Oil Price Shocks and the Optimality of Monetary Policy

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Abstract

The observed tightening of interest rates in the aftermath of the post-World War II oil price hikes led some to argue that U.S. monetary policy exacerbated the recessions induced by oil price shocks. This paper provides a critical evaluation of this claim. Within an estimated dynamic stochastic general equilibrium model with the demand for oil, I contrast Ramsey optimal with estimated monetary policy. I find that monetary policy amplified the negative effect of the oil price shock. The optimal response to the shock would have been to raise inflation and interest rates above what had been seen in the past.

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Since oil prices and interest rates have increased prior to almost every recession in the U.S. after the Second World War. However, energy constitutes a small share of the gross domestic product (GDP), so many observers doubt that oil price shocks themselves could generate sizeable recessions; instead monetary policy is blamed for exacerbating the recessions. A common explanation, such as that offered by Bernanke, Gertler, and Watson (1997), states that the Federal Reserve raises interest rates too much in response to high oil prices, which depresses economic activity, beyond the negative effect of oil price shocks. In this paper, I critically reevaluate this statement,and similar to Bernanke, Gertler, and Watson (1997) and Leduc and Sill (2004), find that monetary policy indeed amplifies the negative effects of oil price shocks. However, an optimal monetary policy response to oil price shocks would raise inflation and interest rates above what has been seen in the past. This latter finding differs drastically from the argument provided by Bernanke, Gertler, and Watson (1997), whereas the former finding contradicts Leduc and Sill (2004) who claim that a better monetary policy is one that stabilizes inflation.

I obtain these results using an estimated theoretical model that features demand for oil. With this model, I can compare the dynamic of the baseline economy with the dynamics of a model, in which monetary policy is socially optimal.[1] The theoretical model combines a medium-scale Dynamic Stochastic General Equilibrium (DSGE) model with a version of the oil demand model by Finn (2000), where oil is necessary for capital use. However, unlike Finn, I assume that oil price is a nonstationary process. The DSGE model features habit formation through consumption, investment costs, endogenous depreciation rates, and sticky prices and wages.

I also estimate the key structural parameters of the model by applying the Markov Chain Monte Carlo (MCMC) estimation approach of Chernozhukov and Hong (2003) to the Impulse Response Function Matching estimator. To obtain empirical impulse responses, I identify an oil price shock in a structural vector autoregressive (SVAR) model of the U.S. business cycle. The SVAR model predicts that in response to an oil price shock, GDP, investments, hours, capital utilization, and real wages fall, while the interest rate and inflation rise. These findings are economically intuitive and in line with existing research (Blanchard and Gali (2007), Dhawan and Jeske (2007), Herrera and Pesavento (2007), Peersman (2005), Leduc and Sill (2004), Rotemberg and Woodford (1996), and Hamilton (1983)).

This research engages most closely with the arguments of Bernanke, Gertler, and Watson (1997,2004) who claim that the large negative effect of historical oil shocks on the U.S.

[1] I focus on the optimal monetary policy from a timeless perspective, *à* la Woodford (2003).

economy is not due to the oil shocks but rather is the result of the systematic tightening of monetary policy in response to oil price increases. To evaluate the effects of oil price shocks that are due to the systematic monetary policy response, Bernanke, Gertler, and Watson (1997) run counterfactual experiments in the VAR model that identifies the oil price shock. The experiments eliminate monetary policy by setting the coefficients of a monetary policy equation in the VAR to zero; thus the authors can study how the response of output to oil shocks changes in this restricted model. Bernanke, Gertler, and Watson (1997) further show that systematic monetary policy is responsible for a substantial fraction of the output drop.[2]

However, empirical VAR models may not be suitable to policy experiments, according to the Lucas critique. That is, the estimates of the coefficients would differ under alternative policies, especially if those policies are very different from the observed ones. Trying to minimize the distortion associated with the Lucas critique, Bernanke, Gertler, and Watson (2004) consider another counterfactual: a *temporary* shutdown in the response of the Federal funds rate to an oil price shock. However, Carlstrom and Fuerst (2006) argue that the Lucas critique must be quantitatively important there as well. In this paper, I overcome the Lucas critique by running counterfactual experiments within a theoretical model of oil price shocks that replicates the predictions of my empirical SVAR model.

Leduc and Sill (2004) and Carlstrom and Fuerst (2006) study a related question in the context of a theoretical model. To evaluate the contribution of systematic monetary policy to the observed output drop after an increase in the price of oil, these authors compare monetary policy rules to an alternative unresponsive, or "constant," monetary policy.[3] Leduc and Sill (2004) find that monetary policy is responsible for 40 percent of the cumulative output drop over five years. Carlstrom and Fuerst (2006) estimate that monetary policy is responsible for 30 percent or less of output drop over a two-year period, depending on the "neutral" monetary policy in place. They emphasize the importance of defining a "neutral" policy correctly. In the context of a theoretical model, it seems more appropriate to compare monetary policy with an alternative that might do the least harm to the society trying

[2]Bernanke, Gertler, and Watson (1997, 2004) find that systematic monetary policy explains almost all of the output drop and attribute approximately 50 percent of the output drop to systematic monetary policy.

[3]According to Leduc and Sill (2004), "constant" policy is a k-percent money growth rule. Carlstrom and Fuerst (2006) find that the estimated output effect of a given monetary policy rule depends crucially on the choice of "constant" monetary policy. In addition to the money growth peg considered by Leduc and Sill (2004), Carlstrom and Fuerst (2006) suggest two other alternatives for constant policy: an interest rate peg and Wickselian monetary policy, which adjusts the interest rate so that the real economy behaves as if there were no nominal rigidities. This policy, however, is not optimal from a welfare point of view because it does not consider that nominal rigidities generate output losses.

to accommodate oil price shocks. Unlike Leduc and Sill (2004) and Carlstrom and Fuerst (2006), I therefore consider the effect of the systematic monetary policy compared with a policy that is optimal from a welfare point of view. I show that, indeed, optimal policy is associated with a smaller output drop than the model predicts. Leduc and Sill (2004) claim they could not find a monetary policy rule that would help to fully eliminate the negative effect of the oil price shock on output. I demonstrate that such a policy is not desirable from the welfare point of view.

Another drawback of the models presented by Leduc and Sill (2004) and Carlstrom and Fuerst (2006) is that their conclusions derive from calibrated, rather than estimated models. For this paper, unlike extant literature, I estimate the parameters of the model to match empirical evidence about the effect of an oil price shock. Moreover, unlike most theoretical papers that model oil prices, I introduce price as a growing, rather than a stationary process. This assumption is in line with most empirical studies in this area.[4] Overall, the estimated model describes the empirical evidence more precisely than have prior theoretical models of the oil price shock.

The remainder of this paper is organized as follows: in Section 1, I describe the empirical strategy to identify the oil price shock. Section 2 presents the theoretical model. In Section 3, I describe the estimation strategy and the results of the estimation of the model parameters. In Section 4, I explain the propagation mechanism for the oil price shock. Section 5 focuses on the monetary policy contribution to the effects of the oil price shock, and Section 6 concludes.

1 An Empirical Model of the Effects of Oil Price Shocks

To estimate the effects of the oil price shock, I rely on an SVAR model. In this model, I use quarterly data about the price of oil P_t^{oil}, output GDP_t, labor H_t, GDP deflator P_t, capital utilization CU_t, real wage rate W_t, personal consumption expenditures C_t, investment I_t and the nominal interest rate R_t over the period 1954:III - 2006:IV.[5] Table 1 provides more details about the data.

I assume that the vector of $n = 9$ random variables Y_t evolves according to the following dynamic process

$$A^0 Y_t = \alpha + A(L) Y_t + \epsilon_t,$$

[4] See Blanchard and Gali (2007) and Bernanke, Gertler, and Watson (1997), among many others.

[5] The choice of the initial date is determined by data availability.

where A^0 is a square matrix of the size 9×9, α is a constant vector of the size $n \times 1$, $A(L) = A_1 L + A_2 L^2 + ... + A_k L^k$, the size of Y_t is 9×1, and k is the number of lags of VAR. Y_t consists of

$$\{\Delta \left(\frac{P_t^{oil}}{P_{t-1}^{oil}}\right)^+, \quad \Delta \log(\frac{GDP_t}{H_t}), \quad \Delta P_t^{GDP}, \quad CU_t \ , \quad H_t, \quad \frac{W_t}{GDP_t}, \quad \frac{C_t}{GDP_t}, \quad \frac{I_t}{GDP_t}, \quad R_t\},$$

with all the variables as logarithms except for the interest rate. The wage rate, consumption, and investment enter Y_t in logarithms as the ratio to GDP to accommodate the long-term growth of these variables.

The Federal funds rate captures the monetary policy reaction to changes in macroeconomic indicators.[6] The Federal funds rate follows the block of macroeconomic variables that presumably do not react contemporaneously to monetary policy changes. This assumption is common in literature estimating monetary policy shocks. I find that the predictions of the model are robust to the position of this variable in the Y_t vector.

The first variable in the SVAR model identifies the oil price shock. Following much research on the asymmetric effects of oil price drops and increases,[7] I include only the increases of the nominal price of oil as the first variable in the VAR. Namely, I substitute negative values with zeros in the variable of the logarithm of oil price growth. Mork (1989) shows that the effect that occurs after increases in the oil price is greater in magnitude than the effect observed after drops in the oil price. Also, Hamilton (2003) finds support for the hypothesis of nonlinearity in the effect of oil prices on the U.S. economy.

Hamilton (1996) also suggests a different oil price indicator to take into account asymmetry in the effect of oil prices, defined as either the difference between the current price of oil and the maximum oil price over the previous year or 0, whichever is greater. The reason Hamilton uses this oil price variable is that the increases observed after 1985 normally have served to correct the previous quarter's oil price drops, and such corrections should have little effect on the economy. Hamilton's indicator helps eliminate such episodes of oil price increases. Defined in this way, Hamilton's oil price variable can capture the negative effect of the price of oil on GDP in a bivariate regression. I use a simpler oil price indicator, *a lá* Mork, because the results of the SVAR model change only marginally with Hamilton's indicator. Thus, the difference between the effect of small and large oil price increases smoothes out in a larger-scale model.

[6] Most VAR models use Federal Funds Rate as indicator of monetary policy (See, for example, Altig, Christiano, Eichenbaum, and Lindé (2004), Bernanke, Gertler, and Watson (1997) among others.) Bernanke and Blinder (1992) and Bernanke and Mihov (1995) show that the Federal Funds rate is a good indicator of the policy stance after 1966.

[7] See Hamilton (2003), Hooker (1996), Lee, Ni, and Ratti (1995), Mork (1989) among others.

However, in contrast with Mork (1989), I use the nominal price of oil to identify the oil price shock. Due to discounting by the price level, the real price of oil is subject to other innovations, such as technology shocks, preference shocks, and so forth. I include the nominal as opposed to the real oil price in my model because I want to identify the macroeconomic effect of the innovation in the nominal oil price.[8]

To identify the exogenous component of the oil price growth process, which I call the oil price shock, I impose short- and long-run restrictions on the dynamics of the oil price variable. Namely, the indicator of the oil price shock may respond to endogenous variables in the VAR. However, unexpected movements in other variables can not affect the nominal oil price contemporaneously (short-run restriction), and no shocks other than the oil price shock can have a long-lasting effect on the nominal price of oil (long-run restriction). The short-run restriction makes it impossible for the shock to affect the economy in the first quarter, which supports the idea of a long lag in the effect of the oil price shock on the economy. For example, Hamilton and Herrera (2004) notice that the major effect of oil price shocks can be observed about one year after the shock. The long-run restriction on the effect of oil recognizes that oil is a non-renewable source of energy, so in the end, the price of oil will be determined by its scarcity. The identification strategy that I use produces an overidentified VAR model. It is supported by tests of overidentifying restrictions versus other plausible alternatives.

As an alternative identification strategy for the oil price shock, I also test the assumption of a fully exogenous oil price process. Theoretical research usually treats the oil price process as exogenous. At the same time, empirical evidence questions the exogeneity of the oil price relative to an economy as large as the Unites States.[9] I find that treating of the oil price as an exogenous process results in impulse responses that are only marginally different from the responses in the SVAR model. However, this identification scheme is not supported by the test of overidentifying restrictions.

Among other alternative identifying restrictions, I consider a long-run and a short-run identification of the oil price shock separately. I find that inducing the short-run restriction is crucial for obtaining the statistically significant impulse response functions that are intuitive from an economic point of view.

I estimate the SVAR model with three lags, that is, an average between the two lags suggested by Bayesian and Hannan-Quinn information criteria and the four lags predicted

[8]Rotemberg and Woodford (1996) and Blanchard and Gali (2007) pursue the same idea.

[9]See Barsky and Kilian (2001) and Kilian (2008).

by the Akaike information criterion. The responses of the model variable are qualitatively similar for models estimated with two, three, or four lags.

