Belief Formation: An Experiment With Outside Observers*

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Abstract

In this paper we investigate the necessary ingredients for an accurate model of belief formation. Using experimental data from a previous experiment, we bring in a new group of subjects whose job it is to predict the action choices of the subjects from the previous experiment. While the rules we consider are all, strictly speaking, adaptive (being based on past observables), we uncover some forward-looking, sophisticated behaviour on the part of our subjects. Going from less to more sophisticated, we find that the following are important components of the belief formation process: the history of play, payoffs (whether real or "imagined" in the sense of Camerer and Ho (1999)) of the observee (*i.e.,* the player whose actions our subjects are predicting) and the payoffs of the observee's opponent. The paper also documents the presence of subject-specific heterogeneity in both initial beliefs and, to varying degrees, almost all of the variables found to influence beliefs.

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1 Introduction

Consider a player in a two-person finite strategy game who has been repeatedly interacting with his opponent for $t-1$ periods. If his opponent has k strategies, what factors would our player take into account when forming his beliefs about the likelihood of his opponent using any one of his k strategies in period t?

This paper is motivated by the fact that despite the large attention given to learning models in the experimental (and theoretical) literature, relatively little attention has been given to models of belief formation.[1,2] Those models that do exist, fictitious play, weighted fictitious play (Cheung and Friedman (1997)), and other adaptive models, consider a very limited number of variables and mostly concentrate on the past actions of opponents or state variables. Few, if any, are forward looking or sophisticated in a significant way.

Such simplified models of beliefs seem unsatisfactory and have been shown to lack consistency at least with beliefs elicited from subjects using a proper scoring rule (see Nyarko and Schotter (2002a,b)). As a result we feel it is of interest to investigate what variables are empirically relevant to people when they form beliefs in a repeated strategic interaction. This is precisely the first goal of this paper: to determine what information subjects make use of when they form and update their beliefs. In doing this we ask the following questions. Do subjects just use past actions of opponents to update their beliefs or do payoffs also matter? If payoffs matter, does the entire payoff matrix matter or simply the payoffs of the person whose actions one is predicting? Are payoffs and actions of the previous period the most relevant variables people use or do distant actions and payoffs resonate as well? How sophisticated are subjects in updating their beliefs? Do subjects pay attention to whether a particular strategy is a best response to the action of his or her opponent in the previous period? The second goal of this paper is to see if there is any uniformity or consensus in the way people update their beliefs. This is obviously important if one seeks to build a single, unified model of belief formation.

To investigate the process of belief formation we employ a unique and new technique which is to bring subjects into the laboratory and show them, period by period, the time

[1]For example, in Camerer and Ho's (1999) EWA model agents do not form beliefs, but instead are attracted to certain actions based on a specific function of observed actions and payoffs. However as has been shown, a significant fraction of subjects in experiments are more sophisticated in their behaviour (see, *e.g.,* Camerer, Ho and Chong (2002) and Ehrblatt, Hyndman, Özbay and Schotter (2007), among others), making an examination of how subjects form beliefs seemingly important and interesting.

[2]There is also a large macro literature concerned with learning and how different learning rules lead to different outcomes. For example, Evans and Honkapohja (1999, 2001) show that the central bank may come to *learn* the high inflation Nash equilibrium, where as Sargent (1999) and Cho, Williams and Sargent (2002) have shown that if more weight is given to recent data, then the economy may periodically settle on a low inflation outcome. In this literature, all of the learning rules are adaptive in the sense that they are based on past data; however, since the authors focus on rational expectations equilibrium, beliefs also become important, making a study of how beliefs are formed all the more interesting.

series of a game that had previously been played by two real subjects. In each round these subject-observers are then asked to assign probabilities to the actions taken by one player in the next period of this interaction. They are then rewarded for their predictions using a proper scoring rule which compares their predictions to the actions taken by the players.

This method has two distinct advantages over the way beliefs have been elicited in the past. First, because one might be concerned that beliefs and action choices may interact with each other in uncontrollable ways (see, *e.g.*, Rutström and Wilcox (2004)), and because we are interested only in beliefs, the subjects in our experiments did not participate in a game, but merely made predictions about the actions of experimental subjects who participated in a separate experiment. This isolates the belief elicitation exercise from the choice exercise. Second, since each observer sees the same time series, we can collect many observations on how subjects form beliefs, while holding the time series that they experienced constant. This design feature aids us greatly in our ability to accomplish our second goal of quantifying subject-specific heterogeneity, since we have many subjects making predictions on the *same* time series of action choices.

In this paper we uncover a number of stylized facts about the way people update their beliefs. For example, we show that while history is an important factor (subjects are very likely to increase their belief on the previously chosen action), payoffs are also important: subjects are more (less) likely to increase their belief on the observee's previously chosen action if that action gave the maximal (minimal) possible payoff given the action of the observee's opponent. We also show that our observers are very likely to increase their belief on that action which would have been a best response to the opponent's choice, even if that action was not chosen. By itself, the former observation would suggest that our subjects held a fairly dim view of the actual players since it would imply that our observers view the players as making decisions based on inertia. However, in combination with the latter two observations, it demonstrates that our observers believe the actual players are motivated by economic considerations and are capable of learning. A reasonable first approximation would be that our observers model the actual players as EWA learners. However, as we show, this would be an incomplete classification since we also show that beliefs can often remain very volatile throughout the entire length of the game and that even the payoffs of the opponent of the player whose actions they are predicting matter. Neither of these features are captured by EWA or other similar models. Finally, we demonstrate that the process of belief formation is a fairly heterogenous one across people. This is true for all of the games we investigate and in virtually all of the variables that we find to be important components of the belief formation process. Moreover, often the primary source of heterogeneity is different in different games.

The rest of the paper is organized as follows. In the next section we provide details on our experimental design as well as details of the specific time series of actions about which our subjects were asked to form beliefs. In Section 3 we first try to show that the beliefs we elicited were meaningful in the sense that they were fairly predictive of actions. We then derive a simple notion of consensus amongst our subjects to see if the process by which people update is similar across people. Overall we find relatively low levels of consensus, except when the game converged very early. However, if we focus only on the belief on the previous period's chosen action, subjects' beliefs appear to move more consistently in the same direction.

Sections 4 and 5 are where we present most of our results. In the former section, we provide an analysis of initial beliefs of our subjects. This is important because it is unlikely that subjects will update beliefs in a similar fashion if they begin the process with very different initial beliefs. Hence, investigating the starting point for the belief formation process is important. Using a method of Haruvy (2002), we are able to estimate modes in initial beliefs. The behavior associated with the two most prominent modes correspond to level-1 and level-2 behavior (see, *e.g.*, Stahl and Wilson (1994, 1995) and Costa-Gomes, Crawford and Broseta (2001)). Level-1 behavior can be thought of as a uniform belief over the actions of the player being observed, while level-2 behavior corresponds to a degenerate belief on the best response to a uniform belief on the actions of the observee's opponent. Substantially fewer subjects have level-3 beliefs, and very few subjects believe that the Nash action will be chosen.

In Section 5 we seek to shed light on the above-mentioned questions concerning how subjects update beliefs. To do this, we first empirically identify those variables which have a significant impact on beliefs and then look at subject heterogeneity. Finally, in Section 6 we provide some concluding remarks. The appendix contains the experimental instructions.

2 The Experiment

Ehrblatt et al. (2007) conducted a number of experiments using two-person 3×3 games in which subjects reported beliefs and chose actions for a number of periods. Their experiments produced different choice sequences from different pairs of players, some converging to the Nash equilibrium while others not. In our experiments, subjects were recruited and brought into the lab. On their computer terminals, subjects saw the period-by-period replay of action choices for one pair of subjects who had previously participated in the experiments of Ehrblatt et al. (2007). In other words, we took the time series of actions of a pair in the previous experiment and played it out period by period. In the instructions, the subjects were informed that the games they were about to see were played in the past by NYU undergraduates so that ambiguity regarding the population was eliminated. The new experiment used the same language and followed the same basic procedures as the old experiments on which they are based. That is, our observers knew that the people they were observing played the game in a fixed pair for 20 periods and that they could see the payoffs of both players. The subjects were shown the time series of two games (one dominance solvable, the other not). Their task was to predict the actions of one of the players in this game for 20 periods as the actions in the time series were revealed to them period by period. Predictions were rewarded with the same quadratic scoring rule used in the Ehrblatt et al experiments. The experiment was programmed in z-Tree (Fischbacher, 2007).

Note that in this experiment subjects do not play a game but are spectators who were asked to make predictions, period by period, about the actions of one of the players whose behavior they were observing. The interesting feature of the experiment was that since all subjects observed the same time series we are able to study the belief formation process of subjects while controlling for the observed actions. In most other belief elicitation experiments that we know of, subject beliefs are elicited pair by pair so that the observed actions are not controlled. Two notable exceptions to this are Palfrey and Wang (2007), who show subjects sequences of choices made by subjects in the experiments of Nyarko and Schotter (2002b) in an attempt to see whether beliefs can be reliably elicited, and Offerman, Sonnemans and Schram (1996) who had a group of spectators predict the contribution of a group of participants in a public goods game; however, since spectators were "paired" with a specific participant, each spectator was actually predicting the contributions of a different group. In our design, the actions observed by all subjects are held constant so we can study the belief formation process in isolation and the consensus (if any) of observing subjects about beliefs.

The games that the subjects saw were as in Figure 1 and were chosen because they had the following properties:

- A unique pure strategy Nash equilibrium in the stage game, which is highlighted in the table.
- Nash payoffs are on the Pareto frontier.
- Payoffs in the Nash equilibrium were not symmetric.

The game labeled "DSG" was dominance solvable, while the game labeled "nDSG" was not. The precise time series of games that subjects saw is reported in Figure 2. In Table 1, we also report whether subjects were asked to predict the actions of the row or column player as well as the number of subjects in each session. As can be seen, subjects each saw two different games. In Session 1, neither of the games converged (indicated by the "-NC" suffix), while in Session 2, both games converged (indicated by the "-C" suffix). In the

Figure 1: Games Used in the Experiments

	$A1$	$A2$	$A3$
$A1$	51, 30	35, 43	93, 21
$A2$	35, 21	25, 16	32, 94
$A3$	68,72	45, 69	13, 62

(1.a) DSG

	$A1$	$A2$	$A3$
$A1$	12, 83	39, 56	42, 45
$A2$	24, 12	12, 42	58,76
$A3$	89, 47	33, 94	44, 59

(1.b) nDSG

dominance solvable game, convergence was in period 6, while in the non-dominance solvable game it was only in period 17.

