Dependence Structures in Chinese and U.S. Financial Markets A Time-varying Conditional Copula Approach

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Abstract

In this paper, we use a Time-Varying Conditional Copula approach (TVCC) to model Chinese and U.S. stock markets' dependence structures with other financial markets. The AR-GARCH-t model is used to examine the marginals, while Normal and Generalized Joe-Clayton copula models are employed to analyze the joint distributions. In this pairwise analysis, both constant and time-varying conditional dependence parameters are estimated by a two-step maximum likelihood method. A comparative analysis of dependence structures in Chinese versus U.S. stock markets is also provided. There are three main Öndings: First, the time-varying-dependence model does not always perform better than constant-dependence model. This result has not previously been reported in the literature. Second, although previous research extensively reports that the lower tail dependence between stock markets tends to be higher than the upper tail dependence, we find a counterexample where the upper tail dependence is much higher than the lower tail dependence in some short periods. Last, Chinese financial market is relatively separate from other international financial markets in contrast to the U.S. market. The tail dependence with other financial markets is much lower in China than in the U.S.

JEL Classification: C51; F36; G15; P52

Keywords: AR-GARCH-t model; Time-varying conditional copula; Dependence structure; Stock market

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1 Introduction

The nature of dependence between returns in financial markets has been a heatedlydebated issue among financial economists in both academia and the investment industry. Understanding the dependence structure will help the investors to identify the opportunities for international portfolio management in terms of asset allocation and pricing. (see Bartram and Dufey (2001) among others) The widely used linear dependence measure is too simple to correctly characterize Önancial return distributions under certain conditions. As Jondeau and Rockinger (2006) point out, when financial returns are non-normal, it is impossible to specify the multivariate distribution relating two or more return series. The copula may be one possible way to overcome the drawbacks of linear dependence measures like the correlation coefficient.

Previous research has investigated how the correlation between stock market returns varies over time. There exists significant asymmetric dependence. For example, Longin and Solnik (1995) examine correlations between stock markets over a long time period using the constant conditional correlation (CCC) model proposed by Bollerslev (1990). They find that correlations are generally higher during more volatile periods and depend upon several economic variables, such as the dividend yield and interest rate. However, tail dependence is not of interest in their paper. After that, Longin and Solnik (2001) find that international stock markets are more correlated in bear markets, using extreme value theory. They find that the multivariate normality of the joint distributions can be rejected in a statistical test. Ang and Chen (2002) propose a test for asymmetric correlation by comparing empirical and model-based conditional correlations. Patton (2004) finds dependence asymmetry of financial returns both in the marginal distributions and in the dependence structure. Patton (2006a, 2006b) develops a theory of conditional copulas and employs time-varying copula models to analyze two foreign exchange rate series. Compared to previous approaches in estimating correlation, the conditional copula model does not require normality in the marginal distributions and can take advantage of the two-step maximum likelihood method, which makes estimation more feasible. Jondeau and Rockinger (2006) model financial returns with time-varying skewed-t GARCH models and then use a time-varying or a switching Gaussian or Student's t copula for the dependence between countries. Okimoto (2007) estimates regime-switching copulas for pairs of US-UK and other G7 countries. Rodriguez (2007) adopts the copula model with Markov switching parameters and finds evidence of changing dependence structures during periods of financial turmoil. Increased tail dependence and asymmetry in times of high volatility characterize Asian countries within a relatively short time period.

As the largest emerging market in the world, China has been experiencing rapid economic growth in last two decades, which has led to a fast growing Chinese stock market. Unfortunately, Chinese financial markets attract less academic attention. In late 1997, Asian countries experienced a significant financial crisis. This financial crisis focused more attention on the study of the dependence between financial markets. Kim (2005) finds that some differences exist in the time path of dependence among Asian countries. The question is whether the degree of dependence between China and other countries is lower than that between other countries, so that Chinese market can be thought to be insulated from future crises. It is also interesting to compare the dependence structure in the largest emerging market, i.e. Chinese stock market with that in the largest developed market, i.e. the U.S. stock market.

This study is devoted to the Chinese and U.S. stock markets. The objectives of this study are as follows: First, we investigate the different dependence structures between the Chinese stock market and other major stock markets using constant conditional copula models. For comparison purposes, we analyze the dependence structures between the U.S. stock market and others. It is, to my knowledge, the first attempt to examine the dependence structures between the Chinese financial market and other major markets. Second, we try to examine the dynamics of general dependence and tail dependence using time-varying conditional copula models. Finally, a comparative analysis between China-related models and U.S.-related models is conducted and some suggestions for practitioners are given.

There are three main findings: First, the time-varying-dependence model does not always perform better than constant-dependence model. This result has not previously been reported in the literature. Second, we find that the upper tail dependence can be much higher than the lower tail dependence in some short periods, which has not been documented in the financial contagion literature. Finally, Chinese financial market is relatively separate from other international financial markets in contrast to the U.S. market. The tail dependence with other financial markets is lower in China than in the U.S. Additionally, we find that the dependence is negatively correlated with physical distance between financial centers. There may be a general level of the dependence among financial markets in developed countries and the dependence among western financial markets have a more groupwise flavor.

This paper is organized as follows. The next section provides a brief review of copulas and conditional copulas. In section 3, we discuss the model specification, including the choice of estimation strategy and specific marginal and copula models. Section 4 presents estimation results for both marginal and copula models. Section 5 concludes.

2 Theory of Conditional Copula

2.1 Copula

It is necessary to understand what a copula is before we can discuss conditional copula. For simplicity, we will focus on only bivariate copulas even though the extension to the multivariate case is straightforward. Suppose we have two random variables Y_1 and Y_2 . Then the joint distribution function can be written as:

$$
F(y_1, y_2) = \Pr(Y_1 \le y_1, Y_2 \le y_2) \tag{1}
$$

where y_1 and y_2 denote the realizations of random variables Y_1 and Y_2 , respectively.

A copula is actually a multivariate joint distribution. It allows the decomposition of a joint distribution into its marginal distributions and its dependence function, i.e. copula function.¹ We may construct the copula function by transforming the random

 $1A$ complete and formal definition of copulas can be found in Nelsen (2006). Also, Joe(1997) provided many nice properties of various copula families.

variables Y_1 and Y_2 to their uniform marginal distributions (CDFs) denoted as F_1 and F_2 , respectively. Formally,

$$
F(y_1, y_2) = \Pr(F_1(Y_1) \le F_1(y_1), F_2(Y_2) \le F_2(y_2))
$$
\n
$$
= C(F_1(y_1), F_2(y_2))
$$
\n(2)

2.2 Conditional Copula

Patton (2006a) summarizes the conditional copula theory. We give a brief review here. Similar to the unconditional case, we have two random variables Y_1 and Y_2 . We introduce a conditioning vector W. Let $F_{Y_1Y_2|W}$ denote the conditional distribution of (Y_1, Y_2) given W, and let the conditional marginal distributions of $Y_1|W$ and $Y_2|W$ be denoted $F_{Y_1|W}$ and $F_{Y_2|W}$, respectively. We assume that $F_{Y_1|W}$, $F_{Y_2|W}$ and $F_{Y_1Y_2|W}$ are all continuous for simplicity.² Theorem 1 on conditional copulas in Patton (2006a) is reproduced below:

