is born out by the data. We therefore collect data for two sets of countries: the benchmark countries defined in section 6.1 above and countries with little private spending on education (public education countries).[17] Since no countries in our sample rely more or less exclusively on privately funded education, the relative performance of the private education model cannot be analyzed empirically. We define 'Public Ed' countries as those with a public share in total educational spending of more than 85%, while all other countries in our sample are considered benchmark countries.[18] We use OECD data on education spending for a total of 36

[16] To ensure compatibility of the welfare indicators, we impose identical initial values for the three stock variables (k, h, and x) across all models in our numerical simulations.

[17] These economic indicators include life expectancy at birth, income, real GDP per capita growth, human-to-physical capital ratio, rate of interest, and GDP-to-capital ratio.

[18] Our sample includes only 36 countries due to the unavailability of the share of private spending in total educational spending for many countries. The group of countries with mixed funding of their education systems include Chile, Republic of Korea, Indonesia, Jamaica, United States, Paraguay, Japan, Australia, Argentina, Canada, Israel and Germany. Countries that use mostly public funds to finance education include United Kingdom, Mexico,

countries (Table B3.1; OECD, 2003), while the economic and demographic indicators are taken from UNDP (2003), World Bank (2003) and version 5.6 of Summers and Heston (1994). All indicators except for the growth rate have been calculated as averages over 1995-2000. The growth rate is calculated as the average annual growth rate of real GDP per capita over 1961-2001 to represent long-run growth rate.

Table 8: Economic Indicators by Mode of Education Funding

	Life-expectancy at birth	*Real GDP per capita*	*Growth Rate of Real per capita GDP*	*Human-Physical Capital Ratio*	*Interest Rate*
Benchmark Countries	74.99	21987	2.96	0.0139	5.94
Public Ed. Countries	67.61	5442	2.50	0.0064	6.79
Mean	70.23	11318.5	2.66	0.0106	6.48

Note: Human-physical capital ratio is defined as number of workers with Secondary Education divided by gross fixed capital formation.

The results, given in Table 8, show that - for four out of five indicators - the predicted relative model performance summarized in Table 6 is supported by the data. In particular, life expectancy at birth, the welfare indicator (as measured by real GDP per capita in international dollar), long-term growth and human-to-physical capital ratio are higher for benchmark countries than for public education countries. In terms of the interest rate - defined as the lending interest rate adjusted for inflation as measured by the GDP deflator -, 'benchmark' countries tend to have lower values than 'public ed' countries, contrary to the prediction from Table 6. However, if the 'rate of interest on deposits' is used instead of the interest rate, the relative group averages are again in line with their predicted magnitudes. Using alternative measures for longevity and the

Spain, Hungary, Czech Republic, The Netherlands, Italy, Iceland, Ireland, Switzerland, Belgium, France, Greece, Austria, Uruguay, Thailand, India, Denmark, Slovakia, Sweden, Finland, Portugal, Turkey and Norway.

human-to-physical capital ratio leads to qualitatively similar results as the ones reported in Table 8.[19]

6.4 Endogenous Fiscal Policy

What would be the preferred tax rate when comparing the benchmark with the public education model? And which of the two models would yield higher levels of welfare? To answer these questions we produce numerical values of the welfare index given in Eq. (30) for various levels of γ_h and the optimally chosen tax rate (in a second-best sense) for each of the two models.[20] Table 9 reports these maximum welfare levels and the corresponding optimal second-best tax rates (in parentheses).

We find that the welfare ranking of two regimes depends on the (exogenous) split between government spending on education versus health. For high levels of public spending on education relative to health ($\gamma_h \geq 0.5$), the public education model is welfare superior, while the reverse is true for low public education spending (values of $\gamma_h < 0.5$). That is, in a world where the government is forced to spend most of its revenues on education, the country is better of if schooling is provided exclusively by the public sector. In contrast, if the government decides to allocate its resources mostly to finance public health, the country is better off with a mixed funding for education. In the extreme case that the government spends all its resources on health and nothing on education, welfare is the same as under the mixed spending regime. The intuition behind these results is the following: When a majority of the public resources is allocated toward financing education, it leaves little funds for health. As a result, longevity is reduced in

[19] The figures in Table 8 represent weighted averages of the two groups. The weight used is population. Using per capita GDP as weight yields qualitatively similar results.