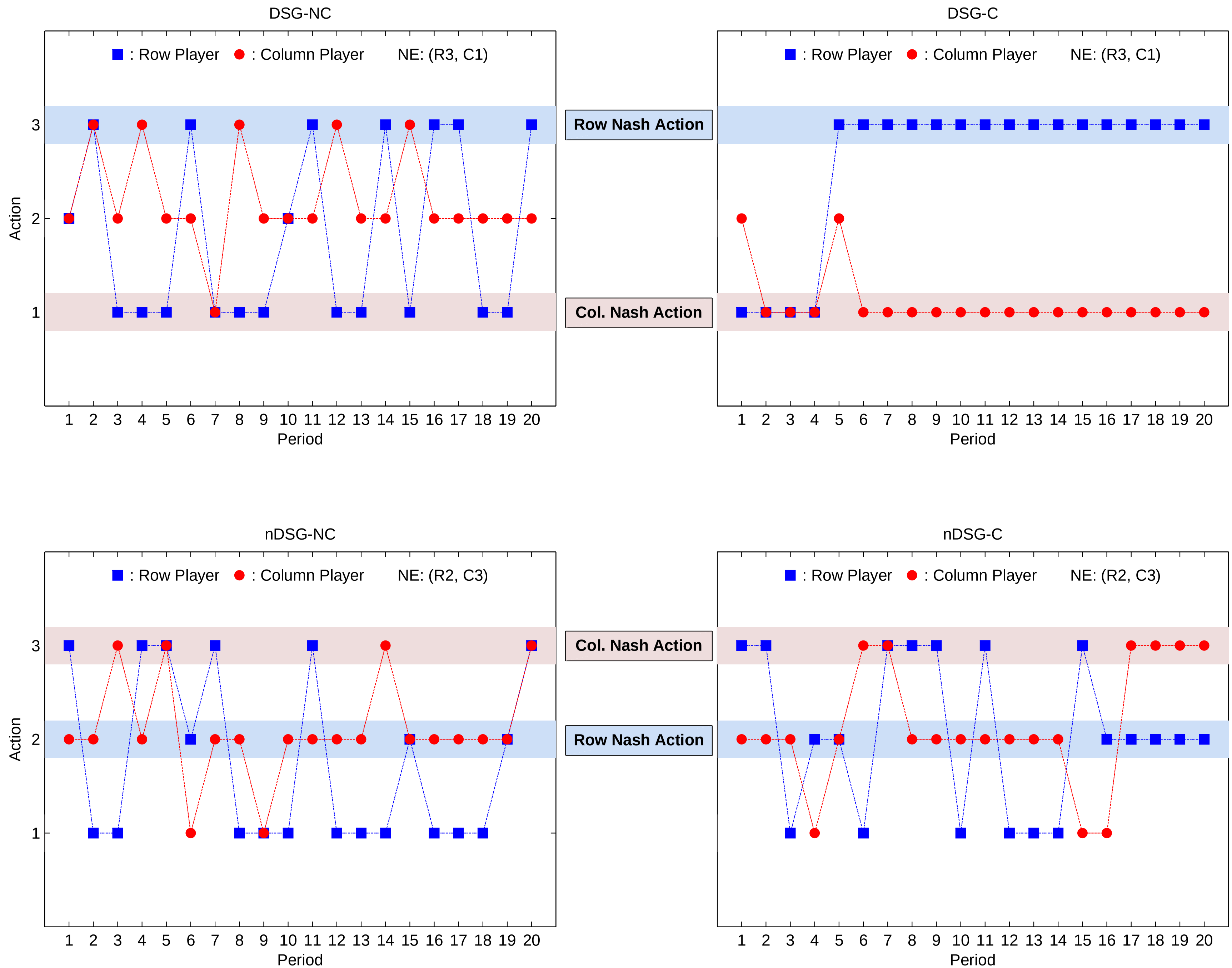
Table 1: Summary of Experimental Sessions

	Session 1		Session 2	
	DSG-NC	nDSG-NC	DSG-C	nDSG-C
Prediction	Column	Row	Column	Column
N	38	38	53	53

3 Beliefs & Consensus

Our main goal in this section is to determine how similarly or differently do subjects form beliefs. This is important because if a general theory of belief formation is to be constructed, then one would either need to know that all people update their beliefs similarly or, if people are heterogenous, how diverse and in what ways they are. Differences in beliefs across subjects could stem from a number of reasons. Two that we will focus on are initial beliefs and the updating process more generally. However, prior to exploring the reasons for heterogeneity in beliefs, we first show that the beliefs we elicited were meaningful, in the sense that our observers' beliefs actually have some predictive power. We then derive a simple notion of consensus in beliefs and try to empirically quantify the amount of consensus amongst our observers. Using our strongest measure of consensus, we actually find a high degree of discord amongst our subjects. Our subsequent analysis suggests that differences in initial beliefs and subject-specific heterogeneity with respect to the parameters which are shown to influence beliefs are the likely culprits. Despite this heterogeneity, we still find many common elements in the variables people use to update their beliefs even if they do so in somewhat different ways.

Figure 2: Time Series of Games Used in Experiments



3.1 Are Beliefs Meaningful?

Our first goal is to demonstrate that the beliefs we have elicited are meaningful. We do this by showing that the beliefs we elicited have a fairly significant amount of predictive power. Define the *hit rate* as follows. For each observer i and each period t, define:[3]

$$H_{it}(b_t(a), a_t) = \begin{cases} 1, & \text{if } b_t(a_t) > \frac{1}{3} \\ \frac{1}{3}, & \text{if } b_t(a) = \frac{1}{3} \, \forall a \\ 0, & \text{o.w.} \end{cases}$$

where $b_t(a)$ is the vector of beliefs at time t and a_t is the chosen action that period. Then, the hit rate is given by

$$\bar{H} = \frac{1}{20N} \sum_{i=1}^{N} \sum_{t=1}^{20} H_{it}(b_t(a), a_t)$$

The hit rate would be $\frac{1}{3}$ if all subjects had uniform beliefs each period. To the extent that $\bar{H} > \frac{1}{3}$, subjects elicited beliefs have predictive power. This information is displayed in Table 2. Notice that in all cases, the hit rates are substantially and significantly above $\frac{1}{3}$ (in all cases, $p < 0.01$), indicating that subjects are predicting with some accuracy the actions of the actual players. We take this as evidence that our subjects were generally trying to accurately predict behavior and that they were, to some extent, successful.

TABLE 2: Summary Statistics on Beliefs

Game	Hit Rate
DSG-NC	0.5439
nDSG-NC	0.5846
DSG-C	0.7805
nDSG-C	0.5887

3.2 A Notion of Consensus

Before examining the belief formation process in detail and trying to uncover the necessary ingredients of a compelling model of beliefs, we first try to capture the amount of consensus between subjects in the belief formation process. While there are many ways in which one

[3] Since subjects could not enter a belief of *exactly* $\frac{1}{3}$, we code all beliefs $b \in \{(b_1, b_2, b_3) : b_i \in [0.33, 0.34] \, \forall i\}$ as being the uniform belief.

could proceed, we choose to be very conservative. Define:

$$D_{i,t}^k = \begin{cases} 1, & \text{if } b_{i,t}^k - b_{i,t-1}^k > 0 \\ 0, & \text{if } b_{i,t}^k - b_{i,t-1}^k = 0 \\ -1, & \text{if } b_{i,t}^k - b_{i,t-1}^k < 0 \end{cases} \tag{1}$$

$D_{i,t}^k$ discretely measures the directional movement in the belief on action k by player i between time $t-1$ and time t. Next define

$$C_{ij}(t) = \begin{cases} 1, & \text{if } D_{i,t}^k = D_{j,t}^k \ \forall k \\ 0, & \text{o.w.} \end{cases} \tag{2}$$

That is, subject i's and subject j's beliefs are *in consensus* in period t if their beliefs move in the same direction (in all dimensions). This is why we say that our measure is conservative: we insist that the belief of each action to move in the same direction.[4] Finally define:

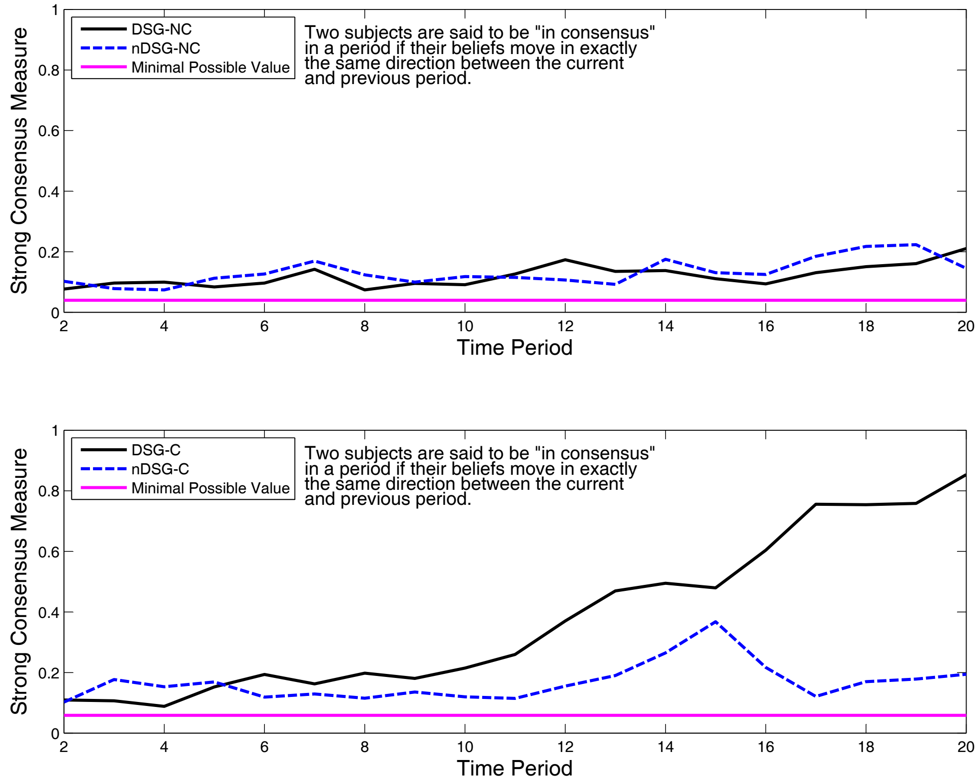
$$C(t) = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j \neq i} C_{ij}(t) \tag{3}$$

The results of this exercise are plotted in Figure 3.(a). One can see that the lowest value our consensus index can take is approximately $\frac{1}{13} \approx 0.0769$,[5] and as the reader can see, with the exception of the game DSG-C, which converged in period 6, the degree of consensus amongst our players is often not much above this value. In Figure 3.(b), we plot the results of a weaker notion of consensus where we only insist that the beliefs on the action that was chosen move in the same direction. Since there are only three possible ways in which the belief on a single action can change, the index is necessarily higher. However, the same pattern emerges, except for the game DSG-C, even this weaker notion of consensus is not much above the minimal possible value. Of course, this somewhat negative result should not be construed to mean that people use completely different models of belief formation. Even if subjects have the same model of belief formation, their initial beliefs or the corresponding parameters of the model may be different, which could lead to a lower level of consensus. We will argue this point below.

