

Trade Liberalization versus Protectionism: Dynamic Welfare Asymmetries

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Abstract

We investigate whether the losses from an increase in trade costs (protectionism) are equal to the gains from a symmetric decrease in trade costs (liberalization). We incorporate dynamics through capital accumulation into a standard Armington trade model and show that the welfare changes are asymmetric: Losses from protectionism are smaller than the gains from liberalization. In contrast, standard static trade models imply that the losses equal the gains. The intuition for asymmetry in our model is that, following protectionism, the economy can coast off of previously accumulated capital stock, so higher trade costs do not imply large losses immediately. We develop an accounting device to decompose the source of welfare asymmetries into three time-varying contributions: share of income allocated to consumption, measured productivity, and capital stock. Asymmetry in capital accumulation is the largest contributing factor, and measured productivity is the smallest.

JEL codes: F13, F11, E22

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1 Introduction

Following decades of globalization after World War II, a vast literature emerged to quantify the welfare gains from trade. In recent years, however, there has been a reversal in trade sentiment: Britain’s 2016 Brexit referendum, the U.S.-China trade war beginning in 2018, and export restrictions on essential medical equipment during the COVID-19 pandemic. Each of these represent a return to protectionism (see Fajgelbaum, Goldberg, Kennedy, and Khandelwal, 2019). These reversals lead to a natural question: Are the losses from protectionist policies equal to the gains from liberalization policies?

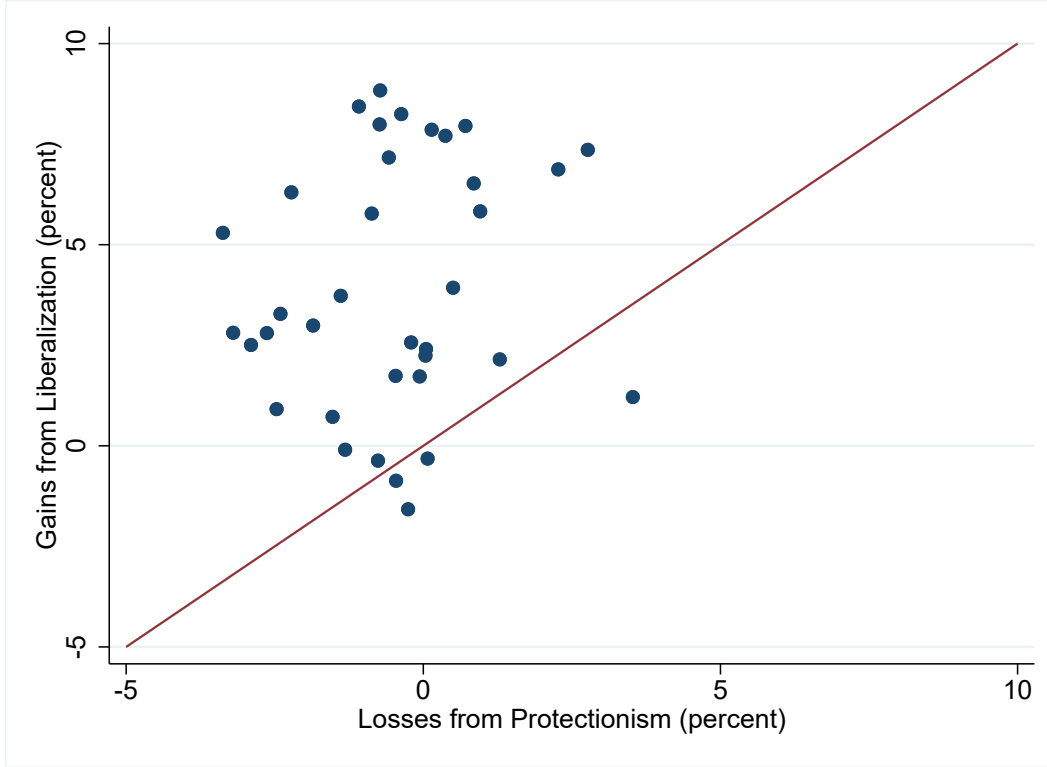
The answer from workhorse static models of trade is yes. In such models, the change in welfare is the change in income due to a change in trade costs; see, for instance, Arkolakis, Costinot, and Rodríguez-Clare (2012) and Waugh and Ravikumar (2016). If the changes in trade costs are symmetric in the protectionism and liberalization scenarios, i.e., the initial distribution of trade costs in one scenario is identical to the final distribution of trade costs in the other, then the welfare changes in the two scenarios would be equal in magnitude.

Evidence from post World War II, however, suggests that the costs of protection are less than the gains from liberalization. Figure 1 illustrates the per-capita income growth after a policy change. Five years after a protectionist policy, countries experience a reduction in income growth. The magnitude, however, is smaller compared with the increase in income growth following a liberalization policy. The difference in magnitude persists even after 10 years. Using a dataset of trade agreements from 1986 to 2016, Ait Benasser (2020) finds that import volumes respond more to liberalizing policies, whereas reversals to protectionism do not erase the gains from liberalization. Evidence of asymmetry has also been noted in spatial economics (e.g., Glaeser and Gyourko, 2005): Cities grow faster than they decay.

We incorporate dynamics through capital accumulation and show that the welfare losses from an increase in trade costs (protectionism) are smaller than the welfare gains from a symmetric decrease in trade costs (liberalization). The intuition is that, following protectionism, the economy can coast off of previous investments. In other words, decades of low trade costs facilitate the accumulation of capital stock, but increasing trade costs does not imply large losses immediately. As a result, in the short run, the magnitude of the growth rate following liberalization exceeds the magnitude of the (declining) growth rate following protectionism. We use a consumption-equivalent measure of welfare changes, as in Lucas (1987), so consumption in the short run has more weight and, hence, the losses from protectionism are smaller.