[20] The private education regime is omitted from the table since it represents the special case of γ_h = 0. For the optimally chosen tax rate (τ = 0.15), the level of welfare achieved under this regime is 14.49, a value that exceeds the maximum welfare levels achievable in either of the other two models (see Table 9).

equilibrium causing disincentives for households to invest in education. In fact, the lower the extent of public spending on health, the higher is the extent of sub-optimality associated with

Table 9: Maximum levels of welfare under the benchmark and public education models

γ_h	**0.05**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
Benchmark model	**14.47 (15.5)**	**14.46 (16.1)**	**14.42 (17.5)**	**14.38 (19.2)**	**14.33 (21.2)**	14.20 (17.9)	13.90 (14.8)	13.31 (12.1)	12.27 (9.47)	10.56 (6.0)
Pub. Edu. model	12.38 (31.3)	12.99 (31.9)	13.57 (32.7)	13.90 (33.4)	14.10 (34.3)	**14.22 (35.5)**	**14.28 (37.3)**	**14.28 (44.5)**	**14.24 (41.9)**	**13.73 (42.7)**

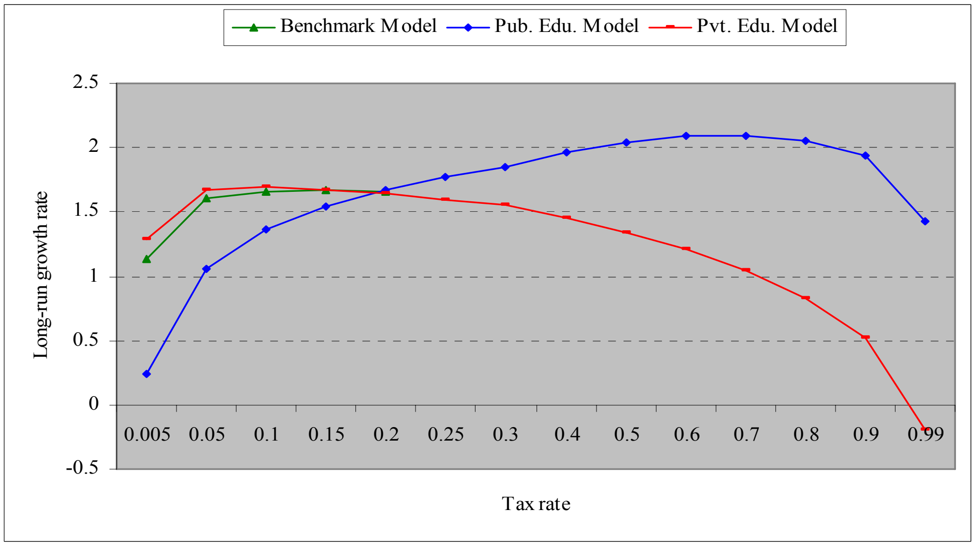
Note: Welfare maximizing second-best tax rates (%) in parenthesis.

private spending on education and the lower is the associated welfare level. Conversely, when most government spending is targeted toward health, private spending on education becomes more and more desirable from society's point of view (because of higher longevity and consequent lower discount rates over time), thus making higher private educational spending optimal.

Lastly, we examine the relationship between the balanced growth rate and the level of taxation and compare the outcome across models. As is typical for models with endogenous growth mechanisms driven by production externalities (see Barro, 1990; Barro and Sala-i-Martin, 1995), we find a hump-shaped relation between the two variables in all models (see Fig. 3).[21] Furthermore, while the growth rates in the benchmark and the private education models peak at relatively low tax rates (between 10 and 15%, approximately), the growth-maximizing tax rate in the public education model turns out to be around 75%. Why is this rate higher in the public education model? As shown in Table 9, the public education model requires a higher welfare-maximizing tax rate (for any level of γ_h) than the corresponding benchmark model.