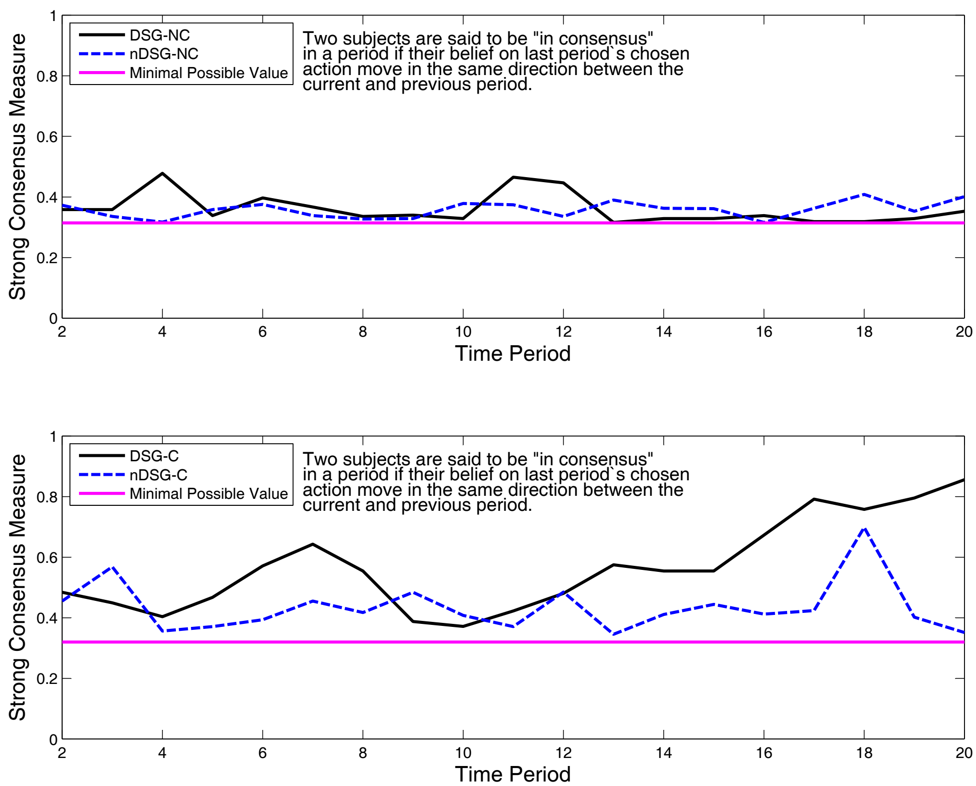
[4] We are actually slightly more lenient in two instances. Consider two subjects, i and j. Suppose, for example that for subject i $D_{i,t}^k = 0$, while for subject j, $D_{j,t}^k > 0$ (resp. $D_{j,t}^k < 0$). We say that beliefs are in consensus if, in addition $b_{i,t-1}^k = 1$ (resp. $b_{i,t-1}^k = 0$). That is if one subject had a belief of zero (or one) and left it unchanged, while another subject had a belief non-degenerate belief but moved it closer to zero (or one) we say they are in consensus.

[5] There are 13 possible ways that beliefs can change; therefore, if we have 13α subjects, where $\alpha \in \mathbb{N}$, one can show that the minimal value our index can take is $\frac{\alpha-1}{13\alpha-1} \nearrow \frac{1}{13}$. In general, the minimal value of the index is increasing in the number of subjects.

Figure 3: Consensus Indices



(a) Strong Consensus Index



(b) Weak Consensus Index

4 Initial Beliefs

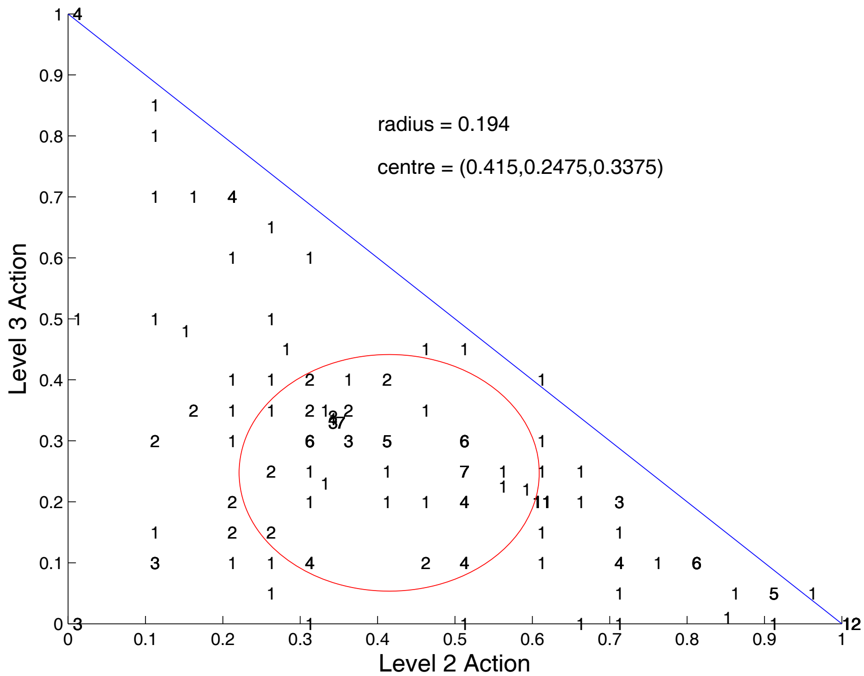
We now turn our attention to an analysis of initial beliefs, with our goal being to quantify the degree of heterogeneity and also provide some economic intuition for the central tendency of the initial beliefs. As stated earlier, one reason why the observed belief updating process of our subjects might differ is that they start the process out with different initial beliefs. If this is true, then it is likely to mask a more broad consensus about how subjects update beliefs since if subject-observer i's initial beliefs are very different than subject-observer j's, and they observe the same action on the part of the observee, then it is likely that subject i may react to that observation in a very different way than subject j despite the fact that such a difference might not appear if they both started out with the same belief vector. Ideally we would like to observe the updating of beliefs of subjects who start the process out with identical beliefs, but we did not try to induce a common initial belief in our experiments.

We organize our initial beliefs data according to the so-called *level-n* theory, which has been used in many forms by Stahl and Wilson (1994, 1995), Costa-Gomes et al. (2001), Haruvy (2002) and Costa-Gomes and Weizsäcker (2008), among others. According to this view, a belief of $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ would correspond to level-1 behaviour, while level-2 behaviour would correspond to a best-response to level-1, and so on. For example, in the game DSG, a level-2 initial belief for the column player would correspond to $(0, 0, 1)$ since $A3$ is the best response to a uniform prior over the row player's choices, while a level-3 belief would correspond to $(0, 1, 0)$.

Figure 4 pools the initial beliefs of our subjects from all four games. On the horizontal axis is the belief on the level-2 action, on the vertical axis is the belief on the level-3 action and the origin represents a degenerate belief on the Nash action. It appears that there are at least two distinct modes of initial beliefs corresponding to level-1 and level-2 behaviour, while there are very few observations near either the Nash action or the level-3 action. Notice also that the smallest circle containing at least half of the observations is centred very close to the level-1 belief, biased slightly in the direction of level-2 beliefs. Using the kernel density estimation method suggested by Haruvy (2002), Figure 5 lends further support to our claim of multiple distinct modes in beliefs, many of which appear consistent with *level-n* reasoning.[6] However, we would like to determine how many of the modes are "real" in a statistical sense. To do this, we conduct the global modes test using the same bootstrap procedure as Haruvy (2002).

[6] Specifically, we estimate the probability density function of initial beliefs with $\hat{f}(x) = \frac{1}{nh^2} \sum_{i=1}^{n} K\left(\frac{x-X_i}{h}\right)$ where $K(u)$ is a 2-dimensional kernel function (in our case, either the Gaussian or biweight kernels). The main estimation difficulty is due to the triangular nature of the domain. To overcome this, the data are reflected about some (in the case of the biweight kernel) or all (in the case of the Gaussian kernel) boundary hyperplanes. The exact procedure for obtaining the kernel density estimates and the bootstrap procedure to follow can be found in Haruvy (2002) and the references cited therein.