We embed a model of trade between many countries, à la Armington, into a neoclassical

Figure 1: Per-capita GDP Growth–Trade Liberalization and Protectionism



Notes: The figure illustrates the 3-year moving average after reform, relative to the pre-reform growth rate. The data in the figure are for Costa Rica, El Salvador, Guatemala, Honduras, Nicaragua, and Peru. Each dot in the figure represents, for each country and each year after the policy reform, the difference between a 3-year moving average growth rate and the pre-reform growth rate (i.e., average growth rate 10 years before the reform). The case of liberalization is on the Y-axis, and the case of (negative of) protectionism is on the X-axis. A positive number on the Y-axis indicates that countries are growing faster post-reform than pre-reform. A positive number on the X-axis indicates the opposite: Countries are growing at a slower rate post-reform than pre-reform. If all observations were to lie at the 45-degree line, that would indicate symmetric gains. Observations that are off the 45-degree line represent asymmetry. Specifically, those above the 45-degree line imply gains from liberalization that are larger than the negative losses from protectionism. This is an unbalanced panel spanning the period 1950-2017.

model of capital accumulation.¹ We then examine the transition path from an “observed” steady state to an autarky steady state and the path in the reverse direction. In both cases, we initiate a one-time, permanent change in trade costs.

We impose balanced trade in each period, which helps us compute the steady state independently from the transition.² We construct two steady states—observed and autarky—such that they are symmetric in the change in trade costs. That is, the *change* in the distribution of trade costs is the same between the two steady states. The welfare changes between the steady states are symmetric, by construction, but the transition paths between

¹All of our results go through if we use Eaton and Kortum (2002) or Krugman (1980) structures for trade since the aggregate intratemporal expressions for prices and trade flows are identical.

²Recent examples of multi-country dynamic models with capital accumulation and balanced trade include Anderson, Larch, and Yotov (2015), Alvarez (2017), Brooks and Pujolas (2018), and Mix (2023).

the two steady states need not be symmetric. The symmetry between the steady states is a critical disciplining feature of our exercise so that any asymmetries that emerge are solely from transitional dynamics.

Each country's steady-state level of consumption, output, investment, capital stock, and measured productivity can be analytically expressed in terms of its fundamental productivity, its home trade share, and some elasticities. In the long run, the investment rate is invariant to the trade regime so changes in consumption are identical to changes in income. In contrast, the share of income allocated to consumption (consumption rate) varies along the transition path. The decoupling of consumption from income implies that changes in income, brought about by changes in measured productivity and capital stock, do not fully describe consumption dynamics. In order to evaluate the consumption dynamics and, hence, dynamic measures of welfare, we turn to computation.

We calibrate the model to data for 136 countries, assuming a steady state prior to the trade reform. The calibrated model is our "open" economy where the bilateral trade volumes are those in 2015. Our closed economy is autarky, one that has prohibitively high trade costs. The protectionism scenario starts from the open steady state and suddenly moves to autarky. The liberalization scenario starts from the autarky steady state and suddenly moves to the open economy.

Our quantitative results are as follows. First, the dynamic gains and losses from the change in trade costs are heterogeneous across countries. Vietnam, for instance, is in the 95th percentile of gains and losses and its welfare change is more than 20 times that of Australia, which is in the 5th percentile. This heterogeneity is common to dynamic models of trade e.g., Alvarez (2017).

Second, the change in welfare due to the change in trade costs is asymmetric: Vietnam gains almost 60% from liberalization but loses 56% by moving to autarky. These differences do not mechanically emerge from either different base values of welfare in the two scenarios or our functional form for preferences. Our consumption-equivalent measure is invariant to such concerns.

Third, we develop an accounting procedure, much in the spirit of growth accounting, to decompose the welfare changes into contributions coming from the consumption rate, the share of spending on domestically produced goods (which implies changes in measured productivity through selection), and the capital stock. Asymmetry in capital accumulation accounts for the largest share of the asymmetry between dynamic gains and losses, followed by asymmetry in consumption rates. Asymmetry in measured productivity accounts for the smallest share.

The asymmetry in capital accumulation stems from the fact that the boost in capital stock

after liberalization exceeds the decline following protectionism. The size and persistence of asymmetry depend on the depreciation rate of capital. Asymmetry is small and disappears quickly when the depreciation rate is high. For instance, when capital fully depreciates every period, it ceases being a durable input to production, so the model becomes static for practical purposes and, thus, has no asymmetry.

The reason for the quantitative importance of capital stock in the welfare asymmetries could be that the previously accumulated capital stock helps smooth consumption. However, in a “Solow” version of our model where the consumption rate is held constant so its contribution to the asymmetry is zero, the contribution of capital stock remains large. It is twice as large as the contribution from measured productivity.

There could be alternative explanations for the asymmetry in the data. One possibility is asymmetry in the underlying shocks: Liberalization shocks may be greater in magnitude than protection shocks. We argue that asymmetries can naturally emerge in the presence of a durable factor of production, even when the magnitudes of the respective shocks are equal. Another possibility is that frictions such as downward nominal wage rigidity yield asymmetric outcomes (see Rodríguez-Clare, Ulate, and Vásquez, 2020). However, such rigidities would imply that the losses from protectionism are less than those relative to a world with no rigidities and, hence, imply more asymmetry in welfare changes relative to our model.

2 Quantitative Framework

The world economy consists of N countries, indexed by n or $i = 1, \dots, N$. Time is discrete and runs from $t = 1, \dots, \infty$.