Theorem 1 Let $F_{Y_1|W}(\cdot|w)$, $F_{Y_2|W}(\cdot|w)$ be the conditional distribution of $Y_1|W=w$ and $Y_2|W=w$, respectively, $F_{Y_1Y_2|W}(\cdot|\omega)$ be the joint conditional distribution of $(Y_1, Y_2)|W =$ w and ω be the support of W. Assume that $F_{Y_1|W}(\cdot|w)$ and $F_{Y_2|W}(\cdot|w)$ are continuous in y_1 and y_2 for all $w \in \omega$. Then there exists a unique conditional copula $C(\cdot | \omega)$ such that

$$
F_{Y_1Y_2|W}(y_1,y_2|\omega) = C(F_{Y_1|W|}(y_1|w), F_{Y_2|W}(y_2|w)|w)
$$

= C(u,v) (3)

$$
\forall (y_1, y_2) \in \bar{R} \times \bar{R} \text{ and } w \in \omega \tag{4}
$$

where $u = F_{Y_1|W}(y_1|w)$ and $v = F_{Y_2|W}(y_2|w)$ are realizations of $U \equiv F_{Y_1|W}(Y_1|w)$ and $V \equiv F_{Y_2\mid W}(Y_2\mid w)$ given $W = w$.

Theorem 1 is virtually an extension of Sklar's Theorem (1959). U and V are the conditional "probability integral transforms" of Y_1 and Y_2 , respectively. Fisher (1932) and Rosenblatt (1952) prove that U and V follow the $Unif(0, 1)$ distribution, regardless of the

²This assumption is not necessary for the properties of copulas to hold. See Nelsen (2006).

original distributions. Most nice properties of copulas come from this probability integral transformation. Patton (2002) shows that a conditional copula has all the properties of an unconditional copula. There are many copula families. In the next section, we will discuss the specific copula functions used in our analysis.

3 Model Specification

3.1 Estimation Strategy

It has been widely accepted that financial time series are generally non-normal and follow Student's t distribution. Moreover, in each marginal distribution, we model serial correlation and heteroskedasticity via the $AR(p) - GARCH(1, 1) - t$ model. After estimating the marginal distributions, we will estimate copula dependence parameters using maximum likelihood method. Let $u \equiv F_{Y_1|W}(y_1|w; \theta_1)$ and $v \equiv F_{Y_2|W}(y_2|w; \theta_2)$, where θ_1 and θ_2 are the vectors of parameters of each margins (or the coefficients of conditioning vector W). Given $C(u, v; \delta) = C(F_{Y_1|W}(y_1|w; \theta_1), F_{Y_2|W}(y_2|w; \theta_2); \delta)$, the copula density is:

$$
c(u, v; \delta) = \frac{\partial^2 C(u, v; \delta)}{\partial u \partial v}
$$
(5)

Hence the joint density of an observation $(y_{1,t}, y_{2,t})$ is:

$$
c(y_{1,t}, y_{2,t}; \delta) = \frac{\partial^2 C(u_t, v_t; \delta)}{\partial u_t \partial v_t} \cdot \frac{\partial u_t}{\partial y_{1,t}} \cdot \frac{\partial v_t}{\partial y_{2,t}} = c(u_t, v_t; \delta) \cdot f_{Y_1|W}(y_{1,t}|w; \theta_1) \cdot f_{Y_2|W}(y_{2,t}|w; \theta_2)
$$
(6)

Therefore, the log-likelihood of a sample can be written as:

$$
L(y_{1,t}, y_{2,t}; \delta, \theta_1, \theta_2) = \sum_{t=1}^T \ln[c(u_t, v_t; \delta) \cdot f_{Y_1|W}(y_{1,t}|w; \theta_1) \cdot f_{Y_2|W}(y_{2,t}|w; \theta_2)]
$$

$$
= \sum_{t=1}^T \ln[c(F_{Y_1|W}(y_{1,t}|w; \theta_1), F_{Y_2|W}(y_{2,t}|w; \theta_2); \delta)
$$

$$
\cdot f_{Y_1|W}(y_{1,t}|w; \theta_1) \cdot f_{Y_2|W}(y_{2,t}|w; \theta_2)]
$$
(7)

$$
= L_C + L_{Y_1} + L_{Y_2} \tag{8}
$$

where $L_C(y_{1,t}, y_{2,t}; \delta, \theta_1, \theta_2) = \sum_{i=1}^{T}$ $\sum_{t=1} \ln c(F_{Y_1|W}(y_{1,t}|w;\theta_1), F_{Y_2|W}(y_{2,t}|w;\theta_2);\delta), L_{Y_1}(y_{1,t};\theta_1) =$ $\sum_{i=1}^{T}$ $\sum_{t=1}^{T} \ln f_{Y_1|W}(y_{1,t}|w; \theta_1), \text{ and } L_{Y_2}(y_{2,t}; \theta_2) = \sum_{t=1}^{T}$ $\sum_{t=1}$ ln $f_{Y_2|W}(y_{2,t}|w;\theta_2)$ are the individual loglikelihood functions of the copula and its two margins.

There are two parametric estimation methods available for copula modeling. One is a one-step procedure, the other is a two-step procedure. The one-step procedure is to estimate all parameters of the marginals and the copula at one time. Maximum likelihood estimation yields $\hat{\theta} = (\hat{\delta}, \hat{\theta}_1, \hat{\theta}_2)$, such that

$$
\hat{\theta} = \arg \max L(y_{1,t}, y_{2,t}; \delta, \theta_1, \theta_2)
$$
\n(9)

However, in some situations, the maximum likelihood estimation may be difficult to conduct due to too many parameters or just the complexity of the model. As Jondeau and Rockinger (2006) point out, the time-varying dependence parameter may be a convoluted expression of many parameters, hence an analytical expression of the gradient of the likelihood might not exist. Therefore, only numerical gradients may be computable, implying a dramatic slowing down of the numerical procedure. In such a case, a two-step maximum likelihood estimation procedure, also known as Inference Functions for Margins method (IFM) is necessary. In this paper, we use an $AR(p) - GARCH(1, 1) - t$ model to estimate the margins, which leads to many parameters. We also allow the dependence parameters to vary over time, hence the number of parameters increases further. Due to the large number of parameters and the complexity of our model, we choose the twostep estimation strategy. This approach, proposed by Shih and Louis (1995) and Joe and Xu (1996), is the maximum likelihood estimation of the dependence parameter given the estimated marginal distributions. In the first step, the parameters in the marginal distributions are estimated as follows:

$$
\tilde{\theta}_k = \arg \max L_{Y_k}(y_{k,t}; \theta_k) \text{ for k=1,2}
$$
\n(10)

In the second step, copula parameter is estimated given $\tilde{\theta}_1$ and $\tilde{\theta}_2$ from the first step:

$$
\tilde{\delta} = \arg \max L_C(y_{1,t}, y_{2,t}; \delta, \tilde{\theta}_1, \tilde{\theta}_2)
$$
\n(11)

Note that the density estimation of each margin does not affect the estimation of the copula parameter in the second step because each margin is actually estimated in the first step and hence constant in the second step. Therefore, we only need to maximize $L_C(y_{1,t}, y_{2,t}; \delta, \tilde{\theta}_1, \tilde{\theta}_2)$ to get the estimate of copula parameter.³ Patton (2006b) has proved that this two-step estimation produces normal and asymptotically efficient parameter estimates.