The impulse responses to a 10 percent oil price shock, as predicted by the SVAR model, appear as purple dashed lines in Figure 1. The dashed-dotted purple lines in the figures indicate the 90 percent confidence bands, computed using a bias-adjusted double bootstrap procedure (Kilian (1998)) based on $10,000$ draws. I show responses as percentage deviations from the trend, except for the interest rate and inflation. The responses for annualized inflation and the Federal funds rate appear as percentage deviations from their mean values. As Figure 1 depicts, GDP, hours, investments and capital utilization fall in response to a 10 percent oil price shock, with U-shaped responses and a trough occurring approximately two years after the shock. Inflation and the Federal funds rate rise, with peak approximately four quarters after the shock, when the Federal funds rate reaches its maximum response of approximately 0.2 percent, and inflation reaches 0.3 percent a year. These results are in line with empirical estimates of the effect of the oil price shock.[10] Real wages and consumption fall, though the consumption response is not significantly different from 0 with a probability of 90 percent.

To determine the role of oil price shocks in fluctuations of macroeconomic variables, I calculate the contribution of the shocks identified by the proposed SVAR model. The numerical results appear in Tables 2 and 3. Table 2 presents a variance decomposition based on historically observed shocks. Each number in the first column of the table estimates the variance produced by the historical sequence of the estimated shock process as a fraction of the unconditional variance of the series. The second column shows the contribution of the oil price shock to the unconditional volatility of the time series. Table 3 shows conditional standard deviations of macroeconomic variables, as well as the ratio of conditional to unconditional variance predicted by the SVAR model.

The results of the variance decomposition exercise suggest that the oil price shock is not the primary, but is still an important source of business cycle fluctuations. As Table 2 reveals, the contribution of the shock to output volatility is approximately 8 percent, similar to the results provided by Blanchard and Gali (2007).[11] The contribution of oil price shocks to the fluctuation of macroeconomic variables is less than that of the technology shocks

[10]See Leduc and Sill (2004), Peersman (2005), and Dhawan and Jeske (2007), among others. The estimated effect of the oil price shock on the Federal funds rate is larger (0.7%) according to Bernanke, Gertler, and Watson (1997) and Hamilton and Herrera (2004), possible because they work with monthly data series.

[11]The results in the second column of Table 2 are comparable to the squared results in Blanchard and Gali's Table 4, because they show the relative standard deviations rather than relative variances.

found by Altig, Christiano, Eichenbaum, and Lindé (2004)–13 and 15 percent for neutral and investment-specific shocks, respectively.[12] Fisher (2007) suggests a much larger estimate of the contribution of investment-specific shocks to the business cycle. However, his results rely on a model with a fewer variables, so the contribution ascribed to each shock should be larger.

2 A Theoretical Model of the Effect of an Oil Price Shock

Many authors have tried to model the effects of oil shocks, yet I know of no study that addresses the problem in a model that can replicate the observed effects of oil price shocks. Early attempts to modify the standard Real Business Cycle (RBC) model by adding oil[13] as another production factor revealed that the share of oil expenditures in GDP was too small to produce significant output drops after an oil price shock. Subsequent research has suggested some improvements; for example, Rotemberg and Woodford (1996) show that imperfect competition can help generate sizeable output drops. In their model, countercyclical markups cause GDP to drop by 2.5 percent in response to a 10 percent oil price shock, whereas models with perfect competition generate output drops of only 0.5 percent. Finn (2000) suggests that a standard RBC model can produce drops in output and wages as large as in Rotemberg and Woodford's imperfect competition model, if the utilization of capital is accompanied by oil usage and the capital depreciation rate varies with the degree of capital utilization. Aguiar-Conraria and Wen (2007) rely on increasing returns to scale in a monopolistically competitive intermediate goods sector. The multiplier-accelerator propagation mechanism of an oil price shock in their model thus is very similar to the monopolistic markups of Rotemberg and Woodford. They also use the amplifying effect of the variable depreciation rate assumption. Both these features improve the ability of energy price shocks to generate sizeable recessions.

Among alternative models of the energy sector, Atkeson and Kehoe (1999) and Wei (2003) rely on a putty-clay mechanism of capital formation and have the potential to explain asymmetric effects of positive and negative oil price shocks. Atkeson and Kehoe (1999) use energy

[12]Estimates of the contribution of technology shocks to business cycle fluctuations are very different across empirical studies. Altig, Christiano, Eichenbaum, and Linde's (2004) is closest to the SVAR model considered herein, which makes these comparisons reasonable.

[13]Because oil and other sources of energy are close substitutes, I use the term "oil" broadly to describe a generalized good that provides energy.

intensity of production as a putty-clay factor to study the different substitutability degrees of energy in the short- and long-run. Atkeson and Kehoe (1999) document that, compared with more standard putty-putty type models, their model generates an even smaller response of output to an oil price shock.[14] Wei (2003) uses a putty-clay model with energy in the production function and variable capital utilization to study the relationship between oil price shocks and stock market prices. Although the response of wages is large enough to account for the effect of the 1973-74 oil price shock, the response of the value added to a 10 percent oil price increase is approximately 0.5 percent.

Although putty-clay models provide some insights into the asymmetric effect of oil price shocks, they are not better suited to study correlation between the oil price and business cycles than more standard RBC models. These models are also difficult to implement in quantitative research due to the high dimensionality problem. Because the focus of this paper is on the recessionary effect of high oil prices, rather than on asymmetry in the effect of the oil shock, I employ a more standard strategy to model oil.

I combine a medium-scale DSGE model with Finn's (2000) approach to modeling oil with the aim of presenting an empirically plausible model of the macroeconomic effect of oil price shocks. In this model, the features inherited from DSGE modeling provide flexibility for generating responses that better fit the responses of the data.

In this model, the economy is inhabited by an infinite number of households, intermediate firms, and competitive final good-producing firms. Oil enters the model through the approach proposed by Finn (2000) and also used by Leduc and Sill (2004). According to this approach, oil expenditures relate to capital utilization in that households must use oil to supply capital services to firms. The amount of oil is proportional to the capital stock and depends on the intensity of capital utilization. I assume that oil is imported from abroad and paid for using final goods, with a zero trade balance in every period. This assumption reflects that the U.S. economy is a net oil importer. Otherwise, I assume that the economy is closed for capital and asset flows.

I also assume that the capital depreciation rate depends on how intensively capital is used. To make the model better fit the data, I introduce nominal and real rigidities, such as price and wage stickiness, habits in consumption, and investment adjustment costs. The monetary authority can intervene by adjusting interest rates on the risk-free bonds. The role of fiscal authorities is restricted to maintaining a balanced budget.

[14] Atkeson and Kehoe (1999) find that doubling the oil price results in only a 5.3 percent drop in output, whereas their putty-putty model delivers a 33 percent output drop in response to the oil price increase.

Although several empirical studies acknowledge that oil prices are nonstationary, existing theoretical models usually model the oil price as a stationary process. In this paper, I assume that the nominal price of oil is an $I(1)$ stochastic process, which helps to replicate the inverse hump-shaped impulse responses observed in the empirical model.

In the model itself, I use capital letters to represent the variables that grow along the balanced growth path (except the interest rate). Lower-case letters indicate stationary variables. Unless mentioned specifically, all variables are expressed in terms of final consumption.

2.1 Firms

The final good is produced by perfectly competitive firms using a continuum of differentiated goods as inputs and the technology defined by the Dixit-Stiglitz aggregation formula, where η is the elasticity of substitution between production factors. Differentiated goods $Y_{i,t}$ for $i \in [0, 1]$ are produced by monopolistically competitive firms using the following production technologies:

$$Y_{i,t} \leq Y(K^d_{i,t}, Z_t h^d_{i,t}) = F(K^d_{i,t}, Z_t h^d_{i,t}) - Z_t \Psi, \tag{1}$$

where capital services and labor, $K^d_{i,t}$ and $h^d_{i,t}$ are the production inputs;[15] Z_t is a neutral labor-augmenting technology, which also applies to the fixed costs $Z_t \Psi_t$;[16] and $F(\cdot, \cdot)$ is homogenous degree one, increasing and concave in its arguments function. I assume that this function features a constant elasticity of substitution in capital and labor:

$$F(K^d_{i,t}, Z_t h^d_{i,t}) = \left[\theta (K^d_{i,t})^{-\varrho} + (1-\theta)(Z_t h^d_{i,t})^{-\varrho}\right]^{-\frac{1}{\varrho}},$$

where θ is a parameter that determines relative factor shares, and ϱ is a factor substitution parameter, such that $\frac{1}{1+\varrho}$ is the elasticity of substitution between production factors. The neutral technology in (1) is a deterministic process growing at a constant rate μ_z according to

$$\frac{Z_t}{Z_{t-1}} = \mu_z.$$

[15] In the symmetric equilibrium, $K^d_{i,t} = u_t K_t$, where u_t determines the intensity of capital utilization, and K_t is the aggregate stock of capital.

[16] The assumption that the neutral technology affects fixed costs is necessary for the existence of a balanced path.

The problem of firm i, $i \in [0, \quad 1]$, is to maximize the present discounted value of its dividend payments:

$$max\{\mathcal{E}_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \Phi_{i,t+s}\}, \tag{2}$$

where $\mathcal{E}_t$ is the expectation conditional on time t, $r_{t,t+s}$ is the stochastic nominal discount factor between periods t and $t+s$, and $\Phi_{i,t}$ represents real dividends paid out to asset holders in period t. The dividends are the net profits after the trade in goods and state-contingent asset markets:

$$\Phi_{i,t} = \frac{P_{i,t}}{P_t} Y_{i,t} - r_t^k K_{i,t}^d - W_t h_{i,t}^d + \frac{X_{i,t}^f}{\pi_t} - \mathcal{E}_t r_{t,t+1} X_{i,t+1}^f.$$

In this formula, $\frac{X_{i,t}^f}{\pi_t} - \mathcal{E}_t r_{r,t+1} X_{i,t+1}^f$ is the net gain from the trade of state-contingent assets, $X_{i,t}^f$, in real terms. I assume that in each period t, the net equilibrium gain of intermediate good firms from trade in the state-contingent market is zero. Thus, the transversality condition in the optimal choice problem of firms will always be satisfied.

Firms are required to satisfy demand for their product, which produces an additional restriction on the firm's choice of production and prices,

$$Y_{i,t} \geq \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} Y_t. \tag{3}$$

In this formula, Y_t is the aggregate demand for the final good, which I introduce in the next subsection.

I model price rigidity following Calvo (1983) and Yun (1996). The probability of not being able to change the price is α. Firms that cannot change the price of their product today can only correct it according to the previous period's rate of inflation up to the degree of indexation χ; that is, the price of firms that can not choose the price optimally is

$$P_t^i = P_{t-1}^i \pi_{t-1}^{\chi},$$

where π refers to inflation, so that $\pi_t = \frac{P_t}{P_{t-1}}$. Firms that can change the price in period t set the price for their product to maximize their objective (2).

2.2 Households

There are infinitely many households in the economy, all of which are alike. Each household maximizes its expected lifetime utility,

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \tilde{U}(C_t - bC_{t-1}, 1 - h_t). \tag{4}$$

In every period t, the intratemporal utility is derived from habits-adjusted consumption, $C_t - bC_{t-1}$, and leisure, $1 - h_t$:

$$\tilde{U}(C_t - bC_{t-1}, 1 - h_t) = \frac{[(C_t - bC_{t-1})^{1-\sigma}(1 - h_t)^{\sigma}]^{1-\varphi} - 1}{1 - \varphi},$$

where b is the parameter that governs habit formation in consumption. Every household supplies infinitely many types of labor, h_t^j, where $j \in [0 \quad 1]$. Different labor types combine according to the Dixit-Stiglitz aggregator with the elasticity of substitution $\tilde{\eta}$, and aggregate labor can be used for production by intermediate goods producers.

Although households have some monopolistic power to set wages, they are required to supply enough labor to meet the demand for it, as reflected in the following restriction:

$$h_t^j \geq \left(\frac{W_t^j}{W_t}\right)^{-\tilde{\eta}} h_t^d, \tag{5}$$

where h_t^d is aggregate labor demand.

Households supply different types of labor in a monopolistically competitive market. Wages are sticky, *à* la Calvo (1983) and Yun (1996), with $\tilde{\alpha}$ equal to the probability of not being able to reset a wage, and partial indexation for the previous wage rate $\tilde{\chi}$. Therefore, if a wage of type i cannot be set optimally in period t, it is indexed to the previous period inflation rate, according to the formula:

$$W_t^j = W_{t-1}^j(\mu_z \pi_{t-1})^{\tilde{\chi}}.$$

In addition to consuming and working, households accumulate capital and rent it to firms. As does Finn (2000), I also assume that capital depreciates at a variable rate $\delta(u_t)$, which depends on the intensity of capital use in period t, u_t. I also assume a quadratic form for the depreciation function:

$$\delta(u_t) \equiv \delta_0 + \omega_0(u_t - u^{ss}) + \omega_1(u_t - u^{ss})^2, \tag{6}$$

where $\omega_0 > 0, \quad \omega_1 > 0$, and u^{ss} is the capital utilization rate in the steady state.