FIGURE 4: Scatter Plot of Initial Beliefs, Organized by "Level"‡



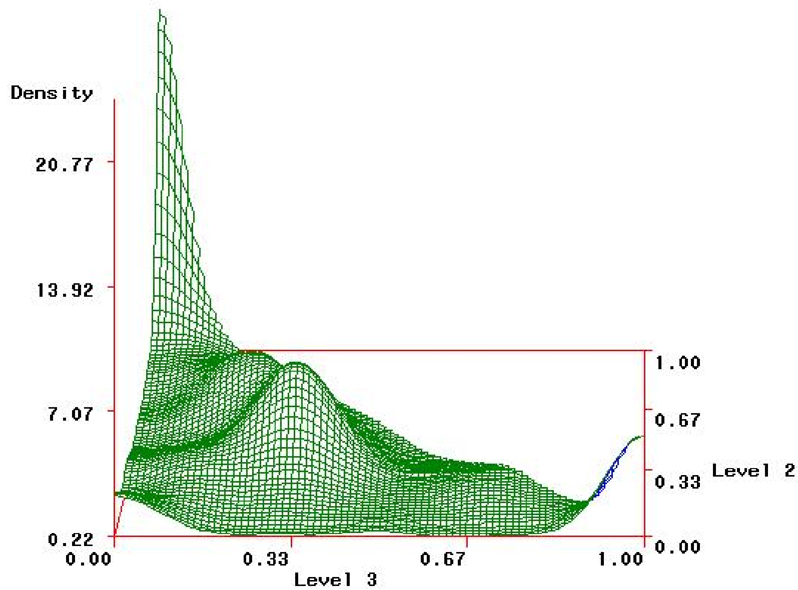
‡ The numbers correspond to the number of times a particular belief was observed in our sample.

Table 3 reports mode test results for our data. Specifically, we conduct a test of the one-sided null hypothesis that there are k or fewer modes against the alternative that there are more than k modes. The intuition for the procedure is as follows: we first search for the smallest bandwidth $\hat{h}_k$ for which there are k modes. Next we repeatedly resample the data and estimate, using bandwidth $\hat{h}_k$, the number of modes found in each of the bootstrap samples. A finding of k or less modes is taken as evidence against the null hypothesis. We conducted the mode tests using only the biweight kernel.[7] Restricting attention to bandwidths smaller than 0.5 (so that the unmodified kernel's support does not simultaneously intersect both boundaries), the null hypotheses of five and seven modes cannot be rejected at even the 10% level of significance, while three, four and six modes are rejected at the 5% level or better. In each case, four of the modes correspond to level-1, level-2, level-3 and Nash behaviours. In the five mode case, the other mode is at $(0.43, 0, 0.57)$, while in the seven mode case, in addition to the aforementioned mode, we also have modes at $(0, 0.56, 0.44)$ and $(0.28, 0.72, 0)$. These modes can be rationalised by assuming that the observer views the actual player as a hybrid of types (*e.g.,* with probability 0.43 a level-2 player and with probability 0.57 a Nash player).[8] Visually inspecting Figures 4 and 5, it seems that most

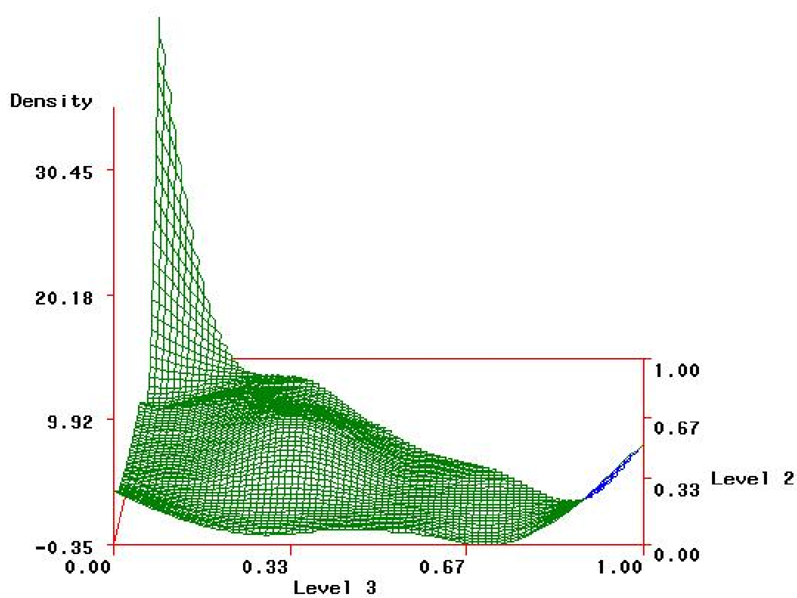
[7]Results would be very similar with the Guassian kernel, but are computationally more demanding due to the laborious reflection procedure.

[8]Recall that the first number in each vector corresponds to the level-2 initial belief, the second number

FIGURE 5: Kernel Density Estimates of Initial Beliefs (Organised by *Level* of Reasoning)



(a) Guassian Kernel



(b) Biweight Kernel

Table 3: Bootstrap Results – Mode Tests (Biweight Kernel)

One Mode		Two Modes	
1	0	1	0
		0	1
$p = 0.347;\ h = 0.735$		$p = 0.117;\ h = 0.714$	

Three Modes		Four Modes	
1	0	1	0
0	0	0	1
0	0	0	0
		0.55	0.45
$p = 0.017;\ h = 0.441$		$p = 0;\ h = 0.399$	

Five Modes		Six Modes	
1	0	1	0
0	1	0	1
0	0	0	0
0.40	0.30	0.40	0.31
0.43	0	0.45	0
		0	0.44
$p = 0.228;\ h = 0.294$		$p = 0.038;\ h = 0.280$	

Seven Modes	
1	0
0	1
0	0
0.36	0.31
0.55	0
0	0.44
0.28	0.72
$p = 0.133;\ h = 0.231$	

The parameter h is the bandwidth used.

of our subjects held either level-1 or level-2 beliefs, with substantially fewer beliefs being consistent with level-3 or Nash equilibrium.

to the level-3 initial belief and the third number to the Nash initial belief.

5 Components of a Belief-Formation Model

In this section we try to identify those features that seem to be important to our subjects as they form their beliefs and report them. In the analysis below, rather than directly positing a structural model of beliefs, we take a more reduced form, regression based approach. This allows us to focus on the impact of observables on beliefs without being tied to a specific (and restrictive) form, while also allowing us to study subject-specific heterogeneity. In addition, we are able to formally examine the qualitative features of existing belief and learning models (such as Cheung and Friedman and EWA).

More precisely, there are many ways in which one could imagine our observers form their beliefs. It could be that our observers are unsophisticated (or hold a particularly dim view of the player whose actions they are predicting) and simply look at the observed history of play and form their beliefs accordingly. If this were so, one could adopt Cheung and Friedman's (1997) model of γ-weighted beliefs. On the other hand, there may be other variables that our subjects take into account when forming their beliefs about the player whose actions they are observing such as what payoff he or she received in the previous period given the action choice, or the payoff that the actual player *could have* received had she chosen differently. Including these variables is analogous to assuming that our observer believes that the player being observed is an EWA learner (see Camerer and Ho (1999)). In this case, our observer calculates the attractions of the actual player and then forms her beliefs based on her account of these attractions. In this way then, while EWA is not exactly a model of beliefs, it can be reinterpreted thusly or at least used to investigate what components are relevant for a proper model of beliefs.[9] Of course, subjects could form beliefs in other ways entirely, or different subjects could focus on different observables in updating their beliefs.

In the sub-sections below we introduce each component that we think may be relevant to a belief formation model one at a time and investigate their significance.

5.1 *The Role of History: Time-Depedent Beliefs*

If observers form beliefs as in Cheung and Friedman (1997), or if they assume that the player they are observing is an adaptive learner (*e.g.,* EWA) and form their beliefs accordingly, in either case, beliefs should become less responsive to current information over time. Here, we test for this dampening effect over time and, because we use Cheung and Friedman's (1997) model of γ-weighted beliefs, we can also see the influence of history on beliefs. Consider then

[9] While the subjects in our experiment do not actually play the game, if players formed beliefs in this way, it would be consistent with the sophisticated types of Camerer et al. (2002) who assume that the naïfs are EWA learners.

the model of γ-weighted beliefs, which is usually expressed as:

$$B^k_{t+1}(\gamma) = \frac{1_t(a^k_j) + \sum_{u=1}^{t-1} \gamma^u 1_{t-u}(a^j_k)}{1 + \sum_{u=1}^{t-1} \gamma^u} \tag{4}$$

where k indexes the particular action and the parameter γ captures the relative weight of history. When $\gamma = 0$ (fictitious play), a decision maker only cares about last period's chosen action, while when $\gamma = 1$ (Cournot), a decision maker gives equal weight to the entire history of play. For any $\gamma \in (0, 1)$ a subject cares about history, but gives declining weight to more distant observations.

For our purposes, a recursive formulation will help illuminate a number of points regarding how beliefs change. Adopting the notation of Camerer and Ho (1999), we can express (4) recursively as follows

$$\begin{aligned} B^k(t) &= \tfrac{\gamma B^k(t-1) + 1_{t-1}(a_k)}{\gamma N(t-1)+1} \\ N(t) &= \gamma N(t-1) + 1 \end{aligned} \tag{5}$$

where $N(0)$ is given (or, sometimes, estimated by the data). Focus attention on the action that was chosen last period and observe that we may rewrite (5) as:

$$\Delta B^k(t) = \frac{1 - B^k(t-1)}{\gamma N(t-1) + 1} \tag{6}$$

Two things are now apparent from (6). First, $\Delta B^k(t) \geq 0$ unless $B^k(t-1) = 1$. Second, for any $\gamma \in (0, 1]$, and a given $B^k(t-1)$, $\Delta B^k(t)$ is decreasing in t, provided that $N(0) \leq \frac{1}{1-\gamma}$.[10] That is, beliefs on last period's chosen action become less responsive to current information as time passes.

Equation (6) suggests a simple regression model of the following form: $\Delta B(t) = \beta_t(1 - B(t-1)) + \epsilon$, where $\beta_t \geq 0$ is a time-dependent parameter (in particular, β_t is decreasing in t). Notice also that for any $\gamma \in (0, 1]$, $\frac{1}{\gamma N(t-1)+1}$ is reasonably well approximated by $\frac{c}{t^2}$, where c is a constant which will depend on γ. Motivated thusly, consider the following model:

$$\Delta B_i(t) = \beta_0 + \beta_1 B_i(t-1) + \beta_2 \left(\frac{1}{t^2}\right) + \beta_3 \left(\frac{B_i(t-1)}{t^2}\right) + \nu_i + \epsilon_{it} \tag{7}$$

where i is a specific individual and we focus our attention solely on last period's chosen action. Notice also that $\Delta B_i(t)$ is a double-censored variable; in particular, $\Delta B_i(t) \in [-B_i(t-1), 1 - B_i(t-1)]$. Therefore, we will employ a random-effects Tobit estimation procedure.