[21] The relevant curve for the benchmark model is truncated at around τ = 20% because for higher tax rates a corner solution with no private spending on education occurs.

Fig. 3: Taxation and long-run growth



Since the balanced growth rate is an important contributor to the overall welfare level, welfare- and growth-maximizing tax rates follow similar pattern. This in turn implies that the tax rate that maximizes growth in the public education model must exceed the growth-maximizing rate in the other models.[22]

6.5 Transitional Dynamics

We examine the dynamical paths of changes in the stocks of physical and human capital (g_k and g_h, respectively) following a positive 10% shock to various model parameters (see Figures 4-10, Appendix A). A change in the value of a parameter can have direct or indirect effects on the growth rates of our state variables in the transition.

As can be seem from the diagrams in Appendix A, the first period responses to parametric shocks often result in overshooting of at least one of the growth rates (g_k or g_h). Fig. 4 shows that in all three models, a positive shock in B leads to overshooting of the growth rate of

[22] To be precise, we find that in our model the welfare maximizing tax rate in the public education model is roughly 35%, while the corresponding numbers for the benchmark and the private education model are 18% and 14%, respectively.

human capital but not that of physical capital during the transition due to the direct positive impact of B on human capital accumulation (see Eq. 11). In addition, there is no change at all in g_k in the first period since a change in B does not affect current but only future saving (see Eqs. 6 and 10). An increase in M (Fig. 5) triggers a response pattern that resembles the previous figure, but with the paths of g_k and g_h reversed. The increase in M increases longevity leading to higher levels of private savings and physical capital accumulation during the transition. In Fig. 6, an increase in τ leads to a crossing of the paths of of g_h and g_k after the first period in the benchmark model reflecting a cyclical pattern of adjustment. The other two models display the familiar transition paths g_k and g_h with one growth rate (g_k) approaching the new equilibrium growth rate from below and the other from above. The slow down in the accumulation of physical capital during the transition is caused by the negative impact of higher taxes on household income and savings. Fig. 7 shows that a positive shock in β (which is tantamount to rise in the 'taste' for child's education) leads to high values of g_h during the transition in both benchmark and private education model, as expected. As a result, g_k approaches its new steady-state level from below. A shock to γ_h is a relevant exercise only in benchmark and public education models. As shown in Figure 8, a rise in γ_h (equivalent to higher spending on education by the government) leads to increased (reduced) growth rates of human capital (physical capital) during the transition period in both models, as expected. Fig. 9 shows that a positive shock to μ – the weight attached to private spending on education in the human capital accumulation function – leads to adjustment paths of both capital stocks that are qualitatively similar to those shown in Fig. 8, as one would expect. However, a positive shock to ν - the weight of public spending on education in human capital accumulation – leads to the opposite transition paths of g_h and g_k in the benchmark and public education models, as shown in Fig. 10. In the public education model, g_h (g_k) approaches its new equilibrium level from above (below), a pattern that is familiar from Fig. 8 and 9. In contrast, the growth rate of human capital (physical capital) is below (above) its new equilibrium level during the transition period in the benchmark model. This is because private households reduce their education spending as a result of the increase in ν which subsequently slows down the overall accumulation of human capital during the transition.

7. Concluding Remarks

We incorporate dual public spending on education and health in a general equilibrium overlapping generations framework in which individuals have lifetime uncertainty. Private decisions regarding saving and expenditure on child's education depend crucially on the incentive effect generated by better health and therefore higher longevity. Health accumulation depends on public funding on health alone, while the accumulation of human capital depends on both private and public funding. We analyze three different models. In the benchmark model, the most general form of human capital accumulation function is used requiring both private and public spending. In the public education model, the government is the sole provider of education, while private individuals are the sole providers in the private education model. The implications of these three models shed important light on policy issues such as the welfare maximizing mix of public versus private spending on education.