Firms In each country, there is a representative retail firm and a representative production firm. The retail firm sources imperfectly substitutable goods from all countries that are differentiated based on their country of origin as in Armington (1969). The quantity that country n sources from country i at time t is denoted by $q_{n,i,t}$, and the aggregate basket made available by the retail firm in country n is given by

$$Q_{n,t} = \left[\sum_{i=1}^N (q_{n,i,t})^{\frac{\theta}{1+\theta}} \right]^{\frac{1+\theta}{\theta}}, \quad (1)$$

where θ is the trade elasticity. The composite good, $Q_{n,t}$, is split between domestic consumption and investment.

The production firm in country n produces the imperfectly substitutable goods using

capital and labor according to

$$y_{n,t} = A_{n,t}(k_{n,t})^\alpha (\ell_{n,t})^{1-\alpha}. \quad (2)$$

The term $A_{n,t}$ is the country-specific fundamental productivity. The inputs $k_{n,t}$ and $\ell_{n,t}$ denote the amounts of capital stock and labor used in production and α denotes capital's share in value added.

Trade International trade is subject to iceberg costs: Destination n must purchase $d_{n,i} \geq 1$ units of the good from origin i in order for one unit to arrive. As a normalization, $d_{n,n} = 1$ for all n . Trade is balanced in each period.

Households Each country admits a representative household of size L_n . Lifetime utility is the discounted sum of per-period flows:

$$U_n \equiv \sum_{t=1}^{\infty} \beta^t \ln \left(\frac{C_{n,t}}{L_n} \right), \quad (3)$$

where $C_{n,t}$ is consumption and $\beta \in (0, 1)$ the discount factor.

The household enters period t with capital stock, $K_{n,t}$. It inelastically supplies both capital and labor to domestic production firms, earning a rental rate $r_{n,t}$ on capital and a wage rate $w_{n,t}$.

Investment in physical capital is $X_{n,t}$. At the end of the period, a fraction δ of the capital stock depreciates. Capital stock at the beginning of the next period is

$$K_{n,t+1} = (1 - \delta)K_{n,t} + X_{n,t}. \quad (4)$$

The household's factor income is spent on consumption and investment, both of which have price $p_{n,t}$. The period budget constraint is thus given by

$$p_{n,t}C_{n,t} + p_{n,t}X_{n,t} = r_{n,t}K_{n,t} + w_{n,t}L_n, \quad (5)$$

2.1 Equilibrium

A competitive equilibrium satisfies the following conditions: i) taking prices as given, the representative household in each country maximizes its lifetime utility subject to its budget constraint and technology for accumulating capital, ii) taking prices as given, firms maximize profits subject to the available technologies, iii) goods and labor markets clear, and (iv) trade

is balanced. At each point in time, we take world GDP as the numéraire: $\sum_n r_{n,t}K_{n,t} + w_{n,t}L_n = 1$ for all t .

The household optimally chooses a consumption path according to the standard intertemporal Euler equation:

$$\frac{C_{n,t+1}}{C_{n,t}} = \beta \left(\frac{r_{n,t+1}}{p_{n,t+1}} + 1 - \delta \right). \quad (6)$$

Prices are determined competitively so that the price faced by country n for a product originating in country i is the country i 's marginal cost, scaled by the bilateral trade cost:

$$p_{n,i,t} = \frac{u_{i,t}d_{n,i,t}}{A_{i,t}}, \quad (7)$$

where $u_{i,t} = (r_{i,t}/\alpha)^\alpha (w_{i,t}/(1-\alpha))^{(1-\alpha)}$ is the unit cost for factor inputs in country i . Firms earn zero profits so factor payments exhaust total revenue:

$$r_{n,t}k_{n,t} = \alpha p_{n,t}y_{n,t}, \quad w_{n,t}\ell_{n,t} = (1-\alpha)p_{n,t}y_{n,t}. \quad (8)$$

Country n 's demand for products originating in country i is

$$q_{n,i,t} = \left(\frac{p_{n,i,t}}{p_{n,t}} \right)^{-(1+\theta)} Q_{n,t}, \quad (9)$$

where $Q_{n,t} = C_{n,t} + X_{n,t}$, and

$$p_{n,t} = \left(\sum_{i=1}^N (p_{n,i,t})^{-\theta} \right)^{\frac{-1}{\theta}} \quad (10)$$

is the price index for the entire basket of goods absorbed.

Market clearing requires that factor supply equals demand:

$$K_{n,t} = k_{n,t} \text{ and } L_n = \ell_{n,t} \quad (11)$$

and each country's gross production is absorbed globally:

$$p_{n,t}y_{n,t} = \sum_{i=1}^N p_{n,i,t}q_{n,i,t}. \quad (12)$$

Balanced trade requires that the final goods market clears in each country:

$$y_{n,t} = Q_{n,t}. \quad (13)$$

2.2 Calibration

We use a set of 136 countries spanning the world income distribution. We assume the world economy is in steady state as of 2015. This year is prior to the Brexit vote and the U.S.-China trade war and, thus, is unlikely to reflect any anticipatory effects of those shocks. Each period in the model corresponds to one year.

The values for the common parameters are reported in Table 1. We set the discount factor $\beta = 0.96$ so that the steady-state real interest rate is about 4 percent. We set the share of capital in value added $\alpha = 0.33$ (Gollin, 2002) and the rate of depreciation $\delta = 0.06$. Finally, we set the trade elasticity $\theta = 4$ (Simonovska and Waugh, 2014).