Investment changes are costly, with the costs $S\left(\frac{I_t}{I_{t-1}}\right)$ per unit of investment. The dynamics of capital can be described as

$$K_{t+1} = (1 - \delta(u_t))K_t + I_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right], \tag{7}$$

where $S(\cdot)$ satisfies the condition that $S(\mu_z) = S(\mu_z)' = 0$ and $S(\mu_z)'' > 0$. This calibration implies that along the balanced-growth path, investments will grow at the same rate as neutral technology, μ_z. Moreover, investing is not costly if the economy is on a balanced growth path. I assume that the functional form of the investment costs function is

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - \mu_z\right)^2,$$

with parameter $\kappa > 0$.[17]

Oil is needed to provide capital services. Households buy oil at an exogenously given price $P^N_{e,t}$. The growth of the nominal oil price, $\mu^N_{Pe,t} = \frac{P^N_{e,t}}{P^N_{e,t-1}}$, is an exogenous stochastic process

$$\log\left(\frac{\mu^N_{Pe,t}}{\mu^N_{Pe}}\right) = \rho_{Pe} \log\left(\frac{\mu^N_{Pe,t-1}}{\mu^N_{Pe}}\right) + \epsilon_{Pe,t}, \tag{8}$$

in which the $\epsilon_{Pe,t}$ - $i.i.d$ stochastic process has zero mean and standard deviation σ_{Pe}.

The amount of oil is proportional to the size of the existing capital stock K_t. This approach to model energy was first used by Finn (2000) in a standard RBC framework. Although oil complements capital, the amount of oil expenditures depends on the intensity of capital utilization, u_t. Thus, the ratio of oil to capital is a function of the capital utilization rate u_t:

$$\frac{E_t}{K_t} = A(u_t). \tag{9}$$

[17] Investment costs are second-order costs that arise only if investments changes relative to the previous period. Christiano, Eichenbaum, and Evans (2005) notice that compared with specifications in prior literature, the more recent second-order investment adjustment costs help achieve a stronger and more persistent effect of monetary policy shock on output, but it does not significantly affect the estimates of the model or the response of inflation to a monetary policy shock.

I assume that $A(u_t)$ is an increasing and convex function, reflecting the idea that the provision of capital services in terms of oil becomes more costly at an increasing rate if capital gets utilized more intensively. Thus, $A(u_t)$ represents a technology that can produce capital services using oil as a production input.

The inverse of this production function, one may retrieve the conditional demand for oil as a function of the capital utilization rate.[18] Because capital utilization directly enters the production technology of intermediate goods, it can be thought of as a production technology determined by capital, labor, and oil. Therefore, the oil price increase will propagate into the economy by negatively affecting the marginal productivity of intermediate production and, consequently, the demands for capital and labor, similar to models that explicitly assume oil is one of the production inputs.[19] As a result, this model encompasses this alternative, oil-in-production, class of model. At the same time, the specification I use allows for an additional transmission channel for the oil price shock, which propagates through the capital services market. According to this mechanism, higher oil prices raise the marginal costs of providing capital services, which decreases the supply of capital services and creates an upward pressure on the rental rate r_t^k. It also creates additional downward pressure on the capital utilization rate compared with the oil-in-production model.

This approach to modeling oil sector was first suggested by Finn (2000), but I also assume that the technology for producing capital services adjusts over time to keep up with growing oil prices. In particular, the more expensive oil is, the more efficient this technology becomes, such that lesser oil expenditures can produce the same amount of capital services. This assumption also induces long-run balanced growth as a result of growing oil prices.[20] Technically speaking, the oil-to-capital requirement $A(u_t)$ is discounted by a process Z_t^* that grows in the long-run at the rate of the real oil price growth, $P_{e,t}$. I model this process as follows:

$$Z_t^* = \alpha_z Z_{t-1}^* + (1 - \alpha_z) P_{e,t}, \tag{10}$$

[18]This requires that $A(u_t)$ is invertible for a range of u_t from 0 to 1.

[19]See, for example, Carlstrom and Fuerst (2006).

[20]This assumption is needed to guarantee the existence of a steady path, though it is not the only way stationarity might be induced in the model. Similar to Fisher (2003) and Altig, Christiano, Eichenbaum, and Lindé (2004), I could assume that the fixed costs of production grow at the rate of the oil price. The permanent increase in the price of oil in my model only has temporary effects on macroeconomic variables (except the quantity of oil), whereas in Fisher's (2003) and Christiano, Eichenbaum and Lindé's specification, the permanent increase in oil price has a permanent effect on some macroeconomic variables.

where $\alpha_z \in [0,1)$. Then, $A(u_t)$ is

$$A(u_t) = \frac{a(u_t)}{Z_t^*}. \tag{11}$$

In (11), I assume that $a(u_t)$ is an increasing and convex quadratic function of the capital utilization rate,

$$a(u_t) = a_0 + \upsilon_0(u_t - u^{ss}) + \upsilon_1(u_t - u^{ss})^2,$$

where $\upsilon_0 > 0$, and $\upsilon_1 > 0$.

The assumption that the oil-to-capital requirement ratio diminishes at the rate of growth of oil prices may be interpreted as follows: the growth of oil prices stimulates the development of technological progress, which helps increase efficient energy use. Alternatively, it may represent a slow transition to alternative, less expensive sources of energy. The longer it takes $A(u_t)$ to respond to the oil price shock, the greater is the negative effect of the shock on the economy. The closer α_z is to 1, the longer it takes the technology to improve. Thus, a permanent increase of the oil price will have a dramatic and almost permanent negative effect on the economy. In contrast, if $\alpha_z = 0$, the technology $A(u_t)$ immediately accommodates the oil price increase.[21] In this case, there is no effect of the oil price shock on macroeconomic variables, because the new technology level decreases the quantity of oil needed to provide capital services by exactly the same proportion as the rise in the oil price, without any effect on capital or capital utilization.

Finally, I assume that markets are complete and introduce state-contingent assets for households. I denote $\mathcal{E}_t r_{t,t+1} X_{t+1}^h$ as the cost of the state-contingent assets acquired at time t, discounted at today's price of consumption, P_t. In turn, $r_{t,t+1}$ is the stochastic nominal discount factor in the period between t and $t+1$.

According to this discussion of household behavior, the intertemporal budget constraint of the household in terms of consumption is:

$$\mathcal{E}_t r_{t,t+1} X_{t+1}^h + C_t + I_t + P_{e,t} E_t = \frac{X_t^h}{\Pi_t} + Tr_t + r_t^k u_t K_t + \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\tilde{\eta}} h_t^d dj + \Phi_t, \tag{12}$$

where Tr_t is a net transfer from the government to the household, and Φ_t is the dividend income the household earns from the ownership in the firms. As for firms, I assume that households are subject to a borrowing constraint that prevents them from engaging in Ponzi games.

[21] This is true unless monetary policy directly responds to the oil price.

2.3 Aggregation, Markets Clearing and Competitive Equilibrium

My focus centers on a symmetric equilibrium in which all firms and households make identical decisions. In particular, monopolistically competitive firms that are able to reset their prices at a given period t all decide on the same price $\tilde{P}_t$. Similarly, the symmetric equilibrium assumes identical wage rates $\tilde{W}_t$ for all labor types able to reset their wages optimally.

The aggregate labor supply is $h_t = \int_j h_t^j$. Due to the real loss associated with sticky wages, some labor supply is wasted. In equilibrium, the loss-adjusted supply for labor $h_t/\tilde{s}_t$ equals the labor demand of firms $\tilde{s}_t$ according to

$$\frac{h_t}{\tilde{s}_t} = h_t^d \tag{13}$$

In formula (13), $\tilde{s}_t \equiv \int_0^1 \left(\frac{W_{i,t}}{W_t}\right)^{-\tilde{\eta}} di$ is the wage dispersion variable. The equilibrium dynamics of this variable can be found in the technical appendix to this paper.

Equilibrium in the market for capital services implies that the supply of capital services $u_t K_t$ equals the aggregate demand for capital from firms $K_t^d = \int_i K_{i,t}^d$,

$$u_t K_t = K_t^d. \tag{14}$$

To recognize that not all of the oil resources consumed by the United States come from abroad, I assume that some oil is domestically produced. In doing so, I follow Rotemberg and Woodford (1996), who rely on the simple assumption that domestic oil production is costless and that the price of domestic oil is determined by the exogenous world oil price. Although these two assumptions are overly simplified, they are designed to capture the negative wealth effect associated with an increase in the price of oil.

Because some oil is imported, the value added can be written as the difference between domestic production and the cost of imported oil,

$$VA_t = \frac{Y_t}{s_t} - s_{im} P_{e,t} E_t,$$

where s_{im} is the share of imported oil in the overall oil expenditures, $Y_t = F(u_t K_t, h_t^d)$ is the aggregate output supply, and $s_t \equiv \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} di$ is the measure of price dispersion in this economy.

The role of fiscal policy is minor and designed to just capture the wealth effect from government activities. I assume that government consumption is constant and financed through lump-sum taxes,

$$TR_t = -G_t. \tag{15}$$

Equilibrium in the market for final goods is determined by the equilibrium of the aggregate demand $C_t + I_t + s_{im} P_{e,t} E_t + G_t$ and supply, which implies that

$$VA_t = C_t + I_t + G_t. \tag{16}$$

The no-arbitrage condition in the state-contingent assets market implies that

$$\lambda_t = \beta R_t \mathcal{E}_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{17}$$

where R_t is the risk-free nominal interest rate.

I further assume that monetary policy can be described by a simple interest rate rule, according to which the interest rate is inertial and responds simultaneously to inflation, the growth of the value added, and the nominal oil price growth:

$$\ln\left(\frac{R_t}{R}\right) = \alpha_R \ln\left(\frac{R_{t-1}}{R}\right) + \alpha_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \alpha_{va} \ln\left(\frac{VA_t}{VA_{t-1}}\right) + \alpha_{Pe} \ln\left(\frac{\mu^N_{Pe,t}}{\mu^N_{Pe}}\right). \tag{18}$$

To solve this model, I modify the non-stationary variables to make them stationary and rewrite the model equilibrium conditions in terms of stationary variables.[22] Table 4 shows these modifications. I use corresponding lower case letters to denote the stationary transformations of nonstationary variables, and I define the symmetric equilibrium in terms of stationary variable transformations, as follows: at any time $t \geq 0$, households maximize the lifetime expected utility, and firms maximize their expected discounted present value of dividend payments. All agents have full information, up to the initial period $t = -1$, and take as given prices that they cannot control.[23] More formally, a competitive equilibrium is defined as a set of stationary processes c_t, h_t, i_t, k_{t+1}, u_t, $\tilde{w}_t$, ϕ_t, k^i_t, h^d_t, $\tilde{p}_t$, mc_t, r^k_t, π_t, w_t, s_t, $\tilde{s}_t$, z^*_t for $t = 0, 1, ..., \infty$, such that, given the monetary policy rule in (18), the initial conditions c_{-1}, i_{-1}, k_0, π_{-1}, s_{-1}, $\tilde{s}_{-1}$, and z^*_{-1} and the exogenous stochastic process for the oil price growth, $\mu^N_{Pe,t}$, these processes satisfy the optimality conditions of firms and households, the

[22]The derivation of the model equilibrium appears in the Technical Appendix to this text, which is available upon request.

[23]For example, the rental rate of capital, the interest rates, the wage rate, inflation, and so forth.

evolution of capital (7), the energy-to-capital technology evolution (10),the market clearing conditions (13), (14), and (16), and the equilibrium in state-contingent assets market (17).

3 Estimation

To obtain realistic predictions from the model, it is important to calibrate it properly. With this aim, I estimate the deep parameters of the model by matching the impulse responses of nine macroeconomic variables to an oil price shock generated by the empirical SVAR model from Section 1 and the impulse responses generated by the model. To generate the impulse responses, I log-linearize the system of equilibrium conditions of the model around the steady state implied by the model parametrization.

3.1 Strategy of Estimation

Some of the model parameters are estimated; others are calibrated using the corresponding data statistics or conventional wisdom. Table 5 summarizes the calibrated parameters.

Parameters specific to the oil sector are calibrated as follows. The share of oil in GDP is fixed at 4.3 percent. Capital utilization is 0.81, which equals the average of the utilization rate in manufacturing according to the data I use in the SVAR model. The quarterly capital depreciation rate is fixed at a conventional value from macroeconomic literature, 0.025. The rate of growth of the nominal oil price corresponds to the average growth rate of West Texas Intermediate (WTI) spot oil price from the data, 0.61 percent, which is 2.48 percent at an annual frequency. Following the estimate of Rotemberg and Woodford (1996), I assume that the share of imported oil in oil expenditures is $s_{im} = 0.5$. The parameter determining the share of capital in production is fixed at the conventional value of 0.36. The share of government expenditures is calibrated, using the data, as 20 percent of the value added.