3.2 Marginal Model

To estimate a bivariate distribution, we need to make an assumption about each univariate marginal distribution first. In this study, we assume each marginal distribution follows an $AR(p) - GARCH(1, 1) - t$ process.⁴ This is a standard model for financial returns introduced by Bollerslev (1987), and which is widely used in the literature; see Patton (2002, 2006a) Jondeau and Rockinger (2006) and Hu (2006) among others. Mathematically,

$$
y_{i,t} = \alpha_i + \sum_{j=1}^p \beta_j y_{i,t-j} + \varepsilon_{i,t} \text{ for } i=1,2
$$
 (12)

$$
\sqrt{\frac{\nu}{\sigma_{i,t}^2(\nu-2)}} \cdot \varepsilon_{i,t} |I_{t-1} \sim t(\nu) \tag{13}
$$

$$
\sigma_{i,t}^2 = a_i + b_i \sigma_{i,t-1}^2 + c_i \varepsilon_{i,t-1}^2 \tag{14}
$$

where $y_{i,t}$ represents univariate stock index return series, α_i is the conditional mean for ith series, $\varepsilon_{i,t}$ is error term in conditional mean equation, $\sigma_{i,t}^2$ is variance, ν is the degree of freedom of Student's t distribution, I_{t-1} is the information set at time $t-1$. We can consider this information set as the conditioning vector W . The standardized residuals

³All estimation in this study is conducted using S-Plus (Finmetrics) and MATLAB.

⁴This stands for autoregressive mean with lag order of p $(AR(p))$ and generalized autoregressive conditional heteroscedesticity (GARCH) variance with Student's t residuals.

are assumed to follow Student's t distribution with degree of freedom ν .

3.3 Copula Model

We will mainly focus on the Normal (Gaussian) and Generalized Joe-Clayton copula $(GJC)^5$ since the former one is a good model to measure general dependence and the latter one is good at modeling both upper and lower tail dependences. These two types of copula models will give us a full picture of dependence structures for financial returns. These results will be discussed in next section.

3.3.1 Normal (Gaussian) Copula

The first copula of interest is the Normal copula, which has the dependence function associated with bivariate normality. It can be written as:

$$
C^{N}(u,v;\rho) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{(1-\rho^2)}} \exp\left\{\frac{-(r^2 - 2\rho rs + s^2)}{2(1-\rho^2)}\right\} dr ds \qquad (15)
$$

where Φ^{-1} is the inverse of the standard normal CDF, ρ is the general dependence parameter.⁶

In this paper, we assume that the functional form of the copula is fixed throughout the sample period while the dependence parameter is time-varying following some evolution equation. We follow Patton (2006a) and assume the following evolution dynamics for ρ_t :

$$
\rho_t = \Lambda \left(\omega_\rho + \beta_\rho \cdot \rho_{t-1} + \alpha_\rho \cdot \frac{1}{10} \sum_{j=1}^{10} [\Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j})] \right) \tag{16}
$$

where $\Lambda(x) = \frac{(1-e^{-x})}{(1+e^{-x})}$ $\frac{(1 + e^{-x})}{(1 + e^{-x})}$ is the modified logistic transformation, aiming to keep ρ_t within $(-1, 1)$ interval. Here we assume that the copula dependence parameter follows an $ARMA(1, 10)$ -type process, in which the autoregressive term $(\beta_{\rho} \cdot \rho_{t-1})$ captures the

 5 This is also called "Symmetrized Joe-Clayton copula" in the literature (see Patton (2006b)). We use "Generalized" to emphasize that this copula provides more áexibility than the regular Joe-Clayton copula in the sense that it is generalized to allow tail dependece to be either symmetric or asymmetric while the regular one contains only asymmetric tail dependence by construction.

⁶Dependence parameter refers to the measure of correlation in copula function. In normal copula, this is ρ .

persistence effect and the last term $(\alpha_\rho \cdot \frac{1}{10})$ $\frac{1}{10} \sum_{i=1}^{10}$ $j=1$ $[\Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j})]$ captures the variation effect in dependence. The functional form of this evolution equation can be changed since is hard to know what the dynamics of the dependence looks like. Here we follow Patton (2006a) to make our results comparable to previous research.⁷

3.3.2 Generalized Joe-Clayton (GJC) Copula

The second copula used in our study is Generalized Joe-Clayton (GJC) copula proposed by Patton $(2006a)$, which is basically a slight modification of original Joe-Clayton (JC) copula.⁸ Joe-Clayton copula proposed by Joe (1997) is a Laplace transformation of Clayton's copula . It is defined as:

$$
C^{JC}(u, v; \tau^U, \tau^L) = 1 - (1 - \{[1 - (1 - u)^{\kappa}]^{-\gamma} + [1 - (1 - v)^{\kappa}]^{-\gamma} - 1\}^{-1/\gamma})^{1/\kappa}
$$
(17)

where $\kappa = 1/\log_2(2 - \tau^U)$, $\gamma = -1/\log_2(\tau^L)$ and $\tau^U \in (0, 1]$, $\tau^L \in (0, 1]$.

Unlike the normal copula, there are two tail dependence parameters, τ^U and τ^L , in this copula function. The upper tail dependence is defined as:

$$
\tau^U = \lim_{\varepsilon \to 1} \Pr[U > \varepsilon | V > \varepsilon] = \lim_{\varepsilon \to 1} \Pr[V > \varepsilon | U > \varepsilon] = \lim_{\varepsilon \to 1} (1 - 2\varepsilon + C(\varepsilon, \varepsilon) / (1 - \varepsilon) \tag{18}
$$

If this limit exists, the copula shows upper tail dependence when $\tau^U \in (0, 1]$ and no tail dependence when $\tau^U = 0$. Similarly, we can define lower tail dependence as:

$$
\tau^L = \lim_{\varepsilon \to 0} \Pr[U \leqslant \varepsilon | V \leqslant \varepsilon] = \lim_{\varepsilon \to 0} \Pr[V \leqslant \varepsilon | U \leqslant \varepsilon] = \lim_{\varepsilon \to 0} \Pr[C(\varepsilon, \varepsilon) / \varepsilon \tag{19}
$$

If this limit exists, the copula shows lower tail dependence when $\tau^L \in (0, 1]$ and no tail dependence when $\tau^L = 0$.

By construction, the Joe-Clayton copula always gives asymmetric tail dependence even if two tail dependence measures are in fact equal. In order to overcome this short-

 7 Actually, we have tried several different evolution equations here, such as including a lag 2 autoregressive term or replacing 10 with 20 in the last term. These modifications did not offer significant improvement in our maximum likelihood estimation, however.

⁸Joe-Clayton copula is also known as the "BB7" copula.

coming, we will use Generalized Joe-Clayton copula, which is given by

$$
C^{GJC}(u, v; \tau^U, \tau^L) = 0.5 \cdot (C^{JC}(u, v; \tau^U, \tau^L) + C^{JC}(1 - u, 1 - v; \tau^U, \tau^L) + u + v - 1)
$$
 (20)

where C^{JC} represents the Joe-Clayton copula. The advantage of the GJC copula is that it can be symmetric when $\tau^U = \tau^L$, whereas the original Joe-Clayton copula still allows asymmetry even though tail dependence is actually symmetric, i.e. $\tau^U = \tau^L$. Consequently, the GJC copula is virtually a generalized version of the Joe-Clayton copula allowing tail dependence to be either asymmetric or symmetric. This property makes the GJC copula more attractive for empirical work because of its generality. The Gumbel and Clayton copulas also capture tail dependence. However, empirical research shows that estimating Gumbel or Clayton copula separately does not produce much different results from estimating the Joe-Clayton copula alone, as reported by Kim (2005).