Table 1: Common parameters

| | | |
|----------|--------------------------------|------|
| β | Discount factor | 0.96 |
| α | Capital's share in value added | 0.33 |
| δ | Rate of capital depreciation | 0.06 |
| θ | Trade elasticity | 4 |

We calibrate the productivity and trade costs using data on bilateral trade flows and geography variables from CEPII, along with aggregate GDP and population from the Penn World Table version 10.0.³ We assume trade costs are a function of geography and include an exporter-specific component, as in Waugh (2010).

$$-\theta \ln(d_{n,i}) = \sum_{p=1}^6 \gamma^p \text{dist}_{n,i}^p + \phi B_{n,i} - e_i.$$

As a result, the ratio of country n 's imports from country i , relative to its domestic purchases, can be expressed as

$$\left(\frac{X_{n,i}}{X_{n,n}} \right) = \exp \left(\ln(S_i) - \ln(S_n) + \sum_{p=1}^6 \gamma^p \text{dist}_{n,i}^p + \phi B_{n,i} - e_i + u_{n,i} \right),$$

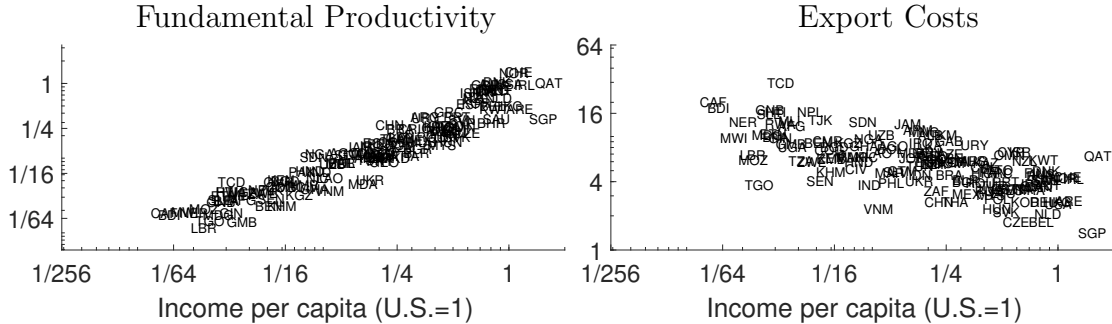
We run a regression using PPML as in Silva and Tenreyro (2006) to handle the zeros in the bilateral trade data. Structurally, $S_n = A_n (r_n^\alpha w_n^{1-\alpha})^{-\theta}$ is the estimated importer fixed effect for country n . Using rental rates, labor compensation, and $\theta = 4$, we recover A_n from the fixed effect. The contribution to trade costs of the distance between country n and i falling into the p^{th} interval (in miles), defined as $[0, 350]$, $[350, 750]$, $[750, 1500]$, $[1500, 3000]$, $[3000, 6000]$, $[6000, \text{maximum})$, is $d_{n,i}^p$. The other gravity control variables in $B_{n,i}$ include common

³<http://www.cepii.fr/> and <https://www.rug.nl/ggdc/productivity/pwt/?lang=en>, (see Feenstra, Inklaar, and Timmer, 2013).

border effect, common currency effect, and regional trade agreement between country n and country i . We estimate $A_i - e_i$ as the exporter fixed for country i , and recover e_i given A_i .

The estimates for productivity and trade costs are depicted in Figure 2. The left panel shows a strong positive correlation between the estimated fundamental productivity and income per capita (each relative to the US). The right panel shows that countries with higher income per capita have lower export costs.

Figure 2: Calibrated Productivity and Trade Costs



Notes: Income per capita is from Penn World Table version 10.0. Fundamental productivity and export costs are the authors’ calculations. Fundamental productivity and income per capita in the U.S. are normalized to one. Export cost is the trade-weighted value of bilateral export costs for each country.

2.3 Model Solution

This model is a simplified version of that in Ravikumar, Santacreu, and Sposi (2019), particularly in that there are no adjustment costs to investment and trade is balanced in each period. Because of these differences we develop a distinct algorithm to compute the exact transition path. Intuitively, we interpret each point in time as a separate dimension in the commodity space, similar to an Arrow-Debreu economy. We restrict the time dimension to be bounded by some limit T , which is “sufficiently large” to ensure that the economy has reached a steady state. Since investment is reversible, we derive a lifetime budget constraint and allocate the present value of lifetime wealth to consumption at each point in time.

To illustrate this idea, we briefly digress and work through some equations that are explicitly incorporated in the solution method. First, we use equations (4) and (5) to derive a lifetime budget constraint, so the present value of lifetime wealth is given by

$$\mathcal{W}_n \equiv \sum_{t=1}^T \frac{w_{n,t}L}{P_{n,t}(1 + \mathcal{R}_{n,t})} + K_{n,1} - \frac{K_{n,T+1}}{1 + \mathcal{R}_{n,T}}, \quad (14)$$

where $K_{n,T}$ is the new steady state to where the economy is headed. The compounded, real,

gross rate of return on investment is defined as

$$1 + \mathcal{R}_{n,t} \equiv \prod_{t'=1}^t \left(\frac{r_{n,t'}}{p_{n,t'}} + 1 - \delta \right).$$

Euler equation (6) implies that the present value of consumption in any period is some share, $\xi_{n,t}$, of lifetime wealth, where $\sum_{t=1}^T \xi_{n,t} = 1$:

$$\frac{C_{n,t}}{1 + \mathcal{R}_{n,t}} = \underbrace{\left(\frac{\beta^t}{\sum_{t'=1}^T \beta^{t'}} \right)}_{\xi_{n,t}} \mathcal{W}_n$$

Detailed steps are outlined in Algorithm A.2, with corresponding equilibrium conditions provided in Table A.2.