The above discussion leads to the following hypotheses:

[10] This is a commonly imposed restriction (see Camerer and Ho (1999)).

Hypothesis 1. *If there is a time dampening effect on beliefs, then:*[11]

(i) $\beta_0 = \beta_1 = 0$
(ii) $\beta_2 > 0$ *&* $\beta_3 < 0$

Hypothesis 2. *If the Cheung-Friedman model of beliefs is correct, then:*

(i) *Hypothesis 1 holds*
(ii) $\beta_2 = -\beta_3$

Table 4: Testing for Time Dependence in Beliefs (Random-Effects Tobit)

	Game			
Parameter	DSG-NC	nDSG-NC	DSG-C	nDSG-C
`cons` / $[\beta_0]$	0.182	0.172	0.186	0.233
	(0.023)	(0.022)	(0.027)	(0.022)
$B(t-1)$ / $[\beta_1]$	-0.440	-0.382	-0.004	-0.283
	(0.042)	(0.043)	(0.034)	(0.037)
$\frac{1}{t^2}$ / $[\beta_2]$	0.156	0.457	0.590**	0.493
	(0.257)	(0.402)	(0.240)	(0.302)
$\frac{B(t-1)}{t^2}$ / $[\beta_3]$	-1.395*	-1.452**	-3.535	-1.323**
	(0.831)	(0.716)	(0.638)	(0.671)
σ_ν	0.057	0.038**	0.081	0.059
	(0.013)	(0.016)	(0.012)	(0.015)
σ_ϵ	0.248	0.258	0.233	0.314
	(0.007)	(0.008)	(0.007)	(0.008)
N	722	722	1007	1007
$L.L.$	-125.2	-160.8	-193.7	-428.2

Highlighted cells significant at 1%; ** Significant at 5%; * Significant at 10%.

The results of our random-effects Tobit estimations are on display in Table 4 and appear to reject both hypotheses. First consider Hypothesis 1. Only for the game DSG-C do we **not** reject the hypothesis that $\beta_1 = 0$; in all other cases, β_0 and β_1 are significantly different from zero at the 1% level. Moreover, regarding part (ii), β_2 is significantly positive at the 5% level only for the game DSG-C (which is unsurprising since the game converged in period 6). Thus we find little evidence to support the conjecture that beliefs become less responsive to

[11] One can see this by examining (6) in more detail: $\Delta B^k(t) = \frac{1}{\gamma N(t-1)+1}\left(1 - B^k(t-1)\right)$. Therefore, $\beta_0 = \beta_1 = 0$, and $\beta_2 = -\beta_3 = c > 0$ where $c > 0$ is some positive constant and $\frac{c}{t^2}$ is meant to approximate $\frac{1}{\gamma N(t-1)+1}$.

new information as time passes.[12] Indeed, for our two non-convergent games, since $\beta_0 > 0$ and $\beta_1 < 0$, there is strong evidence to the contrary.

Consider now Hypothesis 2. Of course, since Hypothesis 1 is rejected, we can immediately reject this hypothesis as well. However, we can say more. For all games we can reject at the 7% level or better the null hypothesis that $\beta_2 = -\beta_3$ ($\chi^2_{DSG-NC}(1) = 3.57$, $\chi^2_{nDSG-NC}(1) = 6.54$, $\chi^2_{DSG-C}(1) = 37.96$, $\chi^2_{nDSG-C}(1) = 3.29$). Indeed, since $|\beta_3| > \beta_2$, it creates the possibility that the belief on last period's chosen action may actually decrease. As a further remark, even if β_0 and β_1 are different from zero, it is also reasonable to expect that $\beta_0 = -\beta_1$. However, in all four games we reject this hypothesis at the 5% level or better.[13] Again, since $|\beta_1| > \beta_0$, it suggests that the belief on last period's chosen action may decline. We will return to this point later.

Interestingly, it seems that the belief formation process is markedly different for convergent and non-convergent games — the only similarity being that neither is supportive of the Cheung-Friedman (or similar) model. In particular, for non-convergent games, the change in beliefs does not appear to dampen over time as evidenced by the significance of β_0 and β_1, and by the lack of significance in β_2 and (to a lesser extent) β_3. In contrast, for convergent games, the dampening effect is more present since β_3 is found to be significantly less than zero and since β_1 is relatively small in absolute value. Moreover, $|\beta_3| > \beta_2$ suggests that beliefs converge to degenerate Nash beliefs rather more quickly than theory would predict for convergent games. That is, once Nash has been played for a couple of periods, subjects seem to understand a sort of regime shift, and beliefs rapidly become degenerate. Finally, that β_2 for the game nDSG-C is significant and greater than β_2 for the game DSG-C is likely due to the later convergence of nDSG-C (period 17 vs. period 6) — hence convergence to degenerate beliefs is slower.

5.2 Other Components Affecting Beliefs

The results above generally lead us to conclude that, in order to adequately explain the belief formation process, one must consider more than the historical actions of the player whose actions are being predicted. We now turn our attention to this task.

5.2.1 Payoffs & Frequencies

As we said above, while Camerer and Ho's (1999) EWA model is a learning model and not a model of beliefs, it does include a number of variables that we think might be important to people as they form beliefs. Hence, for the purposes of analysis let us assume that our

[12]This result is not very surprising, especially for the non-convergent games; both Nyarko and Schotter (2002b) and Ehrblatt et al. (2007) show that beliefs are very volatile throughout long sequences of play.

[13]The respective $\chi^2(1)$ statistics are: 81.6, 55.6, 89.7 and 4.05.

observers believe that the subject's whose actions they are trying to predict are themselves EWA learners. Consider then their model and recall that attractions for each strategy, j, and player i are updated according to:

$$A_i^j(t) = \frac{\phi N(t-1)A_i^j(t-1) + [\delta + (1-\delta)\mathbb{I}(s_i^j, s_i(t))]\pi(s_i^j, s_{-i}(t))}{N(t)} \tag{8}$$

$$N(t) = \rho N(t-1) + 1 \tag{9}$$

Note that in the EWA model attractions are updated recursively as a result of the actions taken last period. Hence, we continue to focus our attention on the belief about the previous period's chosen action. First, observe that attractions (and so beliefs) must increase if last period's chosen action gave the highest possible payoff given the opponent's action choice. Second, provided that $\delta > 0$, attractions (and so beliefs) must decrease if last period's chosen action gave the lowest possible payoff given the opponent's action choice.

Of course, more than just realized payoffs may be at play in determining how beliefs are updated. For example, it could also be that subjects respond quickly to patterns and adjust their beliefs by a greater amount the more an action has previously been played. That is, the frequency of play, beyond that predicted by Cheung and Friedman, influences beliefs.

Formally, we augment (7) to take the new form:

$$\begin{aligned}\Delta B_i(t) &= \beta_0 + \beta_1 B_i(t-1) + \beta_2\left(\tfrac{1}{t^2}\right) + \beta_3\left(\tfrac{B_i(t-1)}{t^2}\right) \\ &+ \beta_4\mathtt{MAX}_{t-1} + \beta_5\mathtt{MIN}_{t-1} + \beta_6\mathtt{freq}(a_{t-1}) + \nu_i + \epsilon_{it}\end{aligned} \tag{10}$$

where $\mathtt{MAX}_{t-1}$ is a dummy variable taking value 1 if the period $t-1$ action gave the player her highest possible payoff given the action of her opponent, $\mathtt{MIN}_{t-1}$ is a dummy variable taking value 1 if the period $t-1$ action gave the player her lowest possible payoff given the action of her opponent and $\mathtt{freq}(a_{t-1})$ is the empirical frequency that action a_{t-1} has been taken up to period $t-1$.

Guided by (8), it is our conjecture that $\beta_4 > 0$ and $\beta_5 < 0$. That is, if the player got his/her maximal (resp. minimal) possible payoff given the action of his/her opponent, our subjects are more (resp. less) likely to increase their belief on last period's chosen action. We also expect $\beta_6 > 0$. That is, the more frequently an action has been played, the greater will be the change in the belief on last period's chosen action. Notice that none of these variables should have any affect on beliefs if our observers form beliefs based on observed history à la Cheung and Friedman while, if our observers view the actual player as an EWA learner, then $\beta_4 > 0$, $\beta_5 < 0$ and $\beta_6 = 0$.

When estimating (10), along with `MAX`, `MIN` and `freq`, we only include those variables which were previously found to be significant at the 5% level (see Table 4). The estimation

Table 5: Other Determinants of Beliefs (Random-Effects Tobit)

	Game			
Parameter	DSG-NC	nDSG-NC	DSG-C	nDSG-C
`cons` / $[\beta_0]$	0.161 (0.038)	0.040 (0.033)	0.027 (0.053)	0.299 (0.044)
$B(t-1)$ / $[\beta_1]$	-0.495 (0.037)	-0.579 (0.039)	N/A	-0.378 (0.037)
$\frac{1}{t^2}$ / $[\beta_2]$	N/A	N/A	0.553 (0.207)	N/A
$\frac{B(t-1)}{t^2}$ / $[\beta_3]$	N/A	-1.731 (0.342)	-3.468 (0.598)	-1.667 (0.455)
`MAX` / $[\beta_4]$	0.019 (0.023)	0.105 (0.022)	0.027 (0.031)	0.101 (0.025)
`MIN` / $[\beta_5]$	-0.083 (0.022)	-0.101 (0.024)	†	-0.076* (0.039)
`freq`(a_{t-1}) / $[\beta_6]$	0.105** (0.054)	0.429 (0.063)	0.187** (0.090)	0.147 (0.049)
σ_ν	0.060 (0.013)	0.043 (0.013)	0.088 (0.013)	0.066 (0.014)
σ_ϵ	0.243 (0.007)	0.231 (0.007)	0.235 (0.007)	0.299 (0.008)
N	722	722	1007	1007
$L.L.$	-111.36	-84.51	-183.97	-392.39

Highlighted cells significant at 1%; ** Significant at 5%; * Significant at 10%.