I calibrate the model so that inflation is equal to the average inflation rate of the GDP deflator. The real interest rate along the balanced growth path is calibrated at 4 percent a year. The implied discount factor then can be derived as $\beta = \mu_z^{(1-(1-\sigma)(1-\varphi))} 1.04^{-0.25}$.[24] The quarterly growth rate of neutral technology is fixed at 0.4 percent (1.61 percent annually), which is the average of the labor productivity growth, calculated as the ratio of GDP to hours in the non-farm business sector over the period of 1954:3 - 2006:4.

[24]With the estimated parameters of the utility function σ and φ from Table 6, the discount factor in the model is 0.9944.

Finally, I fix four parameter values that were difficult to estimate. I set the parameters of the elasticity of substitution in aggregation formulas for final output and labor, η and $\tilde{\eta}$, at 6 and 21 respectively. Similar to other studies that try to estimate these parameters, I find they are weakly identified by the model. I fix the parameter υ_1 of the oil-to-capital requirement function $a(u)$ and ω_1 of the endogenous depreciation rate function $\delta(u)$. The estimation procedure drives both of these parameters to 0, however, the model does not have a unique solution if both υ_1 and ω_1 were set to 0 I find it more reasonable that the increasing use of capital would result in excessive depreciation, whereas energy might be spent in proportion to capital utilization. Therefore, I calibrate depreciation as a convex function and the oil-to-capital requirement ratio as a linear function of capital utilization. I set $\frac{\omega_1}{\omega_0} = 0.01$ and $\upsilon_1 = 0$.

I estimate the vector of 16 structural parameters θ

$$\theta = \{ \rho_{Pe}, \alpha_z, \upsilon_0, b, \kappa, \alpha, \tilde{\alpha}, \chi, \tilde{\chi}, \varrho, \sigma, \varphi, \alpha_R, \alpha_\pi, \alpha_{va}, \alpha_{Pe} \},$$

using bayesian techniques to obtain the estimates. Specifically, I apply the Laplace type estimator (LTE) suggested by Chernozhukov and Hong (2003), who show that the LTE is as efficient as a classical extremum estimator, but may be computationally more attractive. The LTE of the vector θ is defined as

$$\theta = arg \inf_{\zeta \in \Theta} [Q_n(\zeta)],$$

where n is the size of the data sample, $Q_n(\zeta)$ is the quasi-posterior risk function,

$$Q_n(\zeta) = \int_{\theta \in \Theta} \rho_n(\theta - \zeta) p_n(\theta) d\theta;$$

$\rho_n(\cdot)$ is the appropriate penalty function associated with an incorrect choice of parameter;[25] and p_n is the so-called quasi-posterior distribution, defined using the Laplace transformation of the distance function L_n and possibly the prior probability of the parameter θ, $\pi(\theta)$, as follows:

$$p_n(\theta) = \frac{e^{L_n(\theta)} \pi(\theta)}{\int e^{L_n(\theta)} \pi(\theta) d\theta}.$$

The distance function L_n is the weighted sum of squares of differences between the impulse responses generated by the empirical SVAR model ($I\hat{R}F$), and the theoretical model

[25]For the mean estimator, $\rho_n(\cdot)$ is the squared loss function.

($IRF(\theta)$):

$$L_n = -(IRF(\theta) - I\hat{R}F_n)'\hat{V}_n^{-1}(IRF(\theta) - I\hat{R}F_n),$$

where $\hat{V}_n$ is a diagonal weighting matrix with variances of $I\hat{R}F_n$ along the diagonal.[26] I compute the parameter estimates as mean values of a Markov chain sequence of draws from the quasi-posterior distribution of vector θ, generated by the tailored Metropolis Hastings algorithm. I use non-informative priors $\pi(\theta)$ for all parameters in θ. Specifically, I create a sequence of 2 million draws with the starting values $\theta = \{$ 0.28, 89, 0.13, 0.74, 2.68, 0.44, 0.98, 0.74, 0.95, -0.17, 0.04, 1.23, 1.22, 0.06, 0.011$\}$. I use only the second million draws to calculate the mean of θ. The standard errors are calculated as second moments of MCMC chains.

3.2 Estimation Results

The blue solid lines in Figure 1 show impulse responses generated by the theoretical model, shown as percentages of the steady state values, with the exceptions of the interest rate and inflation. The interest rate and inflation are annualized and shown as percentage deviations from the steady state. This graph reveals that the impulse responses generated by the model fit the empirical impulse responses quite well. According to the model, the trough of the value added is approximately 0.5 percent and occurs six or seven quarters after the shock. The predicted dynamics of the value added estimated by the model is quantitatively very close to the empirically observed impulse response of GDP. Capital utilization, investments, hours, and consumption also decrease, and the responses are U-shaped. The dynamics of these variables falls within the 90 percent error bands of the empirical model.

The model shows inflation and the interest rate rise upon the oil price increase. Although the interest rate response is close to the empirical response and shows the size and timing of the peak response to the shock correctly, the response of inflation in the model understates the empirically observed response of the GDP deflator to the shock. According to the theoretical model, inflation peaks between the second or third quarter after the shock, and the maximum response does not exceed 0.1 percent above the the steady state level. In contrast, the empirical response of inflation suggests that inflation peaks at the end of the first year after the shock, at approximately 0.3 percent above the undisturbed inflation level.

[26] I use impulse responses for 20 steps in L_n. An optimal number of steps to consider can be determined by the information criterion for impulse response matching estimator; see Hall, Atsushi, Nason, and Rossi (2007).

The real wage rate falls in response to the oil price shock. The model predicts that the negative response of the wage rate to the oil price shock is 0.05 percent of the steady state, which matches the response of the real wage from the empirical model during the first five quarters after the shock. Although with time the difference becomes more substantial, the response of the real wage rate in the theoretical model is still within the 90 percent error band of the empirical impulse response.

The responses of inflation and the wage rate do not replicate the predictions of the empirical model closely. However, my estimated model of oil price shocks still does a better job replicating the empirical evidence than do models from previous literature that are based on conventional calibrations of model parameters. For example, Leduc and Sill (2004) calibrate their model to replicate the response of output without relying on any estimation strategy. The peak responses of output and the interest rate to a 10 percent oil price shock in Carlstrom and Fuerst (2006) are 0.3 percent and 1.12 percent, respectively, which are smaller than the results from the empirical study of Bernanke, Gertler, and Watson (2004), which provides a source for Carlstrom and Fuerst. Neither study attempts to match the response of other model variables to the shock.

Table 6 presents the estimated parameters of the model developed in Section 2 with standard errors in the parenthesis. The estimates represent mean values of MCMC sequences of the estimated parameters, and the standard errors are standard deviations of these sequences. The autocorrelation of the exogenous process for oil price growth is 0.28. The autocorrelation parameter of the z^* process, α_z, is estimated at 0.88. The parameter for habits b is 0.75, slightly larger than estimates from most DSGE models.[27] The high estimate of habit formation is dictated by the small drop in consumption after the oil price shock. The habit formation parameter has very little effect on the dynamics of other variables. The investment costs parameter is 2.48, close to the estimate of Altig, Christiano, Eichenbaum, and Lindé (2004). The Calvo-Yun price stickiness parameter α is 0.4, which is smaller than results in prior literature. This parameter implies that prices change on average every five months. The Calvo-Yun wage stickiness parameter is 0.97. The high estimate of the wage stickiness parameter emphasizes the importance of wage rigidities, as documented many previous papers.[28] The utility of a representative household depends on the aggregate of different labor types; as Schmitt-Grohé and Uribe (2006) show, a larger parameter of wage

[27] Most research indicates this parameter falls between 0.6 and 0.7, including Christiano, Eichenbaum, and Evans (2005) and Altig, Christiano, Eichenbaum, and Lindé (2004), but does not focus on estimating the effect of oil price shocks.

[28] See Christiano, Ilut, Motto, and Rostagno (2008), and DiCecio (2008), among others.

rigidity is needed to obtain the same Phillips Curve for wages that occurs in models that define utility is defined as a function of a single differentiated type of labor. Because most research is based on the latter approach, the high estimate of the wage stickiness parameter that I obtain is not too different from other estimated DSGE models, such as Smets and Wouters (2007).

The parameters of the monetary policy rule suggest that the interest rate is not inertial and responds to inflation with the coefficient close to 1.16. However, the response of the interest rate to the output gap and oil price is muted and insignificant.

4 Transmission Mechanism of the Oil Price Shock

In this section, I provide intuitive support for some of the results of the paper.

The effect of the oil price shock is transmitted to the economy through the oil-to-capital requirement in (11). The log-linear approximation of oil expenditures is

$$(\widehat{P_t^E A(u_t)}) = \frac{\nu_0}{a_0}\widehat{u}_t - \widehat{z}_t^*, \tag{19}$$

for which I denote $\hat{x}$ as the logarithmic deviation from the steady state, $\hat{x} = \frac{\dot{x}}{x^{ss}}$, and x^{ss} is the steady value of x. The evolution of the stationary energy-to-capital technology process $z_t^* = \frac{Z_t^*}{P_t^E}$ can be written as

$$z_t^* = \frac{\alpha_z z_{t-1}^*}{\mu_{p_{e,t}}} + (1 - \alpha_z) \tag{20}$$

where $\mu_{p_{e,t}} = \frac{P_t^E}{P_{t-1}^E}$ is the growth rate of the real oil price. The log-linear approximation of this equation produces

$$\widehat{z}_t^* = \frac{\alpha_z}{\mu_{Pe}}\widehat{z_{t-1}^*} + \frac{\alpha_z}{\mu_{Pe}}\widehat{\pi}_t - \frac{\alpha_z}{\mu_{Pe}}\widehat{\mu_{Pe,t}^N}. \tag{21}$$

According to (20) and (21), a rise in the price of oil acts as a negative technology shock in the capital market. An increase in the growth rate of the price of oil makes production of capital services less efficient due to higher energy costs. The autoregressive parameter α_z of the z^* process determines first the size of the negative effect on technology, and second how quickly the oil-to-capital technology recovers from the shock. The greater α_z is, the deeper and longer the recession will be. Thus, introducing the price of oil as a non-stationary process may help overcome the problem of energy-enhanced simple RBC models that fail to document the large recessions observed after significant oil price increases. This problem has been emphasized by Rotemberg and Woodford (1996) and Finn (2000).

In the extreme case of immediate adjustment ($\alpha_z = 0$), the effect of the shock in $\mu^N_{Pe,t}$ on z^*_t is nil. Because $z^*_t = \frac{Z^*_t}{P^E_t}$, in response to an increase in P^E_t, Z^*_t increases proportionally; in other words, the oil-to-capital technology immediately and fully adjusts to the shock by absorbing all the negative effect it could create. As a result and at the extreme, the oil price shock has no effect on the economy, provided monetary authority does not change the interest rate or other monetary policy instrument in response to the oil price shock.

Another extreme case is $\alpha_z \to 1$, for which the stationary oil-to-capital technology z^*_t does not respond to an increase in the price of oil at all. As a result, the negative effect of the shock can not be mitigated by a more efficient technology, so the effect of the oil price shock is severe and (almost) permanent.

How does the oil price shock propagate in the economy? It first has an immediate and negative income effect through the household's intertemporal budget constraint. The income effect reduces the demand for consumption and increases the labor supply of households. In addition, the shock deteriorates equilibrium in the market for capital services. The oil price growth enters the first-order conditions of the household with respect to capital utilization, as in (38) in the technical appendix. This equation determines the supply of capital services as a function of its rental rate r^k_t, and its log-linearized approximation (38) can be written as

$$\hat{r^k_t} = \frac{q\omega_0}{r^k}(\hat{q}_t + \frac{\omega_1}{\omega_0}\hat{u}_t) + (1 - \frac{q\omega_0}{r^k})(\frac{\nu_1}{\nu_0}\hat{u}_t - \hat{z^*_t}). \tag{22}$$

Equation (22) thus demonstrates that the fall in z^* as a result of rising oil prices increases the marginal costs of capital services, shrinking its supply. The demand for capital services is determined by the firm's first order condition (25). It is not directly affected by the shock. However, because capital services and labor are substitutes in production, distortions in the capital services market propagate into the labor market and negatively affect labor demand, which may further change the demand for and supply of capital services. General equilibrium determines how capital utilization and labor respond to shock. In the estimated model presented in Section 2, both capital utilization and labor decrease in response to the shock.

In addition to the contemporaneous effect on the macroeconomic variables, the oil price shock distorts the intertemporal choices of households and firms, such that an increase in the price of oil has a negative effect on the return to capital, deteriorating the investment process and the intertemporal optimality conditions by negatively affecting the return on capital.