3 Dynamic Welfare Asymmetries

We investigate asymmetries in dynamic welfare gains from trade under two scenarios: liberalization and protectionism. In the liberalization scenario, we begin in a steady state with autarky levels of trade costs and then permanently reduce the trade costs in period 1 to the calibrated levels. In the protection scenario, we begin in a steady state with the calibrated level trade costs and then permanently increase the trade costs in period 1 to the autarky levels. In both scenarios, we assume that the change in trade cost is a one-time unanticipated permanent event; and after the sudden change, agents have perfect foresight. By design, the changes in the two trade costs are symmetric, in that the initial distribution of trade costs in one scenario is identical to the final distribution of trade costs in the other.

We compute changes in welfare using consumption equivalent units, as in Lucas (1987). Specifically, starting from a steady state at time 0 with consumption C_n^* , we compute the proportionate change in per-period consumption that is required to make the household indifferent from moving to an alternative consumption path $\{C_{n,t}\}_{t=1}^{\infty}$. The compensating factor, or welfare change, is the value of Λ_n that satisfies

$$\sum_{t=1}^{\infty} \beta^t \ln(\Lambda_n C_n^*) = \sum_{t=1}^{\infty} \beta^t \ln(C_{n,t}).$$

We simplify the expression and compute the dynamic gains in log points as

$$\lambda_n \equiv \ln(\Lambda_n) = (1 - \beta) \sum_{t=1}^{\infty} \beta^t \ln \left(\frac{C_{n,t}}{C_n^*} \right).$$

In the liberalization scenario, λ_n will be positive since consumption increases relative to the initial steady state and in the protectionism scenario λ_n will be negative. In the figures that follow, we report the negative of λ_n for ease of illustration.

With balanced trade, we can compute the steady state without knowing the transition path. In particular, the steady-state consumption rate (as a share of output) is

$$\varphi_n^* \equiv \frac{p_n^* C_n^*}{r_n^* K_n^* + w_n^* L_n} = 1 - \frac{\alpha \delta}{\frac{1}{\beta} - (1 - \delta)}. \quad (15)$$

Since this rate depends only on the deep parameters that are common across countries and constant over time, it is invariant to the trade regime. So, consumption is proportional to output in steady state. Thus, the per-capita consumption in steady state is:

$$\frac{C_n^*}{L_n} \propto \left(\frac{A_n}{(\pi_{n,n}^*)^\theta} \right)^{\frac{1}{1-\alpha}} \quad (16)$$

Following changes in trade costs, the only terms that change are the home trade shares and so are sufficient to characterize the change in consumption and, hence, the welfare gains or losses between steady states. Since we construct the change in the distribution of trade costs to be the same in both liberalization and protectionism scenarios, there is no asymmetry when we compare only steady states. That is, the presence of capital does not induce any asymmetry between steady states.⁴

Along the transition path, consumption changes gradually over time following a one-time change in trade costs. Thus, the concept of symmetry in a dynamic context must include all of the time periods. With $\beta < 1$, we choose a terminal period T sufficiently large so that $\beta^T \approx 0$ and the economy is sufficiently “close” to the new steady state. We then compute the dynamic welfare change as

$$\lambda_n = (1 - \beta) \sum_{t=1}^T \beta^t \ln \left(\frac{C_{n,t}}{C_n^*} \right) \quad (17)$$

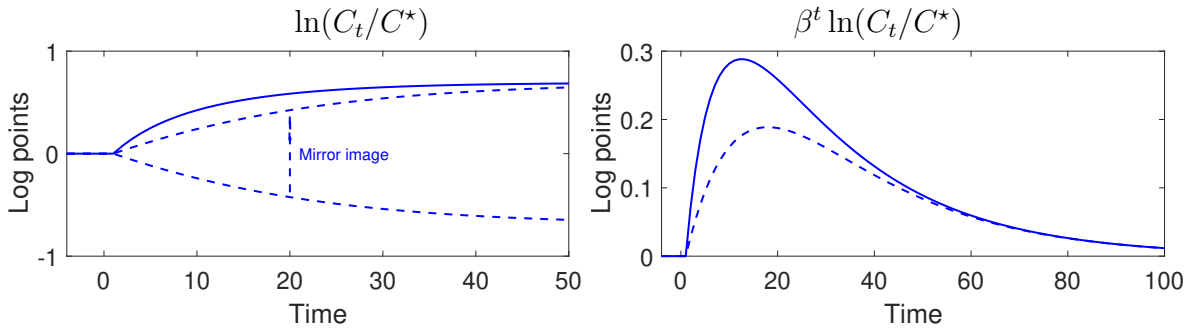
⁴While capital does not give rise to asymmetry between steady states, it is a primary determinant of the welfare changes. Ravikumar, Santacreu, and Sposi (2023) document that the contribution of capital to gains (or losses) is four times that of measured productivity.

Denoting the change in welfare following trade liberalization by λ_n^{Lib} , and that following protectionism by λ_n^{Pro} , we define the dynamic welfare asymmetry as $\zeta_n = \lambda_n^{\text{Lib}} + \lambda_n^{\text{Pro}}$. Note that $\lambda_n^{\text{Lib}} > 0$ and $\lambda_n^{\text{Pro}} < 0$, and welfare is dynamically symmetric when $\zeta_n = 0$.

Figure 3 provides a generic illustration of what transition paths for consumption might look like following permanent trade shocks. The economy is at the initial steady state through period 0. Then, in period 1, the shock hits and the economy begins transitioning to a new steady state. The liberalization path (Lib) begins in a steady state at a value of 1, then gradually transitions to a new steady state of 2. The protectionism path (Pro) begins in a steady state at a value of 2, then gradually transitions to 1.

To investigate the presence of dynamic asymmetries, the left panel involves two steps. The first step is to compute log of the normalized consumption path under liberalization or protectionism relative to the respective initial steady states, $\{\ln(C_t/C^*)\}$, meaning that both paths begin at the value 0. The second step constructs a mirror image of the protectionism path around zero. If the two transition paths are *symmetric*, then the negative of the protection path (increasing dashed curve) would overlap with the liberalization path (solid curve) in the left panel.