By combining ‘productive’ government spending with endogenous length-of-life effects we can investigate not only the short-term (transitional) growth impact of variable longevity, but also the long-run (steady-state) growth and welfare implications for three different education regimes. Another novel assumption of this paper is the simultaneous inclusion of two productive government investments (education and health) and their specific roles in generating and sustaining long-term growth under lifetime uncertainty.

An important result of the paper is that the public education model produces results that are inferior to both the benchmark and the private education model with regard to a number of macroeconomic indicators. For a fixed income tax rate, long-run growth, interest rates, and the ratio of human-to-physical capital are all lower in the public education model than in the other two models, while longevity and welfare are highest in the private education model and lowest in the public education model. The poor performance of the public education model has its root in an income tax rate that is too low relative to its optimal level. In other words, the public education model cannot perform properly if the government cannot raise the funds needed to run both a public health and a public education system. This simple intuition is validated when we endogenize the income tax rate: Welfare-maximizing tax rates are on average more than three times higher in the public education model compared to the benchmark model. In addition, a comparison of welfare levels based on second-best tax rates reveals that the welfare ranking of benchmark versus public education model depends on the (exogenous) relative size of government spending on education and health. For high levels of public spending on education relative to health, the public education model is welfare superior, while the reverse is true for low public education spending.

Our results point to an important policy implication. Faced with a given technology and a level of taxation that is too low relative to its optimal size, a government that cares about longevity, welfare, and long-run growth should leave the funding of education to the private sector, or, at the very least, encourage private participation in funding of education, while using public funds predominantly for the provision of health care services.

References

Barro, Robert (1990). "Government spending in a simple model of endogenous growth," *Journal of Political Economy*, 98(5), p. S103-S125.

Barro, Robert and Xavier Sala-i-Martin (1995). "*Economic Growth*", McGraw-Hill.

Benabou, Roland J. M. (1996). "Heterogeneity, stratification and growth: macroeconomic implications of community structure and school finance", *American Economic Review*, 86, p. 584-609.

Blackburn, K. and G. P. Cpiriani, (1998). "Endogenous fertility, Mortality and growth", *Journal of Population Economics*, 11: p.517-34.

Blackburn, K. and G. P. Cpiriani, (2002). "A model of longevity, fertility and growth", *Journal of Economic Dynamics and Control*, 26: p.187-204.

Blanchard, O. J. (1985). "Debt, deficits and finite horizons", *Journal of Political Economy*, 93, p.223-47.

Boldrin, Michele (1992). "Public education and capital accumulation" *Discussion Papers 1017*, Northwestern University, Center for Mathematical Studies in Economics and Management Science.

Buiter, W., K. Keltzer (1995). "Capital mobolity, fiscal policy and growth under self-financing of human capital formation", *Candian Journal of Economics*, 28, p.S163-S194.

Chakraborty, Shankha (2004). "Endogenous lifetime and economic growth", *Journal of Economic Theory*, 116, p.119-37.

de la Croix, D. and M. Doepke (2004). "Public versus private education when differential fertility matters", *Journal of Development Economics*, 73, p.607-29.

Ehrlich, Isaac, and Francis T. Lui (1991). "Intergenerational trade, longevity and economic growth", *Journal of Political Economy*, 99, p.1029-59.

Fogel, Robert W. (1997). "New findings on secular trends in nutrition and mortality: some implications for population theory", in Mark R. Rosenzweig and O. Stark (eds.), *Handbook of population and family economics*. Handbooks in Economics, 14, Amsterdam, New York and Oxford: Elsevier Science North-Holland, p. 433-81.

Fuster, Luisa (1999). "Effects of uncertain lifetime and annuity insurance on capital accumulation and growth", *Economic Theory*, 13, p.429-45.

Glomm, G. and B. Ravikmar (1992). "Public versus private investment in human capital, endogenous growth and income inequality", *Journal of Political Economy*, 100, p.813-34.

Glomm, G. and B. Ravikmar (1997). "Productive government expenditures and long-run growth", *Journal of Economic Dynamics and Control*, 21, p.183-204.

Heckman, J. J., and P. J. Klenow (1997). "Human Capital Policy", Chicago, IL, University of Chicago Press.