Tail dependence refers to the level of dependence in the upper-right-quadrant tail and lower-left-quadrant tail of a multivariate distribution, hence it is an appropriate measure of the dependence of extreme events. This nice property makes it very useful to examine the joint extreme events in financial returns during high volatility or market crash periods. One explanation of tail dependence in our paper is a probability measure of joint extreme values in two financial markets given one extreme value in one of the two markets.

Similar to the dynamics of ρ_t in the Normal copula, we propose the following evolution equations for τ^U and τ^L , respectively (See Patton (2006) for a more detailed explanation.)

$$
\tau_t^U = \Pi \left(\omega_U + \beta_U \cdot \tau_{t-1}^U + \alpha_U \cdot \frac{1}{10} \sum_{j=1}^{10} |u_{t-j} - v_{t-j}| \right) \tag{21}
$$

$$
\tau_t^L = \Pi \left(\omega_L + \beta_L \cdot \tau_{t-1}^L + \alpha_L \cdot \frac{1}{10} \sum_{j=1}^{10} |u_{t-j} - v_{t-j}| \right)
$$
(22)

where Π is the logistic transformation, used to keep τ^U and τ^L within the $(0,1)$ interval. These dynamics follow an $ARMA(1, 10)$ -type process with an autoregressive term $(\beta \cdot \tau_{t-1})$ and a forcing variable $(\alpha \cdot \frac{1}{10})$ $\frac{1}{10} \sum_{i=1}^{10}$ $\sum_{j=1} |u_{t-j} - v_{t-j}|$, where the autoregressive term represents the persistence effect and the forcing variable captures the variation in depen-

dence. Note that we assume τ^U and τ^L evolve in the same pattern even though it is possible that they follow different dynamics. We use 10 lags in the forcing variable to make the evolution equation comparable with that of the Normal copula.

4 Empirical Results

4.1 Data Description

We examine the interaction between Chinese/U.S. stock indices and each of six other stock indices. The labels are "CHN" for the Shanghai Stock Exchange Composite from China, "DEU" for the DAX from Germany, "FRA" for the CAC 40 from France, "GBR" for the FTSE 100 from the United Kingdom, "HKG" for the Hang Seng Stock Exchange Index from Hong Kong, "JPN" for the Nikkei 225 from Japan and "USA" for the S&P500 from the United States. Daily stock indices are obtained from Datastream from January 2nd, 1991 to December 31st, 2007. The sample consists of 17 years of daily data covering 4434 data points. Table 1 gives summary statistics on all of the stock market returns. As usual, returns are defined as 100 times the log-difference of index values, where P_t is the value of the index at time t. This reduces the sample by one record, yielding 4433 observations. That is,

$$
R_t = 100 \times \log(P_t/P_{t-1}) \tag{23}
$$

We have the following findings: First, in Panel A of Table 1, the average return of Chinese stock market is the highest one followed by the Hong Kong market. In particular, the Japanese stock market shows bad performance considering the negative average return. According to the standard deviation, the most volatile stock market is the Chinese market and the next one is the Hong Kong market, while the less volatile market is the U.S. market. Means of each series are very small relative to their standard deviations. Most of markets exhibit slight negative skewness (i.e. left-skewed) except for China and Japan. China even reaches 6.05, which implies that the distribution is highly right-skewed. All of these results show that the empirical distributions of returns exhibit

non-normal pattern. We also find significant kurtosis in each return series. China displays extremely high kurtosis. This high kurtosis means more of the variance is due to infrequent extreme deviations.

Second, in Panel B of Table 1, results of the Jarque-Bera test strongly reject the null hypothesis of normality, indicating the non-normality of the unconditional distribution of each series. This is one of the reasons why the multivariate normal distribution would be inappropriate. We perform the LM test to examine whether the squared return is serially correlated up to lags 1, 5 and 10. This statistic clearly indicates that ARCH effects are likely to be found in all market returns.⁹ Even if there is one insignificant statistic of ARCH $LM(1)$ test for Chinese stock market, it is statistically significant at the 5% level using lags 5 and 10. Ljung-Box autocorrelation test with correction for heteroskedesticity is also implemented at lags 1, 5 and 10, implying most of return series are serially correlated, at least at one of the lag orders.¹⁰

Finally, in Panel C of Table 1, the unconditional correlation matrix indicates that there exists a rather high dependence between geographically close countries as expected. The correlations between DEU, FRA and GBR are relatively higher than those of other pairs. There are some extra Öndings on the relationship between distance and stock market correlation in this paper. We will discuss this issue in the copula result and further research sections. Unconditional correlations between China and other countries are small, but whether conditional correlations are small or not is still unknown. The linear unconditional correlations in China-related pairs range from - 0.0158 to 0.0511. The ranking from the highest to the lowest is CHN/HKG, CHN/JPN, CHN/DEU, CHN/FRA, CHN/GBR, CHN/USA. The ranking of Spearman correlations remains the same as that of linear correlations. Most of Spearman correlations are less than the linear correlations except CHN/HKG, which actually increases by 53% (from 0.051 to 0.078). For the U.S.-related pairs, the linear correlations range from -0.016 to 0.455. The ranking of linear correlations in descending order is USA/DEU, USA/FRA,

 9 Other lag orders are also used to perform this test, almost all of them show significant ARCH effect. The results are available upon request.

 10 Other lag orders are also used to perform this test, most of them show statistically significant serial correlation at 5% significance level. The results are available upon request.

USA/GBR, USA/HKG, USA/JPN, USA/CHN. However, the ranking of Spearman correlation changes into USA/FRA, USA/GBR, USA/DEU, USA/JPN, USA/HKG, USA/CHN in descending order. Most of Spearman correlations are less than their linear correlations except USA/JPN, which actually increases 7.3% (from 0.109 to 0.117). We can see similar results for Kendall's τ .

The linear correlation is only one way to measure dependence. In order to use it correctly, two conditions must be satisfied: (1) the data in the pairs both come from normal distributions and (2) the data are at least in the same frequency. The first condition is evidently violated in our case, so linear correlation is not effective way to evaluate dependence. Another possibility is to use the Spearman's ρ (Rank) correlation coefficient or Kendall's τ ¹¹. The copula dependence parameter is easily transformed to these rank correlation measures. According to Table 1, Spearman's ρ 's and Kendall's τ 's are a little less than the linear correlations for most pairs.

[Table 1]

4.2 Estimation of the Marginal Models

We use the two-step estimation method in this paper due to the large number of parameters in the time-varying models. First, we select different lag order models for the mean equations based on Akaike Information Criterion (AIC), keeping the conditional variance equation as $GARCH(1,1)$ for each country. We choose $AR(17)$ for CHN, $AR(6)$ for DEU, $AR(7)$ for FRA, $AR(6)$ for GBR, $AR(3)$ for HKG, $AR(1)$ for JPN, and $AR(7)$ for USA. The results for the marginal distributions are reported in Table 2. All coefficients in conditional variance equations are statistically significant at 1% level, indicating strong

$$
\rho = 12 \int \int_{I^2} C(u, v) dC(u, v) - 3
$$

$$
\tau = 4 \int \int_{I^2} C(u, v) dC(u, v) - 1
$$

¹¹As measures of concordance, Spearman's ρ and Kendall's τ can be written with copulas (see Schweizer and Wolff (1981) :

ARCH effects in all of the countries.