† Variable dropped due to collinearity with β_0.

results of (10) are reported in Table 5. Several things are notable about the results. First all of the estimated coefficients are of the expected sign. Second, payoffs matter in the belief formation process. Except for the game DSG-C, which converged quite early to the Nash equilibrium, payoffs seem to matter. That is, in the three other games, at least one of `MAX` or `MIN` has a significant coefficient at the 5% level with the expected sign. Third, in all four cases `freq` has a significantly positive coefficient. That is, the more frequently last period's action has been chosen in the past, the more one increases his/her belief on that action being played again. This suggests that subjects, upon observing an emerging pattern, are actually quick to update their beliefs accordingly. Finally, notice that σ_ν is significantly positive (at the 1% level) in all four games, meaning that subject-specific heterogeneity is still ever-present.

A Different Interpretation and Summary. If one looks at (10) from a slightly different perspective, one can gain additional insight. To make the argument simpler, assume that β_2 (as is always the case) and β_3 (as is the case in non-convergent games) equal zero. Suppose that last period's chosen action was neither the best (so that $\texttt{MAX} = 0$), nor the worst (so that $\texttt{MIN} = 0$) and that the action was never chosen before (so that $\texttt{freq} = 0$). Now ask, under what condition will one's belief actually increase? Working with (10), it is not difficult to see that $\Delta B(t) \geq 0$ if and only if $B(t-1) \leq -\frac{\beta_0}{\beta_1}$. That is there is a threshold: if the belief on last period's chosen action was initially low, then one's belief will increase, while if it was already quite high, then it may actually decrease. This does not seem too surprising for there was nothing particularly special about this action, except that it was actually chosen last period. With this is mind, we can now see the role of `MAX`, `MIN` and `freq` more clearly. For example, if last period's chosen action also gave the maximal possible payoff then, given that $\beta_4 > 0$, it raises the threshold for which the belief will increase: the high payoff provides justification for increasing one's belief on that action. Identical reasoning applies to `freq`, while the opposite holds for `MIN` — since $\beta_5 < 0$, if last period's chosen action gave the worst possible payoff, the threshold will decrease and one's belief is more likely to decline.

What can be concluded thus far? First, history matters: subjects are quite likely to increase their belief on last period's chosen action, and will increase their belief the more this action has been played. Second, payoffs matter: subjects increase (resp. decrease) their belief more on last period's chosen action if it gave the highest (resp. lowest) possible payoff given the action of the player's opponent. Third, even after controlling for payoffs and history, there is a substantial amount of subject-specific heterogeneity. Below, we will attempt to isolate those variables for which heterogeneity is the greatest. Finally, we find little evidence of a dampening effect in changes in beliefs over time. That is, beliefs in period 18 may be as volatile as beliefs in period 2, unless a clear pattern of behaviour has emerged, such as convergence to equilibrium.

5.2.2 *Best Responding & Opponent's Payoffs*

The results presented thus far have mainly focused on the belief about the previous period's chosen action. However, a good model of beliefs should also make predictions about those actions which were not chosen. Of course, instead of focusing on last period's chosen action, we could simply have estimated (10) for each of the actions.[14] While we don't report the results of such estimations, we do note that they are largely consistent with the above results. Indeed, besides obtaining a positive coefficient on a dummy for whether the action was played or not, we often see a positive coefficient on `MAX` and/or a negative coefficient on `MIN` and, though somewhat less often, a positive coefficient on `freq`. That `MAX` and `MIN` are often significant is particularly striking since the action need not have been chosen, which suggests that our observers believe that the actual players possess some *imagination* in the sense of Camerer and Ho (1999).

We follow this line of thought, though somewhat more selectively. Recall (8) above and, in particular, focus on δ. When $\delta > 0$, the payoffs of all actions, whether chosen or not, contribute to the updating of attractions. If the action was not chosen, the contribution is $\delta\pi(s_i^j, s_{-i}(t))$, while if it was chosen, the *undiscounted* payoff enters. Consider the case in which $\delta = 1$ so that players have full imagination and therefore equally weight all possible payoffs, whether realized or not. In this case, the attraction will increase the most on the action that *would have* received the highest payoff, given the strategy of the opponent — that is, on the best response to her opponent's action choice. Therefore, if observers think actions are chosen accordingly (and that $\delta \approx 1$), they should *increase* their belief on the action that was a best response to their opponent's previously chosen action.

So far this does not require a higher amount of sophistication beyond assuming that observee is an EWA learner. In particular, the observer need not make any conjectures about how the observee's opponent chooses actions. However, it may be that the above effect on beliefs is mitigated by concerns about how well or how poorly the observee's opponent fared in the previous period. For example, if a low payment was obtained, the opponent may be less likely to stick with his previously chosen action, making a best response to it (by the observee) less enticing.

Similarly to the analysis of Section 5.2.1, consider the following regression model:

$$\Delta B_i^{br}(t) = \beta_0 + \beta_1 \texttt{PLAY}_{t-1} + \beta_2 \Delta\pi + \beta_3 B_i^{br}(t-1) + \beta_4 \texttt{oMAX}_{t-1} + \beta_5 \texttt{oMIN}_{t-1} + \nu_i + \epsilon_{it} \quad (11)$$

where $\Delta B_i^{br}(t)$ is the change in the belief on that action which was a best response to the

[14] Observe that this would require 12 separate estimations: 3 actions per game times 4 games.

observee's opponent's period $t-1$ choice, $B_i^{br}(t-1)$ is the period $t-1$ belief on that same action, `PLAY` is a dummy variable for whether the observee actually played a best response in period $t-1$ and $\Delta\pi$ is the change in payoff if the observee were to play a best response to her opponent's previously chosen action.[15] Lastly, `oMAX` and `oMIN` take value 1 if the opponent received his maximal and minimal payoff, respectively, given the action of the observee.

Given our previous results that the previous chosen action is somewhat focal to our observers, we expect to see $\beta_0 + \beta_1 > 0$. That is, if the observee played a best response to her opponent's action choice in period $t-1$, we expect that our observers would be likely to increase their belief on that action in period t. On the other hand, if observers are more sophisticated then, at least when $\Delta\pi$ is relatively large, we would expect $\beta_0 + \beta_2\Delta\pi > 0$. That is, if the gain to playing a best response is relatively large, observers are more likely to increase their belief on that action in period t. Additionally, we also expect $\beta_2 > 0$ by itself. Finally, we might also expect to see $\beta_4 > 0$ and $\beta_5 < 0$.

The results of our random-effects Tobit estimations are on display in Table 6. Notice that in all four cases $\beta_0 + \beta_1$ is significantly positive at the 1% level.[16] Indeed, as can be seen in Table 6, for our non-convergent games β_0 is significantly positive — by itself indicating sophistication in the belief updating process. Next, observe that for all four games $\beta_2 > 0$ at the 1% level, implying that observers are more likely to increase their belief on the best response action the greater is the gain from best responding. Combined with β_0, this result implies that $\beta_0 + \beta_2\Delta\pi > 0$ for all values of $\Delta\pi$ for the non-convergent games. For the two convergent games, since $\beta_0 < 0$ matters are slightly different. If we evaluate $\Delta\pi$ at the mean (conditional on $\texttt{PLAY} = 0$), then $\beta_0 + \beta_2\overline{\Delta\pi}$ is not significantly different from zero at the 5% level for the game DSG-C and significantly positive at the 5% level for the game nDSG-C.

Consider now the remaining parameters. In all four games, we find $\beta_3 < 0$, which is consistent with the notion that subjects have a threshold initial belief, above which they do not upwardly adjust their belief based on new information but below which they do. With respect to `oMAX` and `oMIN`, our results are mixed: `oMAX` is only significantly positive for the game DSG-C, while `oMIN` is significantly negative for the two non-convergent games. Finally, notice that σ_ν significantly positive in all four games indicates that subject-specific heterogeneity is also present, and needs to be addressed.

One can think of the coefficient on `PLAY` as capturing the effect of immediate history on beliefs. On the other hand, $\beta_2\Delta\pi$ captures a more sophisticated aspect of belief updating — since the best response action wasn't actually chosen, to increase the belief on it requires imagination. An interesting question is, which of the two effects is more important? If $\beta_1 > \beta_2\Delta\pi$, one could argue that history is the dominant factor. However, we see that

[15] Of course, if the observee was already best responding then $\Delta\pi = 0$; in all other cases, $\Delta\pi > 0$.

[16] The respective $\chi^2(1)$ test statistics are: 46.8, 170.2, 37.6 and 187.2.

Table 6: Testing for Imagination and Sophistication in Beliefs

	Game			
Parameter	DSG-NC	nDSG-NC	DSG-C	nDSG-C
cons / $[\beta_0]$	0.123	0.204	-0.252	-0.062
	(0.047)	(0.032)	(0.060)	(0.055)
PLAY / $[\beta_1]$	0.070	0.143	0.463	0.410
	(0.046)	(0.028)	(0.045)	(0.050)
$\Delta\pi$ / $[\beta_2]$	0.003	0.003	0.021	0.004
	(0.001)	(0.001)	(0.005)	(0.001)
$B(t-1)$ / $[\beta_3]$	-0.374	-0.553	-0.178	-0.382
	(0.041)	(0.037)	(0.034)	(0.033)
oMAX / $[\beta_4]$	-0.034	0.001	0.103	-0.007
	(0.044)	(0.028)	(0.037)	(0.024)
oMIN / $[\beta_5]$	-0.120**	-0.107	†	-0.035
	(0.057)	(0.032)		(0.036)
σ_ν	0.081	0.095	0.089	0.099
	(0.015)	(0.015)	(0.014)	(0.015)
σ_ϵ	0.257	0.230	0.224	0.284
	(0.008)	(0.007)	(0.007)	(0.008)
N	722	722	1007	1007
$L.L.$	-180.20	-83.37	-183.26	-305.91

Highlighted cells significant at 1%; ** Significant at 5%; * Significant at 10%.

† Variable dropped due to collinearity with β_0.

this is not always the case. While this would appear to be the case for the convergent games (perhaps because of the emergence of an easily recognisable pattern — convergence to equilibrium), it is not the case in the non-convergent games: there are realisations of $\Delta\pi$, such that $\beta_2\Delta\pi > \beta_1$.