Figure 3: Illustrative Consumption Paths Under Liberalization and Protectionism



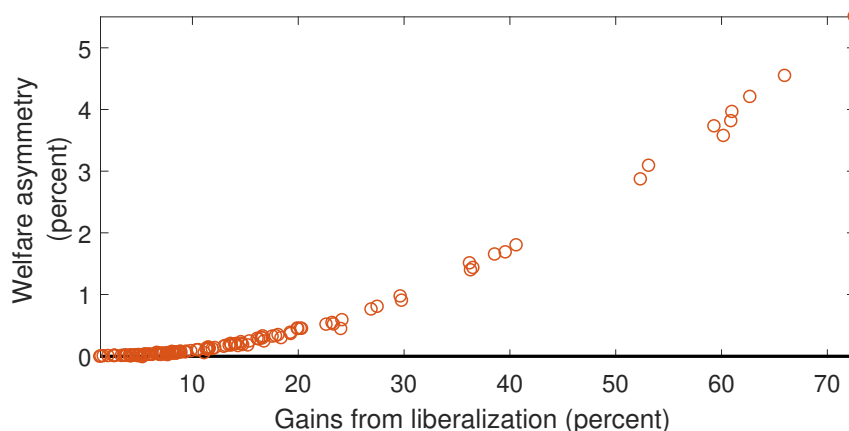
Notes: Units on the vertical axis are log-deviations from initial steady state.

To map the consumption paths into welfare, we need to take into account the role of discounting. The right panel plots the discounted, log-normalized paths, $\{\beta^t \ln(C_t/C^*)\}$, with the protectionism path negated (dashed curve). The area enclosed between each curve and the zero line captures the dynamic welfare change. The area between the solid and dashed curves depicts the dynamic welfare asymmetry. Since the liberalization path lies above the reflected protectionism path in this example, the asymmetry would be positive, meaning that the gains from liberalization exceed the losses from protectionism.

3.1 Quantitative Findings

Among the 136 countries, the dynamic welfare gains from liberalization range from 1.3% to 72.6% (horizontal axis in Figure 4). The gains are small for most countries, but they are skewed. The mean gain is 15.1% and the median is 9.2%. The heterogeneity in gains is due country size (measured by world’s share in total GDP) and trade cost (measured by trade-weighted average import cost). Consider three countries: Brazil, Kyrgyzstan, and Mexico. Brazil and Mexico are similar in size, but the trade cost in Mexico is two thirds of that in Brazil. Thus, Mexico’s gains from liberalization are more than Brazil’s, by a factor of 2.5. Kyrgyzstan and Mexico have similar trade costs, but Mexico is nearly 200 times larger than Kyrgyzstan. As a result, Kyrgyzstan gains 2.5 times more than Mexico.

Figure 4: Welfare Asymmetries and Gains from Liberalization



Notes: Welfare asymmetry is the percentage point difference between the gains from liberalization and losses from protection.

The country’s ranking of losses from protectionism is virtually identical to that of gains from liberalization. When comparing the magnitudes, we find that the gains exceed the losses. That is, the asymmetry is positive, as illustrated in Figure 4. The asymmetry for Vietnam is 3.7% (difference between 59.3% gain and 55.6% loss).

The cross-country distribution of asymmetry is also skewed. The asymmetry is 0% for Australia, 0.06% for Burkina Faso, and 5.5% for Singapore.⁵

In our analysis, recall that the asymmetries reflect only the asymmetric transitional dynamics, not steady states. To get a sense of whether the asymmetries accrue in the short run or the long run, we compute the change in consumption. In particular, for Vietnam, the

⁵Nine countries in our sample have negative asymmetries, i.e., the losses from protectionism exceed the gains from liberalization. However, the negative asymmetries are negligible in magnitude, with the largest negative asymmetry being -0.02% .

log-change in consumption as of period 1, relative to the initial steady state, is 31.6% following liberalization and -27.5% following protectionism, yielding a consumption asymmetry of 4.1% in period 1. The percent change in consumption as of period 5, relative to the initial steady state, is 47.3% following liberalization and -39.8% following protectionism, yielding an asymmetry of 7.5%. The percent change in consumption as of period 10, relative to the initial steady state, is 57.5% following liberalization and -50.8% following protectionism, yielding an asymmetry of 6.7%. That is, the consumption asymmetry initially widens, then shrinks. These numbers indicate that, after liberalization, consumption grows rapidly in the short and medium runs; whereas, after protectionism the loss in consumption accrues more gradually over time.

The intertemporal Euler equation expresses the cumulative log-change in consumption between any two periods t' and t as a function of the cumulative gross rate of return to investment, net of depreciation, between t' and t :

$$\ln(C_{n,t}) - \ln(C_{n,t'}) = (t - t') \ln(\beta) + \sum_{\tau=t'+1}^t \ln\left(\frac{r_{n,\tau}}{p_{n,\tau}} + 1 - \delta\right)$$

The first term on the right-hand side associated with the discount factor does not depend on whether the scenario is liberalization or protectionism. Thus, the asymmetries in consumption are captured by the second term on the right-hand side—dynamics in the cumulative return to investment. This term captures the *dividends* afforded by past investment decisions. Following liberalization, investment jumps and yields a long-lasting flow of returns in the future. Following protectionism, the yield from past investments continues. This is the key reason for the asymmetric dynamics. In order to get a better understanding of how the asymmetries accrue over time, we develop an accounting procedure, much like the one used in growth accounting exercises.