[Table 2]

We then conduct model misspecification test as suggested by Diebold, Gunther and Tay (1998). They examined the correlograms of $(e-\bar{e}), (e-\bar{e})^2, (e-\bar{e})^3$, and $(e-\bar{e})^4$, where e is the probability integral transforms (u and v in our study). Each moment reveals dependence operative through the conditional mean, conditional variance, conditional skewness, and conditional kurtosis. Figure 1 presents the test results. In the AR – $GARCH - t$ case, the correlograms show that there is no serial correlation in the first four moments with few exceptions. So we can conclude that our marginal distribution models for all countries are correctly specified. Put differently, our marginal models are adequate for financial returns.

[Figure 1]

4.3 Estimation of the Copula Models

4.3.1 Results for China-related Copula Models

Table 3 reports China-related Normal and Generalized Joe-Clayton (GJC) copula parameter estimates of both constant and time-varying cases for the purpose of comparison.

Normal Copula In Table 3 Panel A1, the constant dependence measures are significantly different from linear correlations reported in Table 1 Panel C. Specifically, in the CHN/DEU pair it decreases by 29% from 0.007 to 0.005; in the CHN/FRA pair it decreases from 0.002 to -0.004 with the change of sign; in the CHN/GBR pair it is less negative (from -0.002 to -0.001); in the CHN/HKG pair it increases by 69% from 0.051 to 0.086; in the CHN/JPN pair it increases by 38% from 0.029 to 0.04; in the CHN/USA pair it is less negative (from -0.016 to -0.006). These results show that the linear correlation is highly biased due to the inappropriate normality assumption. One of them even changes its sign from positive to negative. We compare these constant-dependence

estimates across pairs and find that the highest constant dependence comes from the CHN/HKG pair, followed by the CHN/JPN and CHN/DEU pairs with all positive signs. This is reasonable since China and Hong Kong have a very close economic relationship. However, all constant dependence levels are relatively low since the highest one is only 0.086, which implies that the returns in the Chinese financial market have not exhibited a high level of dependence. It is noticed that the signs and magnitude of dependence in Normal copula are more consistent with those of Spearman's ρ and Kendall's τ than with linear correlations in Table 1. This again verifies the argument that the linear correlation is inappropriate in certain conditions.

Since the constant case can be considered as a restricted version of the time-varying evolution equation with two restrictions of $\alpha = 0$ and $\beta = 0$, we then perform a formal likelihood ratio test to check which model is preferred. The null is that the restricted version with constant dependence of the model is not rejected as one moves to unrestricted model with time-varying dependence. According to test statistics presented in Panel A2, the null is rejected only in the CHN/HKG pair at the 5% significance level, hence the time-varying model is preferred only in this pair. The constant normal copula models are preferred in all other five pairs at the 5% significance level. We should conclude that the time path of dependence in the CHN/HKG pair derived from the time-varying model would be more informative than others due to a better fit. However, given the fact that the null can be rejected at the 10% significance level in the CHN/DEU and CHN/GBR pairs, the time-varying models of these two pairs could potentially provide some insights on the changes of dependence over time. The dynamics of dependence are captured by the coefficients in the evolution equations. The time path of dependence parameters are presented in Figure 2-7. It can be seen that most of the time paths are close to white noise, but in the CHN/GBR and CHN/HKG pairs they seem to be informative. (see Figure 4 and 5) This is shown in the estimates of evolution equation as the persistence coefficient $\beta's$ are relatively high compared to variation coefficients $\alpha's$, indicating that the timevariation effects dominate in these two pairs. In Figure 1, for the CHN/DEU pair, the dependence is very volatile over time and reaches an extremely high level on March 2007.

In Figure 4, for the CHN/GBR pair, the time path of dependence is relatively clear. It is clear that dependence was increasing throughout last three months of 2007. In Figure 5, for the CHN/HKG pair, we can not find any dramatic change in the dependence level in July 1997 when Hong Kong left British rule though the dependence went up a little bit after July 1st, 1997. One explanation could be that this event was well-anticipated, and thus, is not considered an economic shock. Moreover, we fail to find a significant increase in the dependence level in 1997 and 1998 during Asian financial crisis. This is because China was relatively independent of other financial markets in Asia like Hong Kong, hence the CHN/HKG dependence did not change much during that period. Interestingly, we can not find any significant change for all pairs in December 2002 when A shares were initially open to qualified foreign institutional investors (QFI) . In 2007, the dependence was increasing in general. The CHN/USA pair does not exhibit an informative time path though it reaches an extreme peak in March 2007.

GJC Copula According to Table 3 Panel B1, in the constant tail dependence case, most of the upper and lower tail dependences are close to zero except the CHN/HKG pair. This indicates that China and Hong Kong exhibited some degree of dependence in extreme events as might be expected. In particular, lower tail dependence is slightly higher than upper one, hence there is higher probability of joint extreme events during bear market than during bull market. This is also true for the CHN/JPN pair, even though the magnitude of the tail dependence is less than that of the CHN/HKG pair. For other pairs, there is no observable tail dependence, hence joint extreme events were less likely to happen in these paired countries. Therefore, China was not significantly affected by the extreme events in western stock markets in general. In other words, if western stock markets experience extreme market downturns or upturns, then we should not expect it would happen to China simultaneously if this data is representative.

In Figure 5, for the CHN/HKG pair, even if the constant upper tail dependence is smaller than lower tail dependence, the time path of lower tail dependence is more informative than that of upper tail dependence. We can see that there exists several peaks with the highest one approaching 0.3. This shows that the time-varying model can

give us further insights on changes in the dependence structure throughout the sample period. There is no strong evidence of asymmetric tail dependence in all pairs except the CHN/HKG pair, in which the lower tail dependence is 1.5 times upper tail dependence. We also conduct likelihood ratio tests with four restrictions since we have two separate evolution equations for τ^U and τ^L . The results can be found in Table 3 Panel B2. It turns out that the time-varying models are preferred in the CHN/DEU, CHN/FRA, CHN/GBR and CHN/USA pairs while the constant models fit better in the CHN/HKG and CHN/JPN pairs.

[Table 3]

4.3.2 Results for U.S.-related Copula Models

Table 4 reports US-related Normal and Generalized Joe-Clayton (GJC) copula parameter estimates for both constant and time-varying cases.

Normal Copula In Table 4 Panel A1, we find that the dependence estimates are revised by Normal copula models compared to linear correlations. Specifically, in the USA/DEU pair it decreases by 17% from 0.455 to 0.378; in the USA/FRA pair it decreases by 9% from 0.428 to 0.391; in the USA/GBR pair it decreases by 4% from 0.413 to 0.396. However, in the USA/HKG pair it increases by 6% from 0.110 to 0.117 and in the USA/JPN pair it increases by 8% from 0.109 to 0.118. Just like the China-related pairs, these revisions again show that the linear correlations are biased in non-normal situations. Most constant dependence estimates are closer to the Spearman's correlations and Kendall's τ than linear correlations. Another interesting finding is that the constant dependences are quite close to 0.39 in the first three pairs and close to 0.12 in next two pairs. There may be a general level of dependence within a certain group of countries.