Heston, A., and R. Summers (1994). *Penn World Tables version 5.6.* Revision of Heston, A., and R. Summers. 1991. "The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988", *Quarterly Journal of Economics*, 106, p.327-68.

Kalemli-Ozcan, Sebnem (2002). "Does mortality decline promote economic growth?", *Journal of Economic Growth*, 7, p.411-39.

Knowles, Stephen and P. Dorian Owen (1995). "Health capital and cross-country variation in income per capita in the Mankiw-Romer-Weil model", *Economics Letters*, 48, p.99-106.

Kotlikoff, L. J. and A. Spivak (1981). "The family as an incomplete annuties market", *Journal of Political Economy*, 89, p.372-91.

Lichtenberg, Frank R. (2002). "Sources of U.S. Longevity Increase, 1960-1997", *NBER Working Paper # 8755*, http://www.nber.org/papers/w8755.

Nunes P.A.L.D. and Schokkaert, E. (2003). "Identifying the warm glow effect in contingent valuation", *Journal of Environmental Economics and Management*, 45, p.231-45.

OECD Online Education Database: (2003). Available at:
www.oecd.org/dataoecd/0/13/14483680.xls; accessed on December 5, 2004.

Psacharopoulos, G. and H.A. Patrinos (2004). "Returns to investment in education: A further update", *Education Economics*, 12, p.111-34.

Rebelo, Sergio, (1991). "Long-run policy analysis and long-run growth", *Journal of Political Economy*, 99, p.500 - 21.

Raut, Laxmi K. (2003). Long term effects of preschool investment on school performance and labor market outcome, Mimeo, Dept. of Economics, California State University at Fullerton.

Soares, Jorge (2005). "Public education reform: Community or national funding of education?", *Journal of Monetary Economics*, 52, p.669-97.

Strauss, J. and D. Thomas (1998). "Health, nutrition and economic development", *Journal of Economic Literature*, 36, p.766-817.

Swanson, Charles E. and Kenneth J. Kopecky (1999). "Lifespan and output", *Economic Inquiry*, 37, p.213-25.

United Nations Development Program (2003). *Human Development Report*, Oxford and New York: Oxford University Press.

United Nations Development Program (2004). *Human Development Report*, Oxford and New York: Oxford University Press.

World Bank (2001). *Government Financial Statistics*, IMF, Available at http://www.worldbank.org/research/growth/pdfiles/GDN/GovernmentFinance_6_2001.xls, accessed on December 4, 2004.

World Development Indicators (2003). *WDI CD-ROM*, The World Bank Group.

Yaari, M. E. (1965). "Uncertain lifetime, life insurance, and the theory of consumer", *Review of Economic Studies*, 32, p.137-50.

Appendix A

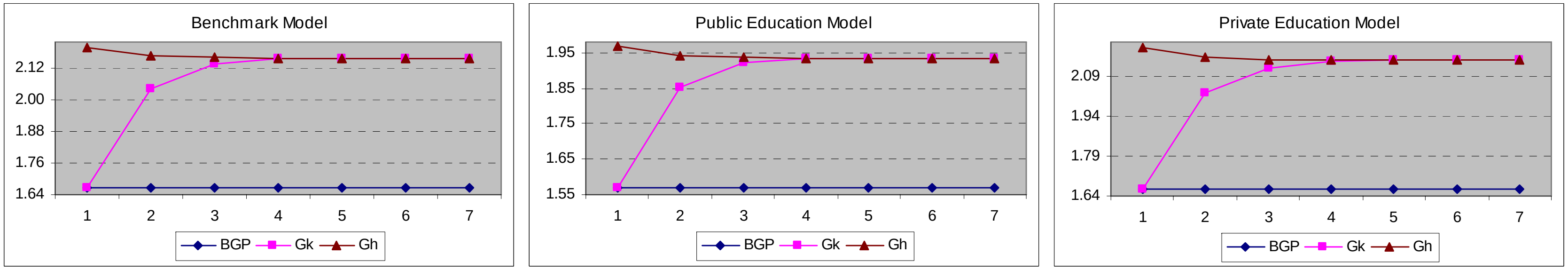


Fig. 4: Increase in Total Productivity Parameter in Human Capital Accumulation (B)