3.2 Accounting Decomposition of the Welfare Asymmetries

In equilibrium, consumption can be expressed as a (potentially time varying) share of output, where output is determined by measured productivity and capital stock. Specifically, we have two equations that characterize the dynamics of aggregate consumption:

$$\begin{aligned} C_{n,t} &= \varphi_{n,t} Y_{n,t} \\ Y_{n,t} &= Z_{n,t} K_{n,t}^\alpha L_n^{1-\alpha} \end{aligned}$$

The term $\varphi_{n,t}$ is the consumption rate (consumption as a share of output) at time t and is determined endogenously along the transition path. The variable $Z_{n,t}$ denotes measured productivity and can be decomposed into an exogenous fundamental productivity and an endogenous component that is described by the home trade share and trade elasticity:

$$Z_{n,t} \propto A_{n,t}(\pi_{n,n,t})^{\frac{-1}{\theta}}$$

Transition dynamics The transition path for consumption, relative to the initial steady state, can be characterized by

$$\frac{C_{n,t}}{C_n^*} = \left(\frac{\varphi_{n,t}}{\varphi_n^*} \right) \left(\frac{\pi_{n,n,t}}{\pi_{n,n}^*} \right)^{\frac{-1}{\theta}} \left(\frac{K_{n,t}}{K_n^*} \right)^\alpha$$

Taking logs and inserting into equation (17), the dynamic welfare change becomes

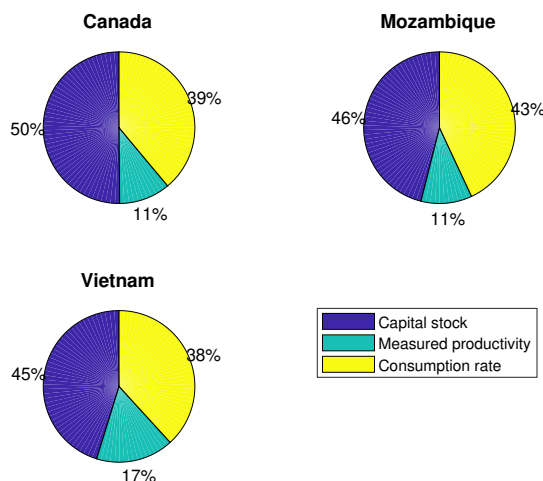
$$\begin{aligned} \lambda_n &= (1 - \beta) \sum_{t=1}^T \beta^t \left[\ln \left(\frac{\varphi_{n,t}}{\varphi_n^*} \right) - \left(\frac{1}{\theta} \right) \ln \left(\frac{\pi_{n,n,t}}{\pi_{n,n}^*} \right) + \alpha \ln \left(\frac{K_{n,t}}{K_n^*} \right) \right] \\ &= (1 - \beta) \underbrace{\sum_{t=1}^T \beta^t \ln \left(\frac{\varphi_{n,t}}{\varphi_n^*} \right)}_{\lambda_n^\varphi} \\ &\quad - (1 - \beta) \underbrace{\left(\frac{1}{\theta} \right) \sum_{t=1}^T \beta^t \ln \left(\frac{\pi_{n,n,t}}{\pi_{n,n}^*} \right)}_{\lambda_n^\pi} \\ &\quad + (1 - \beta) \alpha \underbrace{\sum_{t=1}^T \beta^t \ln \left(\frac{K_{n,t}}{K_n^*} \right)}_{\lambda_n^K} \end{aligned} \tag{18}$$

The first term, λ_n^φ , characterizes the contribution from changes in the consumption share. The second term, λ_n^π , characterizes the contribution from measured productivity. The third term, λ_n^K , captures the contribution from changes in the capital stock. Each of these terms are dynamically co-dependent, so each term has indirect effects on other terms. For example, changes in the consumption rate in one period induce changes in the capital stock in subsequent periods; and differential changes in capital stocks across countries lead to changes in home trade shares. That said, this is just an accounting device similar to the ones used in traditional growth accounting exercises. We use it to illustrate how each component contributes to welfare changes along the transition.

Recall, $\zeta_n = \lambda_n^{\text{Lib}} + \lambda_n^{\text{Pro}}$ is the dynamic welfare asymmetry. If welfare changes are dynamically symmetric then $\zeta_n = 0$. Similarly, we define ζ^φ, ζ^Z , and ζ^K as the contributions to dynamic welfare asymmetries stemming from the three main components: consumption rate, measured productivity, and capital stock. By construction, $\zeta_n = \zeta_n^\varphi + \zeta_n^Z + \zeta_n^K$.

Quantitative findings on the decomposition Figure 5 illustrates the contribution from the three terms in equation (18). For Vietnam, the consumption rate accounts for 38% of the dynamic asymmetry (1.4% out of 3.7%), less than the contribution of 45% from capital stock (1.7% out of 3.7%). The remaining 17% (0.6% out of 3.7%) is accounted for by measured productivity. While the asymmetries vary across countries (0.05% in Canada and 0.3% in Mozambique), the decomposition for all countries in our sample is similar.

Figure 5: Decomposition of Welfare Asymmetries

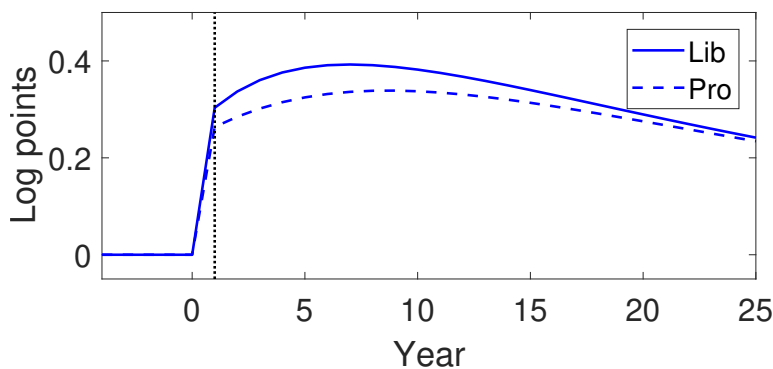


Notes: The welfare asymmetry is 0.05% for Canada, 0.3% for Mozambique, and 3.7% for Vietnam.