Similarly, we then implement likelihood ratio tests to compare constant and timevarying models. The time-varying models are preferred in the USA/DEU and USA/FRA pairs, given that the null hypotheses are strongly rejected at the 5% significance level. For other pairs, the constant models are preferred. Taking a look at coefficients in timevarying equations, the persistence coefficient β 's are significantly higher than variation

coefficients $\alpha' s$ in the USA/DEU and USA/FRA pairs. So persistence effects dominate. In Figure 2 and 3, the USA/DEU and USA/FRA pairs show very clear and similar timevarying paths with significantly increasing dependence in the long run while others do not exhibit this pattern. In the USA/DEU pair, the dependence goes down until November 1993 and goes up thereafter. After September 1997, the dependence was consistently above the constant level at 0.378 with few exceptions and exhibits a more volatile pattern. The time-varying dependence reaches a low of 0 and a high of 0.6. Interestingly, on and shortly after September 11th 2001, there exists some increase in dependence but not as dramatic as we initially expected. Compared to the USA/DEU pair, the USA/FRA pair displays a similar pattern but a smoother time path of dependence. The path reaches two troughs in August 1994 and July 1996 and was gradually increasing during the last two years. The time-varying dependence ranges from 0.34 to 0.46. This interval is 0.12 and hence less than the interval 0.6 of the USA/DEU pair. This smaller range of the USA/FRA pair shows a more stable dependence structure than that of the USA/DEU pair. Beginning from October 1996, the dependence is consistently above its constant level 0.391 with no single exception and becomes even more stable than before considering that the range further reduces to 0.06 (from 0.4 to 0.46). Moreover, the time path of the USA/FRA pair is less volatile than the USA/DEU pair. In the USA/GBR pair, unlike what we expected, the time path is not very informative and moves around the constant level of 0.396. It ranges from -0.15 to 0.15 and reaches the highest peak in March 2007. In the USA/HKG pair, the dependence ranges from 0.06 to 0.2. It also exhibits large variations in relatively short period (within one year). In the USA/JPN pair, the time path is the most volatile one in the U.S.-related pairs, ranging from -0.02 to 0.25. This time path is less informative than others.

GJC Copula According to Panel B, in the constant case, the upper and lower tail dependences are slightly different in levels for the USA/DEU, USA/FRA and USA/GBR pairs. Specifically, in these three pairs, the lower tail dependence are higher than upper tail dependence by 0.016, 0.03, 0.043, respectively. This implies that the limiting probability of U.S. stock market crash, given that German stock market has crashed, is about 8% greater than the joint probability of a market boom, meaning that the stock market is more dependent during market downturns than during market upturns. These findings are consistent with previous research, for example, Longin and Solnik (2001), Patton (2004). In the USA/FRA and USA/GBR pairs, the probabilities of market crash are about 15% and 23% greater than that of market boom, respectively. Therefore, the USA/GBR pair has the most asymmetric tail dependence, followed by the USA/FRA pair, and the USA/DEU pair is less asymmetric. In USA/HKG pair, the lower tail dependence is 900 times upper tail dependence, meaning that the probability of U.S. market downturns, given Hong Kong market downturns, is about 900 times the joint probability of market upturns. This implies that the USA/HKG pair is much more dependent during bear markets than during bull markets, which is an extremely strong asymmetry. In the USA/JPN pair, the lower tail dependence is present while the upper tail dependence is very small. The lower tail dependence is about 40 times upper tail dependence, a strong asymmetry, meaning the probability of joint negative extreme events, given Japanese market has crashed, is 40 times the probability of joint market boom. These tail dependences in USA/HKG and USA/JPN are much more asymmetric than those in other pairs.

For comparison purposes, we perform the likelihood ratio tests with four restrictions. It turns out that all time-varying models are strongly preferred except the USA/HKG pair. In general, the evolutions of time-varying dependence parameters follow different patterns for upper and lower tail dependences. In Figure 2, for the USA/DEU pair, the time path of the upper tail dependence is informative but that of the lower tail dependence is quite noisy. In the plot of the upper tail dependence, we find that its time path is closer to its constant level before August 2000 than after that time. After August 2000, there are five significant peaks. In particular, there is a significant adjustment period for the upper tail dependence from December 2002 though August 2003 when it goes up first and goes back to its constant level. We have similar findings for the USA/FRA pair (see Figure 3). Namely, the time path of the upper tail dependence seems to be informative and very volatile while the time path of the lower tail dependence is close to white noise. In the time path of upper tail dependence, we also find that there are more deviations from the constant level after August 2000 than before. Interestingly, there are four significant peaks after August 2000, among which there are three peaks that happened at the same periods as those in the upper tail dependence of the USA/DEU pair. Surprisingly, in USA/FRA pair, the upper tail dependence could be much higher than the lower tail dependence in some short period, for example, from December 2002 to January 2003. Therefore, although the lower tail dependence is generally higher than the upper tail dependence in constant case, the time-varying-dependence model shows that the joint probability of market upturns (upper tail dependence) could be higher than the joint probability of market downturns (lower tail dependence) in a short time period. This result, to our knowledge, has not previously been documented in the literature.

These two pairs exhibit similar patterns of upper tail dependence, meaning that the upturns in U.S. stock market may have similar effects on German and French stock markets in terms of probability. Also, it is clear that the upper tail dependences are relatively high in several periods in these two pairs, including 9/11 event in 2001, but interestingly it is not the highest peak in dependence path for each pair. In the USA/GBR pair (see Figure 4), the time paths of the lower and upper tail dependences display similar patterns, indicating the symmetric property. There is no significant change in both upper and lower tail dependence. In the USA/HKG pair (see Figure 5), upper tail dependence is very close to zero and lower tail dependence moves around its constant level. In the USA/JPN pair (see Figure 6), lower tail dependence is volatile with three extreme peaks in September 1992, August and October 2005, respectively.

[Table 4]

[Figure 2-7]

4.3.3 Comparative Analysis of Dependence Structures of Chinese and U.S. Financial Markets

First, in general, Chinese financial markets have not been quite as dependent upon other Önancial markets as measured by both general dependence and tail dependence. The fact that most tail dependence parameters are close to zero implies a low possibility of an extreme event in China, given an extreme event in another country. However, the U.S. market is much more correlated with other countries. This is not surprising since the index we are using is from A share market denominated in Chinese yuan which has not allowed to be traded by foreign investors until $2002¹²$ After 2002, only qualified foreign institutional investors (QFII) were permitted to trade in A share market. Consequently, although trade áows between China and western countries increased, the Önancial market in China is relatively separate from international financial markets. However, we might expect that the dependence will increase in the future as it becomes more open to foreign investors. Moreover, the western markets are all developed economies whereas China is thought of as an "emerging" market, hence portfolio managers tend to think of emerging markets as a separate asset class in which to invest, which may explain this low dependence. Another interesting finding is that the Hong Kong market, which traditionally has had a closer economic relationship with mainland China, has higher dependence with U.S. market than with Chinese market. This is because western portfolio managers considered Hong Kong to be "investable" over the entire sample period and were, perhaps, more likely to set their exposure to the Chinese economy through the Hong Kong market rather than investing directly in mainland China.