The return on future capital is determined by the household's future income from renting out its capital services, which equals the expected appreciation of capital less the costs of depreciation. The return on capital also deteriorates by the costs of energy associated with capital use. Thus, the rate of return on capital is

$$\frac{r^k_{t+1}u_{t+1} + q_{t+1}(1-\delta(u_{t+1})) - \frac{a(u_{t+1})}{z^*_{t+1}}}{q_t}. \tag{23}$$

Increasing oil prices decreases future stationary oil-to-capital technology z^* because of the autoregressive nature of this process. As a result, the costs of providing capital services per unit of capital should rise, creating downward pressure on the return to capital. The possibility of adjusting the capital utilization rate has a dual effect on the outcome of the shock. On the one hand, lower capital utilization helps cut down energy expenditures and decrease the rate of capital depreciation, creating an upward pressure on the return on capital. On the other hand, lower utilization of capital reduces the gain from renting out capital services, which tends to decrease the capital rate of return. The return on capital is tied to the real interest rate. According to the preceding analysis, different outcomes may emerge in equilibrium, and it is natural to expect an increase in the real interest rate after a negative technology shock. According to my estimated model, the return on capital and the real interest rate both rise in response to an oil price shock.

5 Optimal Policy Analysis

In this section, I evaluate the contribution of monetary policy to recessions generated by the oil price shock. I choose the welfare approach as my basis for comparisons. I first solve the equilibrium of the model from Section 2, assuming that monetary policy is socially optimal and maximizes the lifetime expected utility of the representative agent. Then I compare the dynamics of the baseline model with the optimal policy model. I also calculate the differences in welfare resulting from the baseline and the optimal policy models.

I focus on the time-invariant optimal monetary policy, or the optimal from a timeless perspective monetary policy, according to Woodford (2003). This definition implies that by the initial period $t = 0$, the economy has been operating for an infinite number of periods, which enables me to disregard the condition of the optimal planner's choice at the initial period $t = 0$ and substitute it with the first-order conditions derived for an arbitrary period $t > 0$. The initial values of state variables thus equal their long-run values. Moreover, I assume that the optimal planner commits to any decisions made in the past.

The optimal (Ramsey) monetary policy is the stationary process for the interest rate $\{R_t\}_{t=0}^{\infty}$ associated with the competitive equilibrium that delivers the maximum utility for the representative agent. Formally, the Ramsey equilibrium is a set of stationary processes c_t, h_t, i_t, k_{t+1}, u_t, $\{x_{t+1,s}^h\}_{s\in S}$, $\tilde{w}_t$, λ_t, ϕ_t, k_t^d, h_t^d, $\tilde{p}_t$, mc_t, $\{x_{t+1,s}^f\}_{s\in S}$, tr_t, $r_{t,t+1}$, r_t^k, π_t, w_t, s_t, $\tilde{s}_t$, z_t^*, and $R_t \geq 1$, for t from 0 to ∞, such that they maximize the objective function of the representative household (4) subject to the first-order conditions in (25)-(29), (33)-(38), and (40) - (42), the capital evolution equation (7), price and real wage dispersion dynamics (30) and (43), energy-to-capital technology evolution (20), market clearing conditions from (13), (14), (16), equilibrium in state-contingent assets market (17), the equilibrium price and wage determination Equations (32) and (45), and the exogenous stochastic process for oil price growth, $\mu_{pe,t}^N$ in (8), with all variables in the information set of the initial period $t = -1$ fixed at their long-run (steady) values. These variables are c_{-1}, i_{-1}, π_{-1}, π_{-2}, w_{-1}, w_{-2}, s_{-1}, $\tilde{s}_{-1}$, z_{-1}^*, λ_{-1}, R_{-1}, and k_0, and period $t = -1$ Lagrange multipliers associated with the constraints of the Ramsey planner appear in the first-order conditions of the optimal planner's problem.

To compute the equilibrium dynamics of the model, I log-linearize the equilibrium conditions of the Ramsey problem around the nonstochastic Ramsey steady state. This steady state delivers the maximum utility to the representative household in the absence of uncertainty. The resulting impulse responses appear in Figure 2, represented by blue lines with stars. The graphs show that in response to a 10 percent oil price shock, the optimal policy raises the interest rate and inflation. The interest rate reaches a maximum of approximately 0.45 percent above the steady state value in the third quarter. Inflation stops rising after the fourth quarter at a peak of 0.4 percent above the steady state. The GDP, investment, capital utilization, and wage rate fall, in a trough observed five to six quarters after the shock. There is no significant drop in consumption, but a slight decrease that never exceeds three basis points. Hours worked rise by approximately 0.1 percent immediately after the shock.

For comparison purposes, Figure 2 also plots the responses of the baseline competitive model (green solid lines). Visual comparison of the impulse responses reveals that monetary policy had a significant negative effect on the dynamic paths of all macroeconomic variables. Compared with the baseline model, the model of optimal monetary policy demonstrates smaller drops in the value added, consumption, investment and capital utilization in response to the oil price shock. Thus, actual monetary policy indeed had a recessionary effect on the economy. This finding confirms the results in Bernanke, Gertler, and Watson (1997) and (2004) that monetary policy exacerbated recessions. It also matches the arguments posed by Leduc and Sill (2004) and Carlstrom and Fuerst (2006), namely, that a different monetary policy could have mitigated past recessions.

Because optimal monetary policy does not completely eliminate the drop in the value added and other variables, it is fallacious to rely on output as a measure of optimality of monetary policy. Leduc and Sill (2004) report that they could not find monetary policy rules that completely eliminate recessions associated with oil price shocks. However, even if these policy rules were available, they could be undesirable due to high costs of maintaining stable production after the shock.

With the optimal monetary policy though, inflation rises more than in the baseline model. Therefore, the optimal policy planner focuses less on stabilizing the inflation that results from high oil prices. This finding contrasts with Leduc and Sill (2004), who find that stabilizing inflation is preferable in an economy subject to oil price shocks. The difference in the results derives from the different calibrations of the degree of wage stickiness in the two models. Whereas Leduc and Sill assume that nominal rigidities are minimal, I find that wage stickiness is especially important for explaining the smooth response of the wage rate to the oil price shock observed from the data. With sticky nominal wages, inflation stabilization results in suboptimal stabilization of the real wage rate. The real wage rate in the optimal policy model falls about twice as much as the baseline model predicts in response to the shock. If wages are sticky, the optimal drop in the real wage rate is achieved through higher inflation.

The greater drop in the real wage rate under the optimal monetary policy increases employment immediately after the shock. The drop in the real wage rate therefore must be large enough to generate a negative wealth effect on consumers that dominates the substitution effect, which in turn raises labor in equilibrium.

The most striking result of the analysis of optimal monetary policy is that the interest rate rises more under the Ramsey optimal policy than in the baseline estimated model. Thus, though Bernanke, Gertler, and Watson (1997) blame monetary policy for raising interest rates excessively, the analysis of the optimal policy suggests just the opposite. The interest rate should have been raised even more to mitigate recessions with the smallest possible costs to the society. At first, this result may seem counterintuitive, because high interest rates often serve as indicators of a contractionary monetary policy. However, it is the real, rather than the nominal, interest rate that is important for the transmission channel of monetary policy, and an increase in the nominal interest rate does not always correspond to an increase in the real interest rate. The dynamics of the real interest rate that the Ramsey planner tries to reproduce roughly equal the dynamics of the interest rate in the corresponding RBC version of the model (without nominal frictions). The real interest rate in the RBC version increases at the shock,[29] with a rise of approximately the same magnitude as that in the baseline model. Because the Ramsey planner allows for more inflation, the nominal interest rate needs to be higher to achieve the desired path for the real interest rate.

The problem for the Ramsey planner is to minimize the real costs of the oil price shock associated with nominal rigidities, such as sticky prices and nominal wages. Thus, the Ramsey planner finds a balance among the responses of the nominal interest rate, inflation, and nominal wages to achieve the desired dynamics for the real wage and the real interest rate.

Is it possible to implement optimal monetary policy in a world in which a monetary policy planner is restricted to follow simple interest rate rules? If monetary policy places a smaller weight on inflation stabilization than that of the baseline estimated policy, it will be reflected in a smaller parameter α_π in the Taylor-type interest rule (18). With more inflation, the nominal interest rate will increase as long as inflation increases more than the the drop in α_π due to the change in the policy. Thus, a policy that pays less attention to the stabilization of inflation related to high oil prices may be a good approximation of the Ramsey optimal solution.

The impulse responses of all the variables, other than inflation and the interest rate, are robust to the choice of parameters for nominal rigidities in the Ramsey model, because no matter how important nominal rigidities are, the Ramsey planner tries to replicate the dynamic allocations of its RBC model counterpart. However, the relative importance of nominal rigidities may change the optimal response of inflation and the nominal interest rate to the oil price shock. To determine the robustness of the optimal responses of these variables, I reestimate the model assuming the Calvo price and wage rigidity parameters set to a more conventional value of 0.75, as estimated by Altig, Christiano, Eichenbaum, and Lindé (2004). The results in Figure 3 show that inflation and the interest rate still increase in the Ramsey optimal solution after the oil price shock. Thus, even with smaller nominal wage rigidities and higher price rigidities, monetary policy was not raising interest rates too aggressively when it responded to high oil prices.

[29] A drop in the real interest rate would be inefficient, because it would stimulate excessive use of costly capital.

To provide numerical estimates of the costs of conducting the "wrong" monetary policy with respect to the oil price shock, I compare how much the value added drops in the baseline model and contrast it with the drop of the value added in the Ramsey planner problem. In turn, I calculate the cumulative drop in the value added over five years:

$$L = -\sum_{t=0}^{19} log(\frac{va_t}{va^{ss}}).$$

I also evaluate the welfare costs of following the estimated monetary policy rule by estimating the conditional welfare expectation for a 10 percent oil price shock and comparing the results across the baseline and the optimal policy models. The first-order approximation cannot capture the differences of welfare in models approximated around the same steady state, so I approximate the model's dynamics up to the second order using the apparatus developed in Schmitt-Grohé and Uribe (2004).

For ease of interpretation, I define the welfare costs as a fraction of the consumption stream in the calibrated model.[30] The welfare costs λ are the minimum share of the lifetime consumption pattern that the representative household would demand to refuse to move to a state of the economy that is associated with the Ramsey equilibrium. Welfare costs λ defined this way satisfy the following relationship:

$$V_0^R = (1+\lambda)^{(1-\sigma)(1-\varphi)} V_0^C + \frac{(1+\lambda)^{(1-\sigma)(1-\varphi)} - 1}{(1-\beta\mu_z^{(1-\sigma)(1-\varphi)})(1-\varphi)}. \quad (24)$$

In this formula, I assume that households do not reoptimize consumption and labor patterns after receiving the transfer from the government. From (24), I can derive λ as

$$\lambda = \left(\frac{V_t^R + \frac{1}{(1-\beta\mu_z^{(1-\sigma)(1-\varphi)})(1-\varphi)}}{V_t^C + \frac{1}{(1-\beta\mu_z^{(1-\sigma)(1-\varphi)})(1-\varphi)}} \right)^{\frac{1}{(1-\sigma)(1-\varphi)}} - 1.$$

In Table 7, I provide the estimates of the cumulative drop in the value added for the baseline calibration of the model, as well as the estimates of the welfare costs. All numbers in the table percentages relative to the outcome of the Ramsey economy. In the first row, I show the estimates for the baseline model. In the second row, I obtain numbers by approximating

[30] This approach is similar to the definition by Schmitt-Grohé and Uribe (2005). However, they define these costs as a share of consumption in the optimal planner's problem. I find it more intuitive to relate the costs to consumption from the calibrated model.

the baseline model around the steady state dictated by the Ramsey optimal solution.

The first column of Table 7 lists the costs of the oil price shock in terms of the cumulative value added drop. The estimates suggest that the baseline monetary policy is responsible for the loss of approximately 59 percent of GDP over a period of five years. This finding is consistent with Bernanke, Gertler, and Watson (1997) and (2004), who report that at least 50 percent of the GDP drop after the 1950s was associated with an incorrect monetary policy. Thus, the disagreement is not about the suboptimality of the monetary policy but rather whether monetary policy was too aggressive or not aggressive enough in raising interest rates when oil prices increased.

The second and the third columns of Table 7 indicate that the response of the estimated monetary policy to a one standard deviation oil price shock leads to the welfare losses of approximately 30 percent of consumption, which amounts to almost \$1800 a year. Thus, the welfare costs represent a significant fraction of the household's consumption expenditures. However, these costs result mostly from actual monetary policy operating at a suboptimal steady state, as becomes clear from the second row of Table 7. The welfare costs of the suboptimal monetary policy are 11 basis points of consumption, or a \$41.2 loss in annual consumption. These estimates reveal the inability of the model to create nonlinearities that might produce significant differences in the second-order approximations.