5.3 Sources of Subject-Specific Heterogeneity

In our regression analysis thus far we have consistently estimated σ_ν significantly positive, indicating that subject-specific heterogeneity is present. We now seek to uncover the sources of subject-specific heterogeneity. Of course, the least restrictive way to estimate (10) or (11) would be to use individual-level data. However, since we only have 19 observations per subject and want to estimate up to 6 parameters, this would surely result in inefficient estimates (especially since the random-effects Tobit model is highly non-linear). Instead we will proceed more modestly. Specifically, we ask the following: If we only allow for subject-specific heterogeneity on a single variable of interest, which variable will lead to the greatest improvement in fit?

Table 5 showed that the variables $B(t-1)$, $\frac{B(t-1)}{t^2}$, `MAX`, `MIN` and `freq` all play a significant role in the belief formation process. Therefore, to answer the aforementioned question, we re-estimate (10) five times — in each of those times, we allow for a subject-specific coefficient on one of the five variables found to affect beliefs.[17] For the first two games, in which we had 38 subjects, this exercise introduces 37 more parameters to estimate, while in the second two games, where we had 53 subjects, it introduces 52 extra variables to estimate.

The results are reported in Table 7. For each game, the table reports the log-likelihood (abbreviated by L.L.) for each of the less restricted models and also indicates whether σ_ν remains significant (indicated by $\surd$) or not (indicated by $\times$) at the 5% level. The highlighted cells indicate the variable that, when one introduces subject-specific coefficients on it, leads to the greatest improvement in fit relative to the baseline case. For example, in the game nDSG-NC, allowing for subject-specific coefficients on `MAX` increases the Log Likelihood by 65%, which is a much greater improvement than any of the other variables. Therefore, in answer to our question: in the game nDSG-NC, if one can only introduce subject-specific coefficients on one variable, it should be introduced on `MAX`. This is also true of the game DSG-C, while for the games DSG-NC and nDSG-C, allowing for subject-specific coefficients on the variable $B(t-1)$ leads to the greatest improvement in fit.

For each of the estimations reported in Table 7, we also conducted a joint test that all of the individual-specific parameters are the same. In general, we were easily able to reject these hypotheses at the 5% level or better. Two exceptions to this are as follows: for

[17] We do not report similar results for estimations of (11) which account for heterogeneity, simply noting that the results are consistent with the reported results based on (10).

Table 7: Accounting For Subject-Specific Heterogeneity‡

Source of Heterogeneity	Game							
	DSG-NC		nDSG-NC		DSG-C		nDSG-C	
	L.L.	σ_ν	L.L.	σ_ν	L.L.	σ_ν	L.L.	σ_ν
baseline	-110.98	$\surd$	-79.80	$\surd$	-176.01	$\surd$	-391.34	$\surd$
$B(t-1)$	-53.11	$\times$	-43.52	$\times$	-94.78	$\times$	-313.67	$\times$
$B(t-1)/t^2$	-83.51	$\surd$	-62.11	$\surd$	-128.27	$\surd$	-361.97	$\times$
`MAX`	-73.79	$\surd$	-28.13	$\times$	-90.26	$\times$	-330.87	$\surd$
`MIN`	-90.77	$\surd$	-44.41	$\surd$	N/A		-366.36	$\surd$
`freq`	-79.05	$\times$	-41.64	$\times$	-103.93	$\times$	-337.20	$\times$

‡ This table reports the Log Likelihood (L.L.) of a series of estimations for each game. For example, the highlighted cell in the first column indicates that if we estimate (10) but introduce subject-specific coefficients for the variable $B(t-1)$, then the log likelihood becomes -53.11. The $\times$ to the right indicates that σ_ν is not significant at the 5% level. In general, the highlighted cell in each column indicates that the corresponding variable leads to the greatest improvement in fit, for that game, amongst the five variables found to influence beliefs.

the games nDSG-NC, DSG-C and nDSG-C we could not reject the null hypothesis for the variable $B(t-1)/t^2$, while for the games DSG-NC and nDSG-C, we could not reject the null hypothesis for the variable `MIN`.[18] Therefore, heterogeneity is present in the most of the variables for all of the games that we considered.

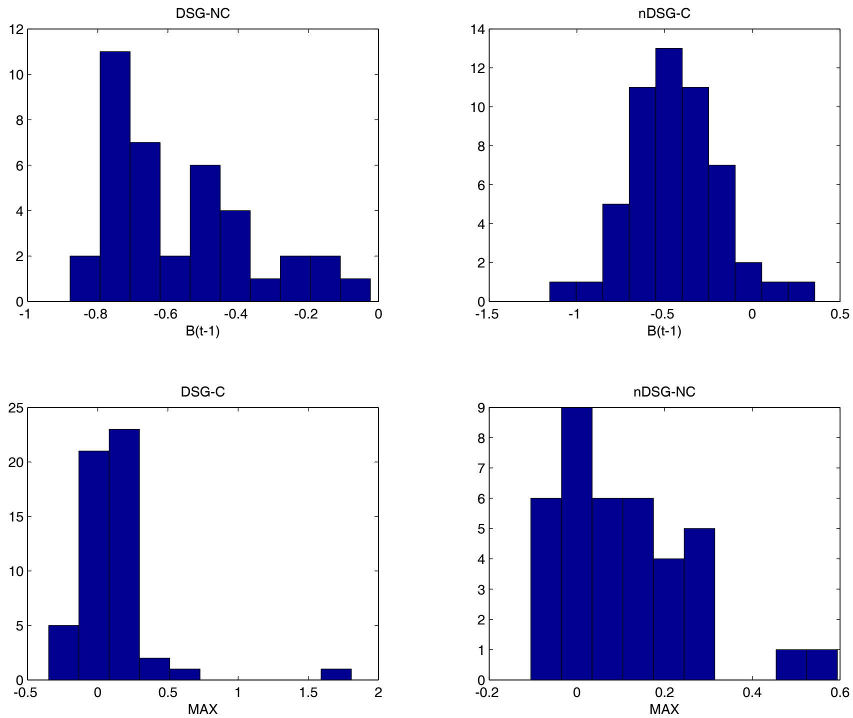
For each game, let us now focus on the variable for which heterogeneity (as quantified by Table 7) is most important. In Figure 6 we plot histograms of the subject-specific estimated coefficients of the variable for which we introduce heterogeneity. In all four cases it seems as though a great deal of variation in the estimated coefficients is present. Importantly, with few exceptions, all of the subject-specific coefficients have the predicted sign. Another way in which to quantify heterogeneity is with respect to significance. For the games DSG-NC, nDSG-NC, DSG-C and nDSG-C, respectively, 35 of 38, 11 of 38, 22 of 53 and 42 of 53 of the subject-specific coefficients are significant at the 5% level. Moreover, of the significant coefficients, all but 8 cases for the game DSG-C are of the expected sign (negative for $B(t-1)$ and positive for `MAX`). Thus, as one might expect, heterogeneity comes in two different forms: some subjects are more sensitive to the particular variable than others, and some subjects do not seem to be influenced at all by the particular variable.

6 Conclusions

We have reported the results of an experiment in which subjects acted as outside observers and predicted the actions of a different group of experimental subjects. We set out with two main goals for this paper: first, to understand how subjects update their beliefs and what

[18]For the games DSG-NC and nDSG-NC the 5% critical value is $\chi^2(37) = 52.19$, while for the games DSG-C and nDSG-C the 5% critical value is $\chi^2(52) = 69.83$.

FIGURE 6: Quantifying Heterogeneity: Histograms of Estimated Parameters



information they make use of in doing so and, second, to try to quantify subject-specific heterogeneity in the belief formation process. We believe that significant progress has been made on both fronts. In particular, we have shown that the following are very important factors influencing how subjects update their beliefs:

- **Past Actions**: Subjects are likely to increase their belief on the action that was chosen in the previous period; however, this tendency declines as the belief on that action become closer to 1 (*i.e.,* estimated coefficient on $B(t-1)$ negative).

- **Observee Payoffs**: Subjects are likely to increase their belief on the action chosen in the previous period if that action led to the highest possible payoff, given the choice of the observee's opponent, and are likely to decrease their belief on the action that was chosen in the previous period if that action led to the lowest possible payoff, given the choice of the observee's opponent (*i.e.,* estimated coefficient on `MAX` positive and on `MIN` negative).

- **Payoff Gain From Best Responding**: The larger the gain from best responding to the opponent's previous choice, the more our observers increase their belief on the action that was a best response to the previously chosen action (*i.e.,* estimated coefficient on $\Delta\pi$ positive).

- **Frequency of Past Actions**: The more frequently an action has been played in the past, the more do our observers increase their belief on that action (*i.e.,* estimated coefficient on `freq` positive).

- **Observee's Opponent's Payoffs**: Somewhat less robust than the payoffs received by the observee, the payoffs of his/her opponent also influence belief updating. In particular, if the opponent received a very high payoff, our observers were more likely to increase their belief on the best response to the opponent's action choice in the previous period. Similarly, if the opponent received a very low payoff, our observers are more likely to reduce their belief on the best response action (*i.e.,* estimated coefficient on `oMAX` positive and on `oMIN` negative).

While all of these factors are adaptive, since they are all backward-looking, they represent differing degrees of sophistication. If only past actions mattered, it would be as if the actual game doesn't have an influence on how decision makers form beliefs. However, when combined with knowledge of payoffs — both real and imagined — we see that our observers view the player's whose actions they are predicting, at the very least, as adaptive learners (and here, EWA might be a sensible first approximation). However, our results have shown that there is still more to belief updating. First, patterns appear to be important, with beliefs being updated very rapidly on frequently chosen actions. Second, in some cases, there is a further degree of sophistication: the payoffs of the observee's opponent also influence beliefs in predictable and intuitive ways.

Thus a successful model of belief formation must take into account these factors. However, such a model, as we have shown, will also be plagued by substantial amounts of subject-specific heterogeneity in both initial beliefs and the updating rules. Regarding the former, we found that there are essentially two large sub-populations: those with uniform (or level-1) initial beliefs and those with level-2 initial beliefs; that is, subjects who have a degenerate belief on the action that is a best response to a uniform belief. Higher levels of reasoning in initial beliefs, including the Nash belief, are much less frequent. If this were the only source of heterogeneity, it may not pose a substantial problem to one attempting to write a model of beliefs. However, as the results of Section 5.3 showed, there is subject-specific heterogeneity present in almost all of the variables which were found to influence beliefs. Moreover, there is heterogeneity within the heterogeneity. That is, in some games, heterogeneity in say `MAX` is more important than $B(t-1)$ and vice-versa; in some games, there may be heterogeneity with respect to payoffs, while in others not. What this means, is that the belief formation process itself is very sensitive to both the game and the history of play for the particular game: if a game converges, fairly quickly subjects seem to recognise this and follow a different process.