Second, in the Figure 2 Normal case, the dependence is increasing in the long run for the USA/DEU pair, whereas the dependence in the CHN/DEU pair is close to white noise with an exception of a significant peak in early 2007. In Figure 3 Normal case, the USA/FRA pair shows a very clear dependence path (similar pattern to USA/DEU), but the CHN/FRA pair shows just noise. In Figure 4-6, both China-related and U.S.-related

 12 There is another B share market denominated in U.S. dollars. We don't use the index from the B share market since it is not a good representative index of Chinese financial market for two reasons: 1) it is very small in terms of market value compared to A share market and 2) domestic residents in mainland China were not allowed to invest in the B share market until 2001.

pairs show very volatile dependence without smooth paths. In Figure 7, the USA/CHN pair shows a very low dependence level with a volatile time path. Therefore, there is no much comovement between Chinese and U.S. stock markets. To sum up, the time paths of dependence in the USA/DEU and USA/FRA pairs are smoother than those in the China-related pairs. So U.S. market comovement with these two countries will be more traceable than China. In contrast, for Britain, Hong Kong, and Japan-related pairs, China exhibits smoother time paths of dependence. In general, one may expect that the closer is the economic relationship between two countries, the clearer and more traceable the time paths. This clear and smooth time path will be useful in forecasting future dependence structure.

Last, Table 5 reports results of model comparisons. We can see that in Panel A, for the China-related pairs, constant models dominate in normal copula though time-varying models are preferred for three pairs each in GJC copula. In Panel B, for the U.S.-related pairs, constant models got four checks in normal copula while time-varying GJC copula models got five checks out of six. It seems that, in general, constant models dominate in the China-related pairs whereas time-varying models dominate in the U.S.-related pairs. However, strictly speaking, model preference varies across different pairs and there is no general preference on model selection between constant and time-varying models. This implies that we have to analyze dependence structures on a case by case basis. There is no common preference in copula models.

[Table 5]

In addition to these empirical findings above, we also find that there exists a negative relationship between physical distance and financial market dependence with few exceptions, i.e. the greater the air distance between financial centers, the lower the dependence or correlation between financial markets. These results are reported in Table 6 and Figure 8. The distance measure is defined as the air distance in statute miles between financial centers in these countries. We scale it down by dividing the original values by 10⁵ in order to make it comparable with dependence values in terms of magnitude. The

Önancial centers are Shanghai in China, Frankfurt in Germany, Paris in France, London in Britain, Hong Kong, Tokyo in Japan and New York in U.S. Panel A reports this negative relationship between dependence and distance in China-related pairs while Panel B describes the negative association between them in U.S.-related pairs.

One significant exception is the comparison between USA/CHN and USA/HKG pairs since greater distance between New York, U.S. and Hong Kong (compared to the distance between New York, U.S. and Shanghai, China) is associated with higher depndence between U.S. and Hong Kong markets. This exception implies that other factors apart from the air distance may dominate the effect of distance on dependence, such as the market capital size or gross domestic product (GDP), etc. Therefore, we define M_i and M_j as the financial market capital size (or country GDP) in each country i and j and assume that they are positively correlated with financial market dependence DEF_{ij} (such as linear correlation, Spearman's ρ , Kendall's τ , and copula dependence estimates) in absolute terms, and also define S_{ij} as the physical distance between financial centers of country i and j . It is natural to propose a gravity model of financial market as follows:

$$
DEP_{ij} = G\frac{M_i M_j}{S_{ij}}\tag{24}
$$

where G is a constant. This model is in line with the spirit of the trade gravity model first proposed by Isard (1954). We can test the model by OLS regression. After loglinearization, the regression equation should be $log(DEF_{ij}) = \beta_0 + \beta_1 log M_i + \beta_2 log M_j +$ $\beta_3 \log S_{ij} + \varepsilon$, where $\beta_0 = \log G$. We expect that β_1 and β_2 are both positive and β_3 is negative. Table 6 Panel C reports the regression results. All coefficients show the correct signs as we expected. This result is consistent with the findings by Flavin *et al.* (2002) and Huang $et \ al.$ (2006). F-statistics indicate that all coefficients are jointly significant at 1% level in all of the four regressions. Moreover, we fail to reject the null hypothesis that the coefficient of air distance is -1 in all of the four regressions. This implies that the perfectly negative correlation between dependence and distance is present. R^2 is reported in the last row. The regression with copula dependence fits best. If our gravity model is correctly specified, than copula dependence estimate should be the most desirable measure

of dependence. In further research, it can be tested by more extensive cross-section data.

[Table 6]

[Figure 8]

5 Concluding Remarks

Dependence structure is an important issue in financial contagion. Linear correlation, though it provides an easy and convenient way to describe comovement between two random variables, is not an appropriate dependence measure and may be highly biased in certain non-normal situations. In particular, the multivariate distributions with complex dynamic features make linear correlation be an improper measure. In addition, asymmetric dependence in equity markets and foreign exchange markets is also documented in recent papers, such as Longin and Solnik (2001), Ang and Chen (2002), Patton (2006a) and Rodriguez (2007). These features can be easily captured in copula models with tail dependence parameters. Therefore, the copula is a powerful and attractive tool to analyze the dependence between margins since it does not require the assumption of normality in the marginals. Recently, copula theory has been extended to a time-varying conditional copula model by Patton (2006a), which contains a conditioning vector and allows the dependence parameter to vary over time. This model provides insights into the dynamics of the dependence structure, which can help us to better understand the áuctuations in dependence structure.

In this study, we used the time-varying conditional copula model to study dependence structures in Chinese versus U.S. stock markets. In order to use the copula, we needed to correctly model marginal distribution for each series. The standard $AR(p)$ $GARCH(1, 1) - t$ model is employed to estimate these conditional marginal distributions. The test suggested by Diebold, Gunther and Tay (1998) is implemented to examine the model misspecification of conditional marginal distributions. After that, two different copulas are considered: Normal copula with general dependence and Generalized JoeClayton copula with upper and lower tail dependence. Moreover, the dependence parameters are allowed to vary over time and ARMA-type evolution equations are proposed for each dependence parameter. The time paths of dependence for each pair are showed and analyzed. The following conclusions and implications can be reached:

First, after examining dependence structures in Chinese versus U.S. financial markets, we have three empirical findings as follows: 1) Due to the low general dependence between Chinese and U.S. Önancial markets, the downturns in the U.S. Önancial markets will less likely affect Chinese stock market than other countries. Moreover, given the very low tail dependence, extreme events in the U.S. financial markets will not influence Chinese financial markets either. This is also true for the effects of other western countries (for example, Germany, France, Britain, and Japan) on China. However, Hong Kong has some impact on Chinese market at both general and tail dependence levels. 2) U.S. stock market is closely associated with European markets, such as Germany, France and Britain, in terms of the general dependence and tail dependence. This implies that there is high probability that the downside in U.S. financial markets and the downside in other European markets will happen simultaneously. Hence we would expect strong comovement in Europe during U.S. recessions and downturns in financial markets. An interesting finding is that the USA/HKG and USA/JPN pairs display similar dependence patterns in terms of general dependence, upper and lower tail dependence. In addition, we find that there may be a general level of dependence (say 0.39 for the USA/DEU, USA/FRA, USA/GBR pairs and 0.118 for the USA/HKG and USA/JPN pairs) among financial markets in developed countries. The dependence among western financial markets have a more groupwise áavor, for example,. the USA/DEU and USA/FRA pairs demonstrate similar patterns of time-varying dependence. 3) Compared to U.S. stock market, Chinese market is relatively independent of other major international financial markets, except Hong Kong. This suggests that we should consider Chinese market as a good candidate in our portfolio to reduce risk when western markets experience downturns. During the ongoing global financial crisis, we would suggest investors to increase weights on financial assets from Chinese financial markets in their portfolio for diversification purpose. This will help them to diversify away risk or at least reduce their loss. This model can be used in conditional asset allocation and Value-at-Risk contexts in a non-normal financial world. The conditional tail dependence may provide some useful information for portfolio weighting hence reduce exposure to downside risk. It also provide some insights on international portfolio management for global hedge fund.