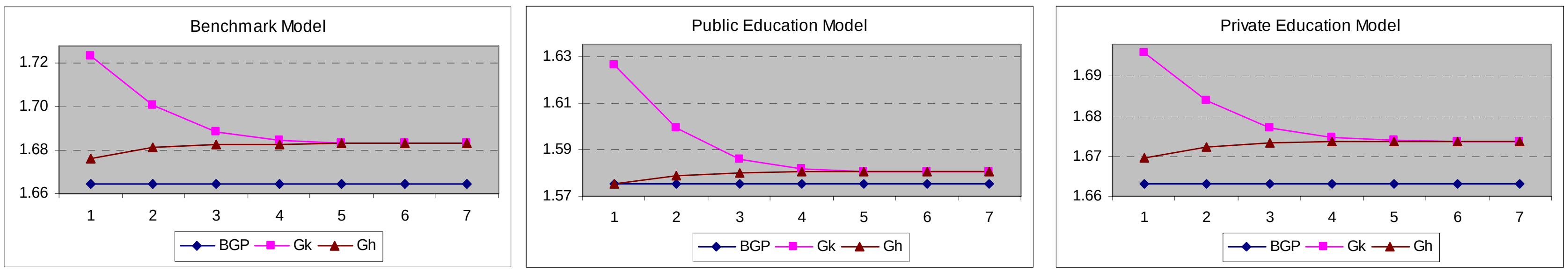


Fig. 5: Increase in Total Productivity Parameter in Health Accumulation (M)

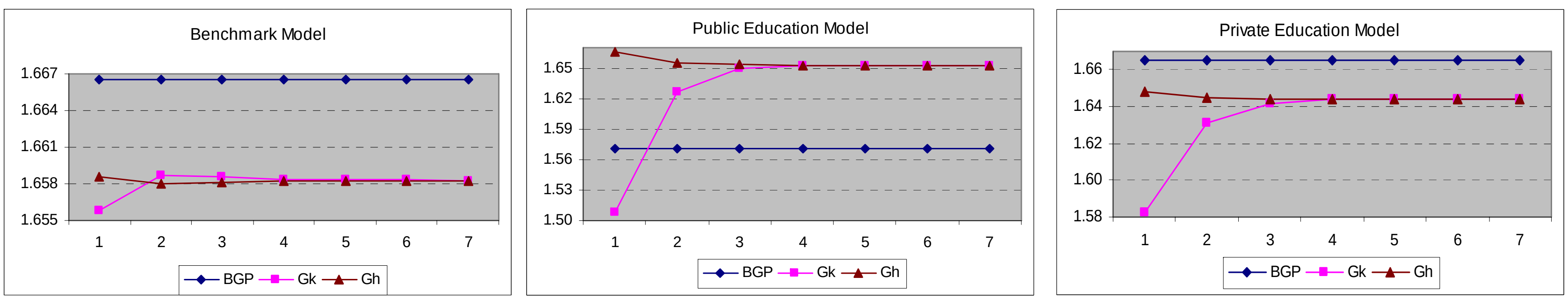


Fig. 6: Increase in Tax Rate (τ)

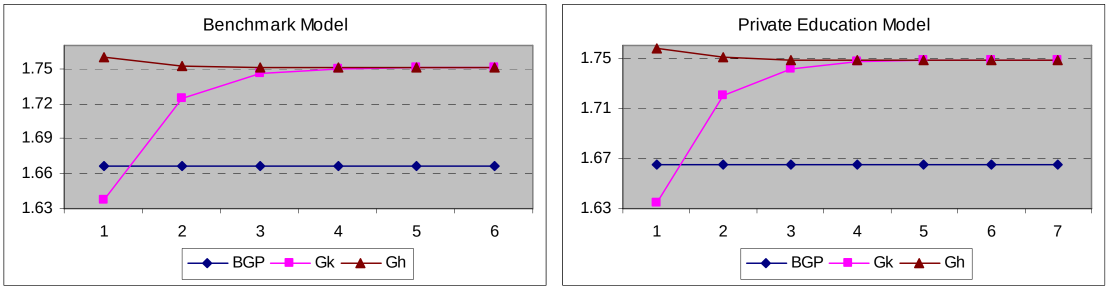


Fig. 7: Increase in the Altruism parameter (β)