6 Conclusion

With this paper, I critically evaluate the statements of Bernanke, Gertler, and Watson (1997) that the excessive tightening of monetary policy in response to the post-World War II oil price shocks exacerbated recessions. I find that actual monetary policy was indeed responsible for the major portion of output drops in a comparison with a monetary policy that maximizes social welfare. However, contrary to the prevailing views, the optimal response of monetary policy to the shock would raise both inflation and interest rates above what has been seen in the past.

This research offers two major contributions to the literature on monetary effects and oil shocks. First, I present an estimated DSGE model of the effect of the oil price shock on the U.S. economy. The estimation of model parameters is critical for obtaining a satisfactory overall picture of the economy's dynamics in response to volatile oil prices. Although still not perfect, the proposed model explains the macroeconomic dynamics of the empirical SVAR model better than do existing models of the oil sector. Second, my application of the welfare approach extends existing analyses of the systematic monetary policy contribution to the adverse effects of oil price shocks. Using the Ramsey optimal monetary policy as an alternative enables to assess the optimality of monetary policy relative to the best possible scenario, rather than some arbitrary, and not necessarily preferable, monetary policy alternative.

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7 Appendix

Table 1: Data

Data	Description	Source
P_t^{oil}	West Texas Intermediate spot oil price[a]	FRED
GDP_t	Seasonally adjusted quarterly GDP, per capita[b]	NIPA, BEA
$Hours_t$	Seasonally adjusted quarterly index of hours of all persons in the non-farm business sector, 1992 = 100, per capita	FRED
π_t	First difference of the log of the GDP deflator. The GDP deflator is calculated as the ratio of nominal to real GDP; both series are seasonally adjusted at annual rates	NIPA, BEA
CU_t	Seasonally adjusted capacity utilization in the manufacturing sector, in percentages/100, quarterly data,	FRED
W_t	Seasonally adjusted quarterly index of the compensation per hour in the non-farm business sector, 1992 = 100	FRED
C_t	Consumption of services and nondurables excluding gasoline, fuel oil and the consumption of other energy goods, plus government consumption expenditures;[c] quarterly; seasonally adjusted at annual rates; per capita	NIPA, BEA
I_t	Nominal investment, consumption of durable goods plus private investment, quarterly and seasonally adjusted at annual rates, per capita	NIPA, BEA
R_t	Effective Federal Funds Rate, monthly data aggregated into quarterly using simple geometric averaging over three months	FRED

Notes: FRED is the database of the Federal Reserve Bank of St. Louis; NIPA is the database of the Bureau of Economic Analysis.

[a]Monthly data from the Dow Jones & Company; aggregated into quarterly data using a geometric average over three months.

[b]Per capita series are obtained from monthly labor force statistics, from www.bls.org. Monthly data are aggregated into quarterly by simple averaging over three months.

[c]Consumption of energy is excluded from the measure of consumption because fuel consumption is not modeled in the theoretical framework.

Table 2: Historical contribution of the oil price shock

	Conditional Standard Deviation	Percentage of Unconditional Variance Explained by Oil Price Shock
Oil Price	11.01	77.1
log(GDP)	0.41	8.05
Inflation	0.34	8.90
Interest Rate	0.28	3.29
log(CU)	0.83	5.59
log(H)	0.46	8.05
log(W)	0.11	1.88
log(C)	0.14	3.51
log(I)	1.47	6.73

Notes: The variance decomposition is based on the HP-filtered time series implied by the SVAR model based on historically observed shocks. Each number in the first column of the table shows the estimate of the variance of time series produced by the SVAR model, where all the shocks except oil price are shut down, as a fraction of the variance of the original time series. The second column shows the ratio of the variance of the time series generated in this way to the unconditional variance of the time series, in percentages.

Table 3: Contribution of the oil price shock to macroeconomic volatilities based on SVAR model

	Conditional Standard Deviation	Unconditional Standard Deviation	Percentage of Unconditional Variance Explained by Oil Price Shock
Oil Price	8.03	8.27	95.2
GDP growth	0.12	0.22	3.73
Inflation	0.034	0.52	4.62
Interest Rate	0.028	0.51	2.39
log(CU)	0.021	1.65	8.43
log(H)	0.036	1.54	9.14
W growth	0.038	0.11	1.25
C growth	0.030	0.10	1.20
I growth	0.32	0.74	4.87

Notes: The first two columns show the conditional and unconditional standard deviations of macroeconomic variables implied by the SVAR model. Conditional standard deviations of macroeconomic variables are based on a 1 standard deviation oil price shock. The third column shows the squared ratio of numbers in columns 2 and 3.

Table 4: Stationary transformation of the variables

New Variable	Transformed Variable	Transformation
	The problem of firms	
$\phi_{i,t}$, $k^d_{i,t+1}$	$\Phi_{i,t}$, $K^d_{i,t+1}$	divided by Z_t
	The problem of households	
c_t, x^h_t, w_t, w^i_t,	C_t, X^h_t, W_t, W^i_t	divided by Z_t
ϕ_t, tr_t, k_{t+1}, i_t, e_t	Φ_t, Tr_t, K_{t+1}, I_t, $P_{e,t}E_t$	
z^*_t	Z^*_t	divided by $P_{e,t}$
$a(u_t)$	$A(u_t)$	multiplied by $P_{e,t}$
$\tilde{p}_t$	$\tilde{P}_t$	divided P_t
$\tilde{w}_t$	$\tilde{W}_t$	divided W_t

Table 5: Calibrated parameters

Parameter	Notation	Value
Nominal oil price growth rate (annualized)	$\mu^N_{P_e}$	2.48%
Share of oil in value added	SE	4.3%
Share of oil in overall oil expenditures	s_{im}	50%
Capital utilization rate	u	81%
Capital depreciation rate	δ	2.5%
Real interest rate (annualized)	R/π	4%
Inflation (annualized)	π	3.57%
Neutral technology growth rate(annualized)	μ_z	1.61%
Capital share in production	θ	36%
Share of government expenditures in value added	SG	20%
Shadow price of investment	q	1
Elasticity of substitution, interm. goods	η	6
Elasticity of substitution, differentiated labor types	$\tilde{\eta}$	21
Convexity of energy-to-capital function $a(u_t)$	υ_1	0
Convexity of the depreciation function $\delta(u_t)$	ω_1	1

Table 6: Estimates of the parameters

Note: s.e. indicates standard error.

Parameter	Notation	Estimate	(s.e.)
Autocorrelation of oil price process	ρ_{Pe}	0.28	(0.02)
Parameter of Z^* process	α_z	0.88	(0.01)
Parameter 1 of capital utilization	$\upsilon_0 * 100$	0.13	(0.02)
Habit parameter for consumption	B	0.75	(0.09)
Investment costs parameter	κ	2.48	(0.26)
Probability of not being able to reoptimize the price	α	0.40	(0.22)
Probability of not being able to reoptimize the wage	$\tilde{\alpha}$	0.97	(0.01)
Price indexation	χ	0.76	(0.01)
Wage indexation	$\tilde{\chi}$	0.89	(0.05)
Production: factor elasticity of substitution parameter	ϱ	-0.16	(0.05)
Utility:	σ	0.1	(0.03)
Utility:	φ	1.71	(0.41)
Monetary policy rule:	α_R	0.28	(0.16)
Monetary policy rule:	α_π	1.16	(0.23)
Monetary policy rule:	α_y	0.086	(0.056)
Monetary policy rule:	α_{p^e}	0.0012	(0.002)

Table 7: Welfare Costs of the Oil Price Shock

Policy	L, Percentage	λ, Percentage	$c_{2006}\lambda$, \$
Non-Ramsey steady state	59	31.57	11,844.6
Ramsey steady state	28.1	0.11	41.3

Notes: λ is welfare costs in terms of consumption, c_{2006} are per capita consumption expenditures in 2006, and L is the cumulative loss of value added over five years, expressed in percentages relative to the Ramsey value added loss.

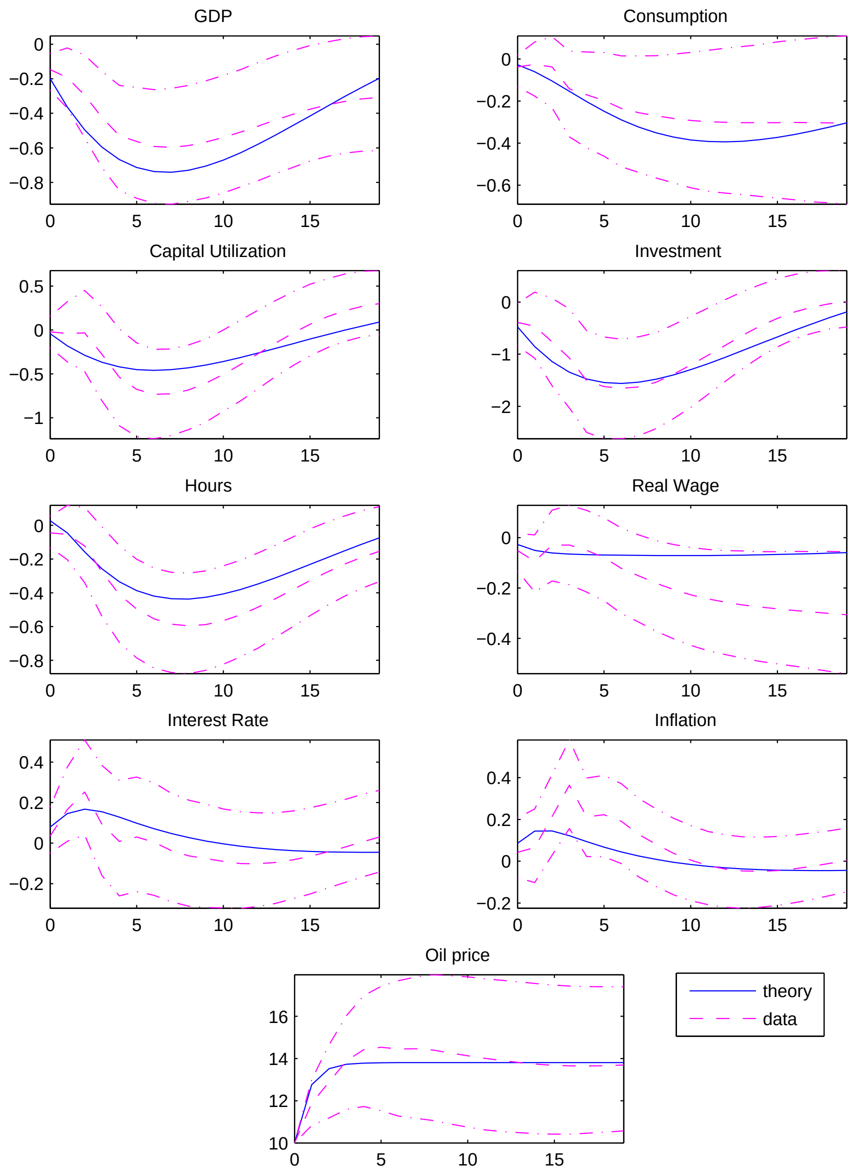

Figure 1: Impulse responses: theoretical and empirical (SVAR) models

Notes: Figure shows impulse responses to a 10 percent oil price shock. All variables except the interest rate and inflation are shown as percentage deviations from their long-run trends. The responses of the interest rate and inflation are presented as percentages of the annualized values relative to their long-run values. Quarters of a year are shown along the horizontal axis. The confidence intervals for the impulse responses of the SVAR model are calculated using the double bootstrap method proposed by Kilian (1998) and represent the 5^{th} and 95^{th} percentage quantiles of the bootstrapped distribution.