Second, the time-varying model provides very important information on the time path of dependence. It shows us that the dependence could be quite volatile and deviates from its constant level frequently, hence the constant model may not be correctly describe the fluctuations in dependence. Notwithstanding the fact that time-varying model is, loosely speaking, more informative than constant model in terms of explaining the changes in the dependence, the time-varying model does not always perform better than constant model. In some situations, the constant model is adequate enough to fully disclose the dependence structure, such as USA/HKG pair in our research.

Third, the asymmetric behavior in tail dependence does not mean that the lower tail dependence is always higher than upper tail dependence. It could be the other way around. In this paper, we find that the upper tail dependence is much higher than the lower tail dependence from December 2002 to January 2003 in the USA/FRA pair. This Önding, to our knowledge, has not been documented in previous research.

Last, but not least, an interesting finding is that the greater is the physical distance, the lower the dependence, at least in China-related pairs. This is, in spirit, similar to the intuition suggested by the gravity model of trade. This model can be tested empirically by more extensive cross-section regression. Also, it could be asked how can the time-varying copula model add values to Value-at-Risk calculation in contrast to the currently-used constant copula model. Moreover, one could also include time dummies to check whether or not dependence level has significantly changed before and after some significant events, for example, the 9/11 event in 2001. In addition, one can employ Monte Carlo simulation method to examine the sensitivity of dependence estimates to different copula models. We leave these topics for further research.

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Table 1 Summary Statistics on Daily Returns

Notes:

1. This table presents summary statistics of each index series. The data are daily percentage return, i.e. 100 times the log-differences of daily stock index returns. The sample period runs 17 years from January 2nd, 1991 to December 31st, 2007, yielding 4433 observations in total. Panel A presents descriptive statistics.

2. Panel B reports test results. Under the null hypothesis of normality, the Jarque-Bera test statistics has a Chi-square distribution with fixed degree of freedom 2. The ARCH LM test of Engle (1982) with null hypothesis of no ARCH effect is conducted using 1, 5 and 10 lags with 1, 5 and 10 degree of freedom, respectively. Tests using other number of lags give the same results. QW statistic is the Ljung-Box statistics for serial correlation, corrected for heteroscedesticity, computed at 1, 5 and 10 lags, respectively. The asterisks, (*) (**) and (***) indicate a rejection of the null hypothesis at the 1%, 5% and 10% levels, respectively. P‐values are reported in parentheses in Panel B.

3. Panel C reports the linear, Spearman's rho and Kendall's tau correlations between two country index returns. P‐values are reported in parentheses in Panel C. The asterisks, $(*)$ (**) and (***) indicate a rejection of the null hypothesis of no correlation at the 1%, 5% and 10% levels, respectively.

Notes: We report maximum likelihood estimates, with standard errors in parentheses, of the parameters of the marginal distribution models for each stock index return series. The asterisks, (*) (**) and (***) indicate a rejection of the null hypothesis at the 1%, 5% and 10% levels, respectively.

Table 3 Results for China‐related Copula Models

Notes: This table reports copula constant-dependence estimates and time-varying-dependence estimates. Copula log-likelihood and AIC are also reported. The Likelihood Ratio (p) Statistic test the null hypothesis that the restricted version (with constant dependence) of a model is not rejected as one moves from restricted model to unrestricted model (with time-varying dependence) where the parameter p is the number of restrictions under the null. So we have two restrictions in Normal copula and four restrictions in GJC copula. P-values are reported in parentheses. The asterisks, (*) (**) and (***) indicate a rejection of the null hypothesis at the 1%, 5% and 10% levels, respectively.

Table 4 Results for U.S.‐related Copula Models

Notes: This table reports copula constant-dependence estimates and time-varying-dependence estimates. Copula log-likelihood and AIC are also reported. The Likelihood Ratio (p) Statistic test the null hypothesis that the restricted version (with constant dependence) of a model is not rejected as one moves from restricted model to unrestricted model (with time-varying dependence) where the parameter p is the number of restrictions under the null. So we have two restrictions in Normal copula and four restrictions in GJC copula. P-values are reported in parentheses. The asterisks, $(*)$ (**) and $(***)$ indicate a rejection of the null hypothesis at the 1%, 5% and 10% levels, respectively.

Notes: This table is based on the results from likelihood ratio tests at 5% significance level for competing models in Table 3 and 4, where "C" means the

constant model is preferred while "V" indicates the time‐varying model is preferred. The asterisks indicate a rejection of the null hypothesis at the 10% level.

Table 6 Dependence Measures and Air Distance

Panel C: Regression Results of Stock Market Gravity Model

Regression Equation: $log(DEP_{ij}) = \beta_0 + \beta_1 log(GDP_i) + \beta_2 log(GDP_j) + \beta_3 log(S_{ij}) + \varepsilon$

Note s:

1. In Table 6, we reproduce Pearson's correlations, Spearman's rhos, Kendall's taus, and Normal copula dependence estimates, as well as air distances for comparison purpose. Panel A reports data for China-related pairs while Panel B reports data for U.S.-related pairs.

2. The air distance data comes from www.infoplease.com. The distance measure is defined as the air distance in statute miles between financial centers and it is scaled down after dividing it by 10⁵. In general, the greater is the air distance, the lower the dependence.

3. The average GDP data comes from International Monetary Fund, World Economic Outlook Database, October 2008. It is defined as the average Gross domestic product over 17 years (from 1991 to 2007, the same as our sample period) based on purchasing-power-parity (PPP) valuation in billions of current international dollars. (http://www.imf.org/external/pubs/ft/weo/2008/02/index.htm)

4. Panel C presents regression results of stock market gravity model, where the regression equation is $log(DEF_{ij}) = \beta_0 + \beta_1 log(GDP_i) + \beta_2 log(GDP_i) +$ $\beta_3 log(S_{ij}) + \varepsilon$. Standard errors are reported under the parameter estimates. F-statistics show that the three explanatory variables are jointly significant in all four regressions. We also test the null hypothesis of the coefficient of distance being -1 and we cannot reject the null at 10% significance level in all four regressions. P‐values are reported in parenthesis. R‐square is reported in the last row.

USA

Notes: z is the probability integral transform of residuals from each country's marginal model. These figures show sample autocorrelations of $(e - \bar{e})$, $(e - \bar{e})^2$, $(e - \bar{e})^3$ and $(e - \bar{e})^4$ for each country. This test is suggested by Diebold, Gunther and Tay (1998).

CHN/DEU

CHN/FRA

CHN/HKG

CHN/JPN

USA/CHN

Panel B

Notes: These two graphs exhibit the negative relationship between air distance and dependence measures. The air distance data comes from www.infoplease.com. The distance measure is defined as the air distance in statute miles between financial centers and it is scaled down after dividing it by 105. The dependence measures are defined as the absolute values of original dependence. Panel A describes China-related pairs while Panel B reports U.S.‐related pairs. Red dotted line represents air distance while solid lines represent various dependence measures, including Pearson's correlation, Spearman's rho, Kendall's tau, and the copula dependence estimates from our normal copula models.