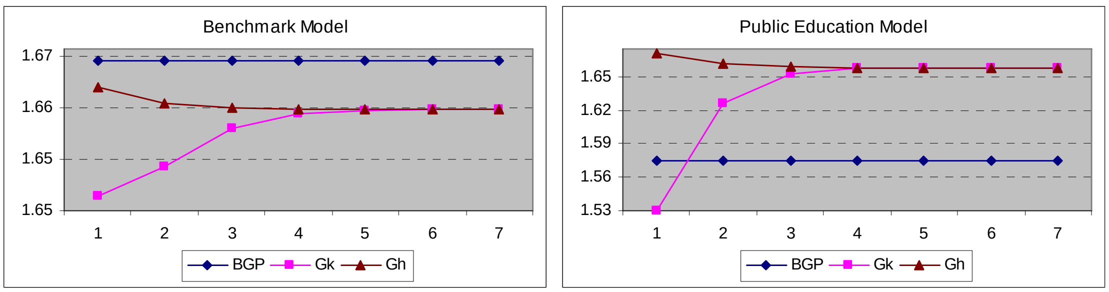


Fig. 8: Increase in the Share of Public Revenue spent on Education (γ_h)

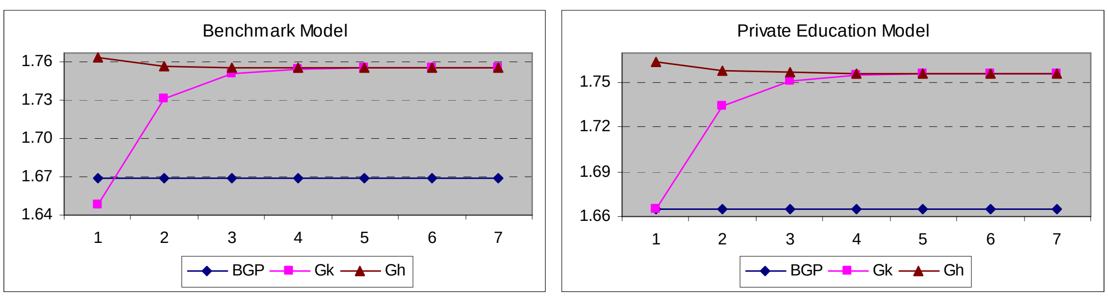


Fig. 9: Increase in the Weight of Private Spending on Education in Human Capital Accumulation (μ)

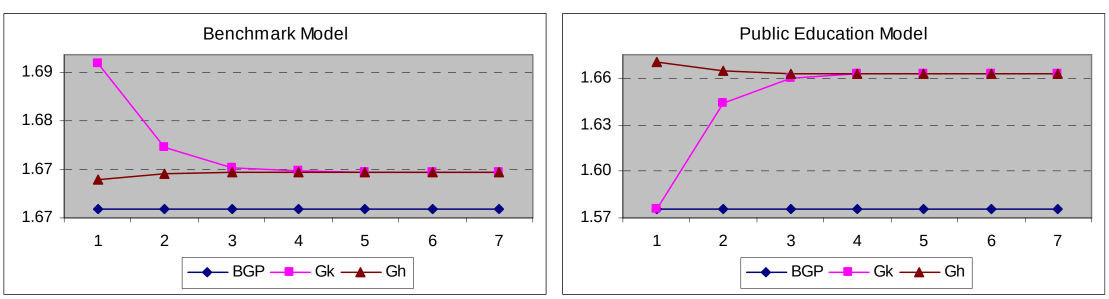


Fig. 10: Increase in the Weight of Public Spending on Education in Human Capital Accumulation (ν)