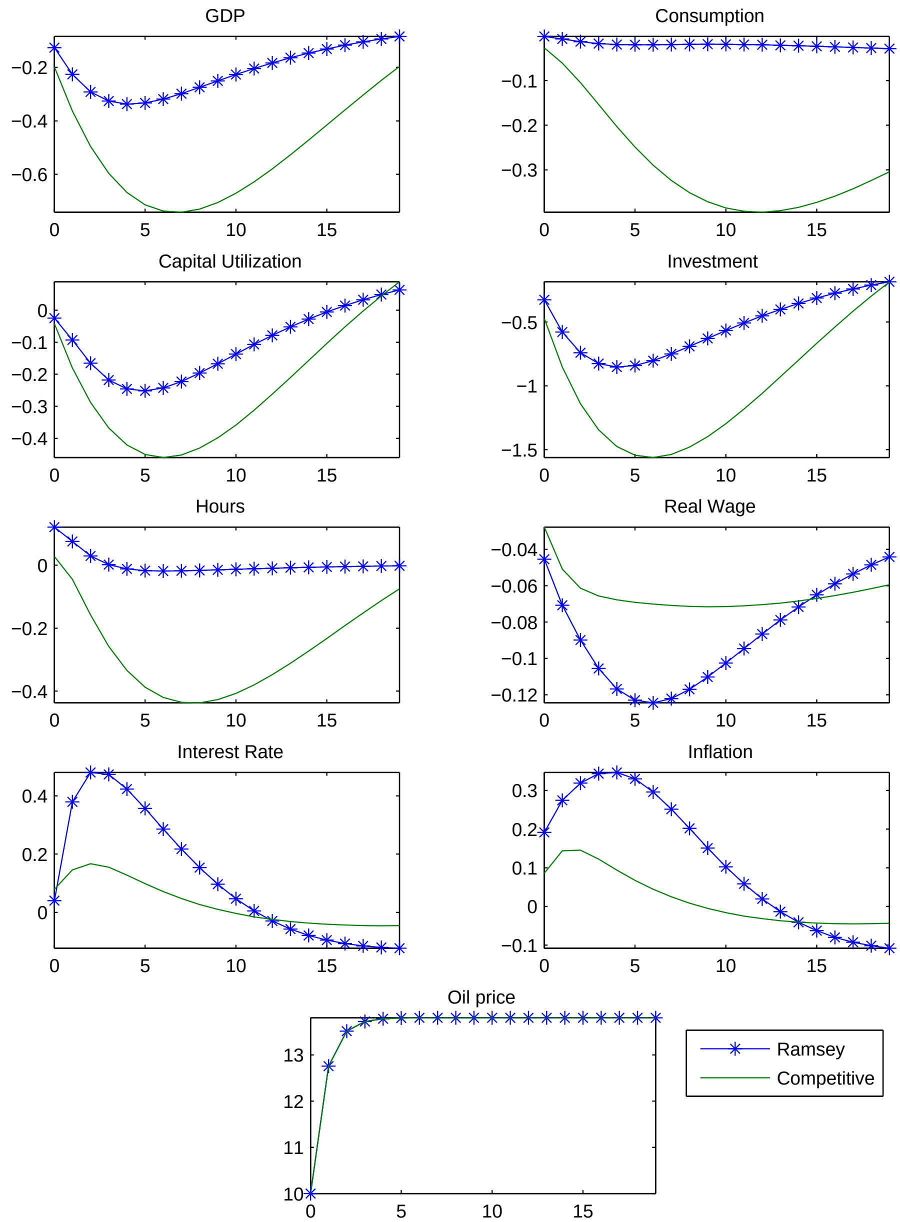

Figure 2: Impulse responses: competitive and Ramsey economies

Notes: Figure shows the impulse responses to a 10 percent oil price shock as the percentage deviations from the trend along the vertical axis for all variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, and quarters of the year along the horizontal axis.

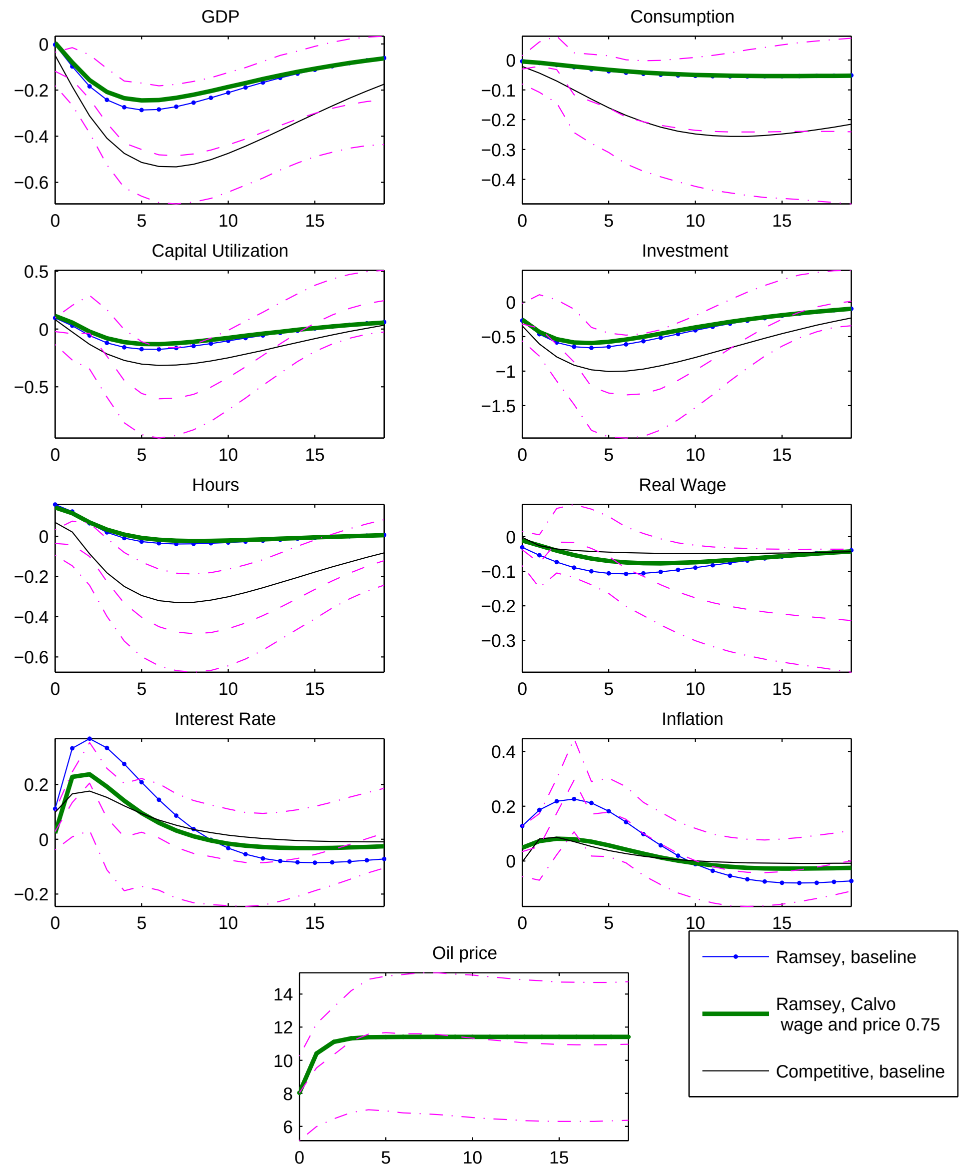

Figure 3: Robustness of the optimal monetary policy

Notes: Figure presents the test of the robustness of the optimal policy to the calibration of the Calvo-Yun price rigidity parameter. The graphs show the impulse responses to a 10 percent oil price shock as the percentage deviations from the trend along the vertical axis for all variables except the interest rate and inflation, percentages of the annualized values as deviations from the averages for the interest rate and inflation, and quarters of the year along the horizontal axis.

7.1 Model Equilibrium

Firms

Denoting $\beta^t mc_{i,t}$ as the Lagrange multiplier on the production function constraint of the firm i in Equation (1), the first-order necessary conditions for the choice of $K^d_{i,t}$ and $h^d_{i,t}$ of firm i respectively, are

$$mc_{i,t}F_1(K^d_{i,t}, z_t h^d_{i,t}) = r^k_t,$$

and

$$mc_{i,t}z_t F_2(K^d_{i,t}, z_t h^d_{i,t}) = W_t.$$

To treat the steady growth of neutral technology, I rewrite the problem of the firms in terms of the stationary variables in the first row of Table 4.

With lowercase letters denoting the stationary modifications of the corresponding capital letter variables, the first-order conditions with respect to $k^d_{i,t}$ and $h^d_{i,t}$ are

$$mc_{i,t}F_1(\frac{k^d_{i,t}}{\mu_{z,t}}, h^d_{i,t}), = r^k_t \tag{25}$$

and

$$mc_{i,t}F_2(\frac{k^d_{i,t}}{\mu_{z,t}}, h^d_{i,t}). = w_t \tag{26}$$

The solution to the problem of optimal price choice can be written as a solution to

$$x^1_t = x^2_t \tag{27}$$

where

$$x^1_t = \frac{\eta}{\eta-1} y_t mc_t \tilde{p}_t^{-\eta-1} + \alpha\beta\mathcal{E}_t \mu_{z,t+1}^{(1-\sigma)(1-\varphi)} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_{t+1}}{\pi^\chi_t}\right)^{\eta} \left(\frac{\tilde{p}_{t+1}}{\tilde{p}_t}\right)^{\eta+1} x^1_{t+1}, \tag{28}$$

$$x^2_t = y_t \tilde{p}_t^{-\eta} + \alpha\beta\mathcal{E}_t \mu_{z,t+1}^{(1-\sigma)(1-\varphi)} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_{t+1}}{\pi^\chi_t}\right)^{\eta-1} \left(\frac{\tilde{p}_{t+1}}{\tilde{p}_t}\right)^{\eta} x^2_{t+1}, \tag{29}$$

and

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t}.$$

The derivation of these formulas coincides with the technical appendix by Schmitt-Grohé and Uribe (2005).

The price dispersion variable s_t follows a dynamic process,

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha \left(\frac{\pi_t}{\pi_{t-1}^{\chi}} \right)^{\eta} s_{t-1}. \tag{30}$$

The equilibrium dynamics of the price level in the symmetric equilibrium can be derived as

$$P_t^{1-\eta} = \alpha (P_{t-1}\pi_{t-1}^{\chi})^{1-\eta} + (1-\alpha)\tilde{P}_t^{1-\eta}. \tag{31}$$

Denoting $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$, Equation (31) produces

$$\tilde{p}_t = \left(\frac{1 - \alpha \pi_t^{\eta-1} \pi_{t-1}^{\chi(1-\eta)}}{1-\alpha} \right)^{1/(1-\eta)}. \tag{32}$$

Households. For the sake of convenience, the Lagrange multipliers on the budget constraint, (12), aggregate labor supply requirement derived from (5), capital accumulation (7), and oil-to-capital constraint (9) are denoted $\beta^t \Lambda_t$, $\frac{\Lambda_t W_t}{\tilde{\mu}_t}$, $\beta^t \Lambda_t Q_t$, and $\beta^t \Lambda_t \Xi_t$, respectively. Then, Λ_t is the marginal utility of wealth, $\tilde{\mu}_t$ is the average wage markup of the household, Q_t is the shadow price of future capital, and Ξ_t is the shadow price of energy. The Lagrangian of the household's problem can be written as:

$$\begin{aligned}
\mathcal{L} = \mathcal{E}_0 \textstyle\sum_{t=0}^{\infty} \beta^t \{ & \tilde{U}(C_t - bC_{t-1}, 1-h_t) + \Lambda_t [\tfrac{X_t}{\pi_t} + Tr_t + r_t^k u_t K_t \\
& + h_t^d \textstyle\int_0^1 W_t^i \left(\frac{W_t^i}{W_t} \right)^{-\tilde{\eta}} di + \Phi_t - r_{t,t+1} X_{t+1}^h - C_t - I_t - P_t^E E_t] \\
& + \tfrac{\Lambda_t W_t}{\tilde{\mu}_t} [h_t - h_t^d \textstyle\int_0^1 \left(\frac{W_t^i}{W_t} \right)^{-\tilde{\eta}} di] + \Lambda_t Q_t [(1-\delta(u_t))K_t \\
& + I_t [1 - S\left(\frac{I_t}{I_{t-1}} \right)] - K_{t+1}] + \Lambda_t \Xi_t [E_t - A(u_t) K_t]. \}
\end{aligned}$$