Of course, that subjects have different beliefs is not inherently negative. To some extent, modern financial markets hinge on different people holding different beliefs on a given asset, otherwise there would be many fewer trading opportunities. In the present setting, it would be quite interesting to see how one's beliefs would change if they were able to observe a signal on the beliefs of other subjects. One possible way to do this would be to first elicit beliefs in the usual way and then open a prediction market where subjects could buy and sell contracts on action choices which would pay a fixed amount if the action was chosen and zero if it was not chosen. In this way, the price on each action contract would reflect the market belief, which could then be contrasted with the initial belief. Moreover, given initial beliefs and the observed prices, it would be interesting to observe subjects' trading behaviour. This is one avenue of future research that we are now pursuing.

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Appendix A: Instructions

The following are the instructions used in the experiment reported in the paper.

General Instructions

Welcome and thank you for coming today to participate in this experiment. The purpose of this experiment is to learn how people make decisions in certain very simple settings.

After this experiment, another experiment will take place. The precise details of that experiment will be explained to you at the appropriate time. Depending on your choices you will earn money, which will be paid at the end of the experiment. The exact method of calculating your final payment will be described below.

We ask that you remain silent throughout the experiment. If, at any time, you have a question, please ask the session coordinator. Failure to comply with these instructions means that you will be asked to leave the experiment and all earnings will be forfeited.

In the experiment it is more convenient to work with points rather than dollars. At the end of the experiment, the total number of points earned will be converted to dollars. The exact conversion factor is the following:

$$20 \text{ points} = \$1.00$$

A Previous Experiment

In a previous experiment, we had two subjects play the following game for 20 periods.

	$A1$	$A2$	$A3$
$A1$	$51, 30$	$35, 43$	$93, 21$
$A2$	$35, 21$	$25, 16$	$32, 94$
$A3$	$68, 72$	$45, 69$	$13, 62$

One of the subjects had the role of the **row** player, while the other had the role of the **column** player. In each of 20 periods, the two subjects **simultaneously** chose an action — either A1, A2 or A3. The actions taken by the row and column players in each period determine the payoffs for that period. Each of the nine boxes above represent the nine possible action combinations. In each box, the **first** entry represents the payoff for the **row** player, while the **second** entry represents the payoff for the **column** player.

To understand how to calculate the payoffs for this game, suppose that the row player chose **A2** and the column player chose **A3**. In this case, the row player would have earned 32 points and the column player would have earned 94 points.

The subjects who have played this game before were recruited just as you were today by the CESS lab recruiting program. Hence they are NYU undergraduates just as you are. They played the game for 20 periods and we have recorded their choices in each of the 20 periods of their interaction. That means that in each of the 20 periods the row player has made one of his or her three possible choices A1, A2, or A3 as has the column chooser. Your task in this experiment is to predict the actions of the **COLUMN** player in each of the 20 periods of his or her interaction with the row player he or she was matched with. We stress that these two subjects were paired with each other for the entire 20 periods. We will now explain this task to you in more detail as well as how you will be paid for your decisions.

Predicting Other People's Choices

In each period, but before learning what actually happened, you will be asked the following three questions which will appear on the computer screen in front of you:

- On a scale from 0 to 100, how likely do you think it is that the COLUMN player will take action $A1$?
- On a scale from 0 to 100, how likely do you think it is that the COLUMN player will take action $A2$?
- On a scale from 0 to 100, how likely do you think it is that the COLUMN player will take action $A3$?

*Your response to each question must be a number between 0 and 100. Moreover, the sum of the three numbers that you provide **must** be **exactly** 100.*

For example, suppose that you think there is a 30% chance that the COLUMN player will take action $A1$, a 25% chance that the COLUMN player will take action $A2$ and a 45% chance that the COLUMN player will take action $A3$. In this case, you will enter 30 in the first box on the left-hand side of the screen, 25 in the second box and 45 in the and third box. The exact computer screen you will see is given below.

After you have submitted your predictions, you will be taken to a waiting screen on which you will see the actions actually chosen by both the ROW and the COLUMN players. Based on your predictions and the action actually chosen by the COLUMN player, you will earn experimental points according to a specific payoff function, which we now explain. Suppose your predictions are as in the above example. Furthermore, suppose that in the current period the COLUMN player actually chose $A2$. In that case your payoff for predicting the COLUMN player's action will be:

$$\text{Payoff} = 5\left[2 - \left(\frac{30}{100}\right)^2 - \left(1 - \frac{25}{100}\right)^2 - \left(\frac{45}{100}\right)^2\right]$$

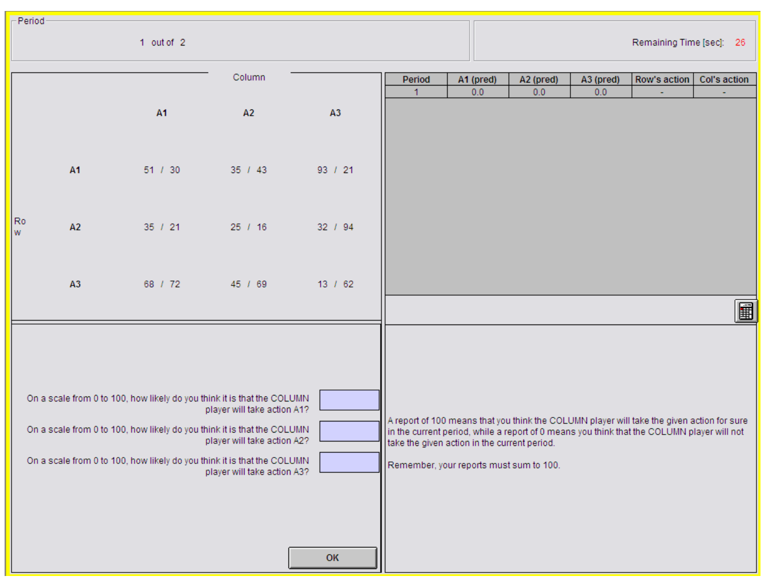
In other words, we will give you a fixed amount of 10 points from which we will subtract an amount which depends on how inaccurate your prediction was. To do this, we find out what choice the COLUMN player made. We then take the number you assigned to that choice – in this case 25% on $A2$ – subtract it from 100%, square it and multiply by 5. Next, we take the number you assigned to the choices not made by the COLUMN player – in this case the 30% you assigned to $A1$ and the 45% you assigned to $A3$ – square them and multiply by 5. These three squared numbers will then be subtracted from the 25 points we initially gave you to determine your final point payoff. Your point payoff will then be converted into dollars at the conversion factor as given above.

Note that since your prediction is made before you know the choices of both the row and column players, the best thing you can do to maximize the expected size of your prediction payoff is to simply state your true prediction about what you think the COLUMN player will do. Any other prediction will decrease the amount you can expect to earn as a payoff.

Note also that you cannot lose points from making predictions. The worst thing that could happen is you predict that the COLUMN player will choose one particular action (*e.g.,* $A2$) with 100% certainty *but* it turns out that the COLUMN player actually chose a different action (*e.g.,* $A3$). In this case, you will earn 0 points. In all other situations, you will earn a strictly positive number of points.

The Computer Screen

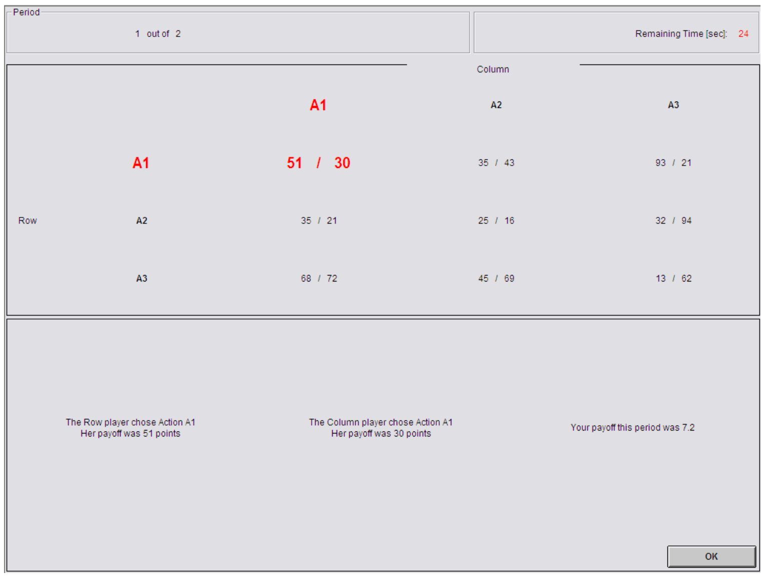
On your computer screen, in each period you will see the following screen:



You make your predictions by entering a response to each question on the bottom left-hand side of the computer screen. To submit your predictions simply press [OK]; you will then be taken to a waiting screen, which will be shown below. **Your responses to these three questions must each be numbers between 0 and 100 and the three numbers must sum to 100. Your response may contain**

at most 1 number after the decimal point. On the bottom right-hand side, you will see a reminder message as well as all of your previous predictions and a calculator button, while on the upper right-hand side of the computer screen you will see the actions chosen by the ROW and COLUMN players in each of the **previous** periods as well as your past prediction.

After you have made your predictions, you will be taken to a waiting screen. On this screen, you will see the actions that the ROW and COLUMN players **actually** made for that period as well as the number of experimental points they earned for that period. You will also see the number of points that you earned for making your predictions.



In this example, the row player chose action A1 and the column player also chose A1. For this period, the ROW player earned 51 points while the COLUMN player earned 30 points. At the beginning of the next round, at the right-hand side of the screen, it will be marked that each player chose A1 in period 1.

This concludes one round. In every round, except the 20$^{\text{th}}$, a new round will proceed in exactly the same manner.

Final Payment

Your final payment for the experiment will be determined as follows. We will sum the number of points you earned in each of the 20 rounds that you played. This number will then be converted back into dollars at the rate of $\$1 = 20$ points. This will be combined with your \$7 participation fee to come up with your final payment. Payments will be made privately at the conclusion of the two experiments.