To solve the problem, it is convenient to rewrite the Lagrangian in terms of stationary transformations of the variables that grow over time in steady equilibrium due to the exogenous growth of the oil price and neutral technology. Table 4 depicts these transformations. In addition, I transform the Lagrange multipliers Λ_t and Ξ_t to get $\lambda_t = \Lambda_t Z_t^{1-(1-\sigma)(1-\varphi)}$ and $\xi_t = \Xi_t / P_{e,t}$. With stationary variables, it is:

$$\mathcal{L} = \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t z_t^{(1-\sigma)(1-\varphi)}[U(c_t - bc_{t-1}/\mu_{z,t}, 1-h_t) + \lambda_t[\tfrac{x_t}{\pi_t \mu_{z_t}} + tr_t + r_t^k u_t \tfrac{k_t}{\mu_{z,t}}$$
$$+h_t^d \int_0^1 w_t^i \left(\tfrac{w_t^i}{w_t}\right)^{-\tilde{\eta}} di + \phi_t - r_{t,t+1} x_{t+1}^h - c_t - i_t - e_t]$$
$$+\tfrac{\lambda_t w_t}{\tilde{\mu}_t}[h_t - h_t^d \int_0^1 \left(\tfrac{w_t^i}{w_t}\right)^{-\tilde{\eta}} di] + \lambda_t q_t[(1-\delta(u_t))\tfrac{k_t}{\mu_{z,t}}$$
$$+i_t[1 - S\left(\tfrac{i_t}{i_{t-1}}\mu_{z,t}\right)] - k_{t+1}] + \lambda_t \xi_t[e_t - \tfrac{a(u_t)}{z_t^*}\tfrac{k_t}{\mu_{z,t}}]].$$

for which I denote $U(c_t - bc_{t-1}/\mu_{z,t}, 1-h_t) = \frac{\tilde{U}(C_t - bC_{t-1}, 1-h_t)}{Z_t^{(1-\sigma)(1-\varphi)}}$. The first order conditions using the stationary variables c_t, h_t, k_{t+1}, i_t, x_{t+1}^h, u_t, and e_t for all $t \geq 0$ can be written as

$$U_{t,1} - \beta b \mathcal{E}_t \frac{U_{t+1,1}}{\mu_{z,t+1}^{1-(1-\sigma)(1-\varphi)}} = \lambda_t. \tag{33}$$

$$-U_{t,2} = \frac{\lambda_t w_t}{\tilde{\mu}_t}. \tag{34}$$

$$\lambda_t q_t = \beta \mathcal{E}_t \frac{\lambda_{t+1}}{\mu_{z,t+1}^{1-(1-\sigma)(1-\varphi)}}[r_{t+1}^k u_{t+1} + q_{t+1}(1-\delta(u_{t+1})) - \xi_{t+1}\frac{a(u_{t+1})}{z_{t+1}^*}]. \tag{35}$$

$$\lambda_t = \lambda_t q_t \left[1 - S\left(\tfrac{i_t}{i_{t-1}}\right) - \left(\tfrac{i_t}{i_{t-1}}\right) S'\left(\tfrac{i_t}{i_{t-1}}\right)\right] + \beta \mathcal{E}_t \tfrac{\lambda_{t+1} q_{t+1}}{\mu_{z,t+1}^{1-(1-\sigma)(1-\varphi)}} \left(\tfrac{i_{t+1}}{i_t}\right)^2 S'\left(\tfrac{i_{t+1}}{i_t}\right). \tag{36}$$

$$\lambda_t r_{t,t+1} = \beta \mathcal{E}_t \frac{\lambda_{t+1}}{\mu_{z,t+1}^{1-(1-\sigma)(1-\varphi)} \pi_{t+1}}. \tag{37}$$

$$r_t^k = q_t \delta'(u_t) + \xi_t \frac{a'(u_t)}{z_t^*}. \tag{38}$$

$$\xi_t = 1. \tag{39}$$

In equations (33) and (34), $U_{t,1}$ and $U_{t,2}$ are the derivatives of the utiliy funtion with respect to the first and second argument, respectively.

The optimal wage of the labor types for which the wage can be set optimally is defined by the following optimality condition:

$$f_t^1 = f_t^2, \tag{40}$$

where

$$f_t^1 = \tfrac{\tilde{\eta}-1}{\tilde{\eta}} \tilde{w}_t \lambda_t \left(\tfrac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} h_t^d + \tilde{\alpha} \beta \mathcal{E}_t \mu_{z,t+1}^{(1-\sigma)(1-\varphi)} \left(\tfrac{\pi_{t+1}\mu_{z,t}}{(\pi_t \mu_z)^{\tilde{\chi}}}\right)^{\tilde{\eta}-1} \left(\tfrac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}-1} f_{t+1}^1, \tag{41}$$

and

$$f_t^2 = -U_{t,2}\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} h_t^d + \tilde{\alpha}\beta\mathcal{E}_t\mu_{z,t+1}^{(1-\sigma)(1-\varphi)}\left(\frac{\pi_{t+1}\mu_{z,t}}{(\pi_t\mu_z)^{\tilde{\chi}}}\right)^{\tilde{\eta}}\left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}} f_{t+1}^2. \tag{42}$$

The derivation of these formulas coincides with that in the technical appendix to Schmitt-Grohé and Uribe (2005).

The dynamics of $\tilde{s}_t$ is as follows:

$$\tilde{s}_t = (1-\tilde{\alpha})\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\alpha}\left(\frac{w_{t-1}}{w_t}\right)^{-\tilde{\eta}}\left(\frac{\mu_{z,t}\pi_t}{(\mu_z\pi_{t-1})^{\chi}}\right)^{\tilde{\eta}}\tilde{s}_{t-1}, \tag{43}$$

where $\tilde{w}_t = \frac{\tilde{W}_t}{W_t}$ is the wage rate chosen by labor types able to reset their wage rates relative to the aggregate wage rate for the economy in period t.

The average wage of those labor types that cannot reset their wages is equal to the previous period wage rate. Thus, the equilibrium dynamics of the aggregate wage rate dynamics is

$$W_t^{1-\tilde{\eta}} = (1-\tilde{\alpha})\tilde{W}_t^{1-\tilde{\eta}} + \tilde{\alpha}W_{t-1}^{1-\tilde{\eta}}\left(\frac{(\mu_z\pi_{t-1})^{\tilde{\chi}}}{\pi_t}\right)^{1-\tilde{\eta}}. \tag{44}$$

The optimal real wage choice in stationary terms, $\tilde{w}_t = \frac{\tilde{W}_t}{Z_t}$, is

$$\tilde{w}_t = \left(\frac{w_t^{1-\tilde{\eta}} - \tilde{\alpha}w_{t-1}^{1-\tilde{\eta}}\left(\frac{\mu_z\pi_{t-1}^{\tilde{\chi}}}{\mu_{z,t}\pi_t}\right)^{1-\tilde{\eta}}}{1-\tilde{\alpha}}\right)^{1/(1-\tilde{\eta})}. \tag{45}$$

7.2 Steady State

The quarterly real interest rate is calibrated to produce an annualized interest of 4 percent:

$$r = 1.04^{0.25}.$$

$$\beta = \mu_z^{1-(1-\phi)(1-\sigma)}r^{-1}.$$

The nominal interest rate is

$$R = \pi r.$$

The real price of oil is the ratio of the nominal price of oil DN to inflation

$$p_e = \frac{DN}{\pi}.$$

The steady value of the stationary stochastic process z_t^* is derived from Equation (20),

$$z^* = \frac{1-\alpha_z}{1-\frac{\alpha_z}{p_e}}.$$

The relative price of optimizing firms can be obtained from Equation (32),

$$\tilde{p} = \left(\frac{(1-\alpha\pi^{(\eta-1)(1-\chi)}}{1-\alpha)} \right)^{1/(1-\eta)}.$$

The relative wage rate of optimizing firms is derived from Equation(45) as

$$\frac{\tilde{w}}{w} = \left(\frac{1-\tilde{\alpha}(\mu_z\pi)^{(\tilde{\chi}-1)(1-\tilde{\eta})}}{(1-\tilde{\alpha})} \right)^{1/(1-\tilde{\eta})},$$

$$\tilde{\mu} = \frac{\tilde{\eta}}{\tilde{\eta}-1} \left(\frac{1-\frac{\tilde{\alpha}\mu_z}{r}(\mu_z\pi)^{(1-\tilde{\chi})(\tilde{\eta}-1)}}{1-\frac{\tilde{\alpha}\mu_z}{r}(\mu_z\pi)^{(1-\tilde{\chi})\tilde{\eta}}} \right) \left(\frac{\tilde{w}}{w} \right)^{-1}.$$

$$mc = \tilde{p}\frac{\eta-1}{\eta} \frac{1-\frac{\alpha\mu_z}{r}\pi^{\eta(1-\chi)}}{\left(1-\frac{\alpha\mu_z}{r}\pi^{(\chi-1)(1-\eta)}\right)}.$$

$$\tilde{s} = (1-\tilde{\alpha})\frac{\left(\frac{\tilde{w}}{w}\right)^{-\tilde{\eta}}}{1-\tilde{\alpha}(\mu_z\pi)^{(1-\tilde{\chi})\tilde{\eta}}}.$$

$$s = \frac{(1-\alpha)\tilde{p}^{-\eta}}{1-\alpha\pi^{(1-\chi)\eta}}.$$

From the definition of SE, which is the share of energy expenditures in value added,

$$SE = \frac{p_e e}{va} = \frac{a_0 k/\mu_z}{z^* va} = \frac{a_0}{z^*}\frac{k}{\mu_z y},$$

$$va = \frac{y-\Psi}{s} - s_{im}p_e e.$$

Also, Ψ is calibrated so that monopolistically competitive firms earn no profits in the steady state, and thus $\Psi = (1-smc)pf$, and $y - \Psi = smcy$, so that $va = mcy - s_{im}p_e e$ and

$$SE = \frac{p_e e}{va}.$$

Now, we can get

$$p_e e = \frac{a_0}{z^*} = \frac{SE}{1+SEs_{im}}\frac{mc\mu_z y}{k},$$

from which $\frac{\mu_z y}{k}$ can be found from the household's first-order condition with respect to k_{t+1} in Equation (35) and the firm's first-order condition with respect to capital in Equation (25), by solving the following nonlinear equation:

$$\left(\frac{\mu_z y}{k}\right)^{1+\varrho} \frac{mc\theta}{u^{\varrho}} - SEmc\left(\frac{\mu_z y}{k}\right) = (\frac{1}{r} - 1 + \delta)q.$$

When $\frac{\mu_z y}{k}$ is known, the ratio $\frac{uk}{h^d \mu_z}$ can be derived from the production function as

$$\frac{uk}{h^d \mu_z} = \left(\frac{(\frac{\mu_z y}{k}/u)^{-\varrho} - \theta}{1-\theta}\right)^{1/\varrho}. \tag{46}$$

Now, the aggregate production (not taking into account fixed costs) and the wage rate can be determined, as

$$\frac{y}{h^d} = \left(\theta\left(\frac{uk}{h^d \mu_z}\right)^{-\varrho} + 1 - \theta\right)^{-1/\varrho}.$$

The ratio $\frac{k}{h^d}$ can be trivially found from Equation (46) as

$$\frac{k}{h^d} = \left(\frac{uk}{\mu_z h^d}\right)\frac{\mu_z}{u}.$$

$$\frac{iv}{h^d} = (\mu_z - (1-\delta))\left(\frac{uk}{\mu_z h^d}\right)/u.$$

$$\frac{p_e e}{h^d} = \frac{a_0\left(\frac{uk}{\mu_z h^d}\right)}{uz^*}.$$

$$\frac{va}{h^d} = \frac{output}{h^d} - s_{im}\left(\frac{p_e e}{h^d}\right).$$

$$\frac{g}{h^d} = SG\left(\frac{va}{h^d}\right).$$

$$\frac{c}{h^d} = \left(\frac{va}{h^d}\right) - \left(\frac{iv}{h^d}\right) - \left(\frac{g}{h^d}\right).$$

Labor demand can be calculated from Equations (33) and (34) as

$$h^d = \left(\tilde{s} + \left(\frac{c}{h^d}\right)\frac{1-b/\mu_z}{1-b/r}\frac{\tilde{\mu}}{w(\frac{1}{\sigma}-1)}\right)^{-1}.$$

$$h = h^d \tilde{s}.$$

The real wage rate is determined by the first-order condition of a firm (26) as

$$w = mc(1-\theta)\left(1-\theta+\theta\left(\frac{uk}{h^d\mu_z}\right)^{-\varrho}\right)^{-1-1/\varrho}.$$

The steady state energy-to-capital ratio in stationary terms is

$$a_0 = z^* \frac{SE}{1+SEs_{im}} mc \frac{\mu_z y}{k}.$$

Fixed costs of production are calibrated to ensure zero profits of firms in steady state,

$$\psi = (1-smc)yh^d.$$

The aggregate supply of goods in equilibrium (taking into account the real losses from price dispersion) is

$$output = \frac{y-\psi}{s}.$$

The rental rate of capital services is calculated using Equation (38)

$$r^k = \frac{\frac{q}{r} - (1-\delta) + \frac{a_0}{z^*}}{u}.$$

Given δ_0, I find ω_0 from the first-order condition (35),

$$\omega_0 = \frac{r^k - \frac{v_0}{z^*}}{q}.$$

Steady state capital can be trivially found from

$$k = \left(\frac{uk}{\mu_z h^d}\right)\frac{\mu_z h^d}{u}.$$

Investment, energy expenditures, government expenditures, consumption, and value added, expressed in terms of consumption, are

$$iv = \left(1 - \frac{1-\delta}{\mu_z}\right)k,$$

$$p_e e = \frac{a_0 k}{\mu_z z^*},$$

$$va = output - s_{im} p_e e,$$

$$c = va - iv - g,$$

and

$$g = SGva.$$

I derive λ from the first-order condition of the household that relates to the optimal choice of labor, (34)

$$\lambda = \frac{\tilde{\mu}\sigma}{(1-h)w}\left((c - bc/\mu_z)^{1-\sigma}(1-h)^{\sigma}\right)^{1-\phi}.$$

Then,

$$f_2 = \frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{\tilde{W}\lambda(\frac{\tilde{W}}{W})^{-\tilde{\eta}} h^d}{1 - \tilde{\alpha} r \mu_z (\mu_z \pi)^{(1-\tilde{\chi})(\tilde{\eta}-1)}}.$$

$$x_2 = output \frac{\tilde{p}^{-\eta}}{1 - \alpha r \mu_z \pi^{(\chi-1)(1-\eta)}}.$$