As noted earlier, nine countries in our sample have negligible negative asymmetries. Negative asymmetries emerge as a result of measured productivity dynamics. Following a move to autarky, the home trade share jumps to one instantaneously and remains there, meaning that measured productivity declines once and for all. Following liberalization, however, the home trade share drops on impact, then undergoes small gradual increases or decreases over time, depending on whether the country experiences faster or slower capital accumulation relative to the rest of the world. Countries experiencing slower capital accumulation, all else equal, experience a decline in their share of global output and thus experience a decrease in their home trade share and an increase in measured productivity. For such countries, because the future is discounted, their welfare is impacted more by the drop in measured productivity after autarky than the rise in measured productivity after liberalization.

Figure 6 illustrates how the asymmetries emerge for Vietnam. The solid line corresponds to the liberalization scenario, and the dashed line to the protectionism scenario. The entirety of the asymmetry occurs within the first 25 periods. The initial boost in consumption under liberalization exceeds the initial drop in consumption under protection in absolute value. For a few periods the growth rate of consumption under liberalization continues to exceed the rate of decline under protection. Eventually the relationship between the two growth rates reverses. The reason for the eventual reversal is that the two steady states are symmetric, so the “distance” covered between steady states is the same in both directions. Since the future is discounted, the growth asymmetries in the beginning of the transition carry more weight, so that the dynamic gains from liberalization exceed the losses from protection.

Figure 6: Consumption Paths for Vietnam

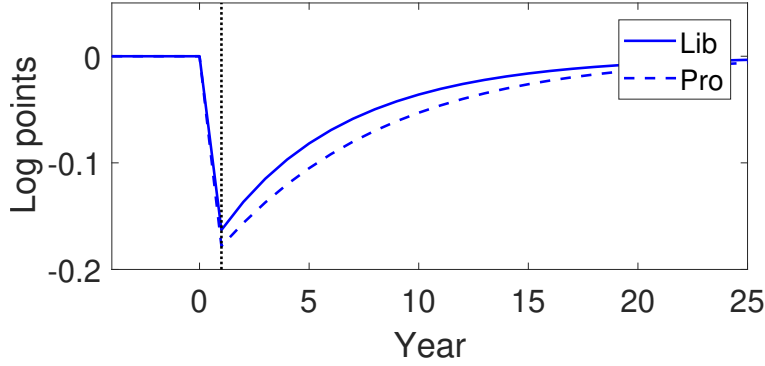


Notes: Discounted log-deviations from initial steady state for Vietnam. “Lib” refers to transition path under trade liberalization following a change from autarky to observed trade costs. “Pro” refers to *negative* transition path under protectionism following a change from observed trade costs to autarky. The “shock” occurs in period 1.

Asymmetry in consumption The asymmetry in the consumption path could arise from two (extreme) scenarios: (i) asymmetry in the share of output allocated to consumption but no asymmetry in output and (ii) asymmetry in output but no asymmetry in the consumption share. Our results indicate that the scenario in Figure 6 is somewhere in between. To assess (i), Figure 7 illustrates the consumption share. It shows that the share “mimics” the consumption path in Figure 6: The path is asymmetric, and the asymmetry is more in the short run and persists for 25 periods. To assess (ii), Figure 8 illustrates asymmetry in the two components of output—measured productivity and capital stock. There is indeed asymmetry in the path of output and it is predominantly due to the asymmetry in capital stock.

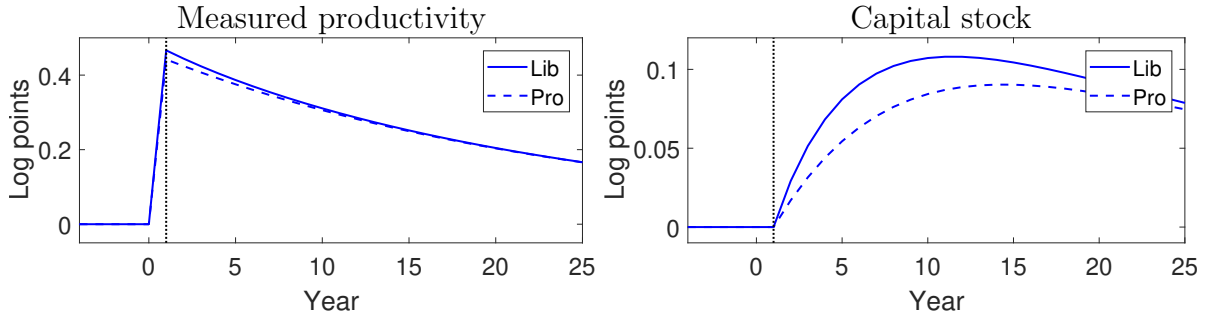
The transition paths for measured productivity, which are accounted for entirely by

Figure 7: Consumption Rate for Vietnam



Notes: Discounted log-deviations from initial steady state for Vietnam. Consumption rate is the ratio of consumption to GDP, $\varphi_{n,t}$. “Lib” refers to transition path under trade liberalization following a change from autarky to observed trade costs. “Pro” refers to *negative* transition path under protectionism following a change from observed trade costs to autarky. The “shock” occurs in period 1.

Figure 8: Measured Productivity and Capital Stock for Vietnam



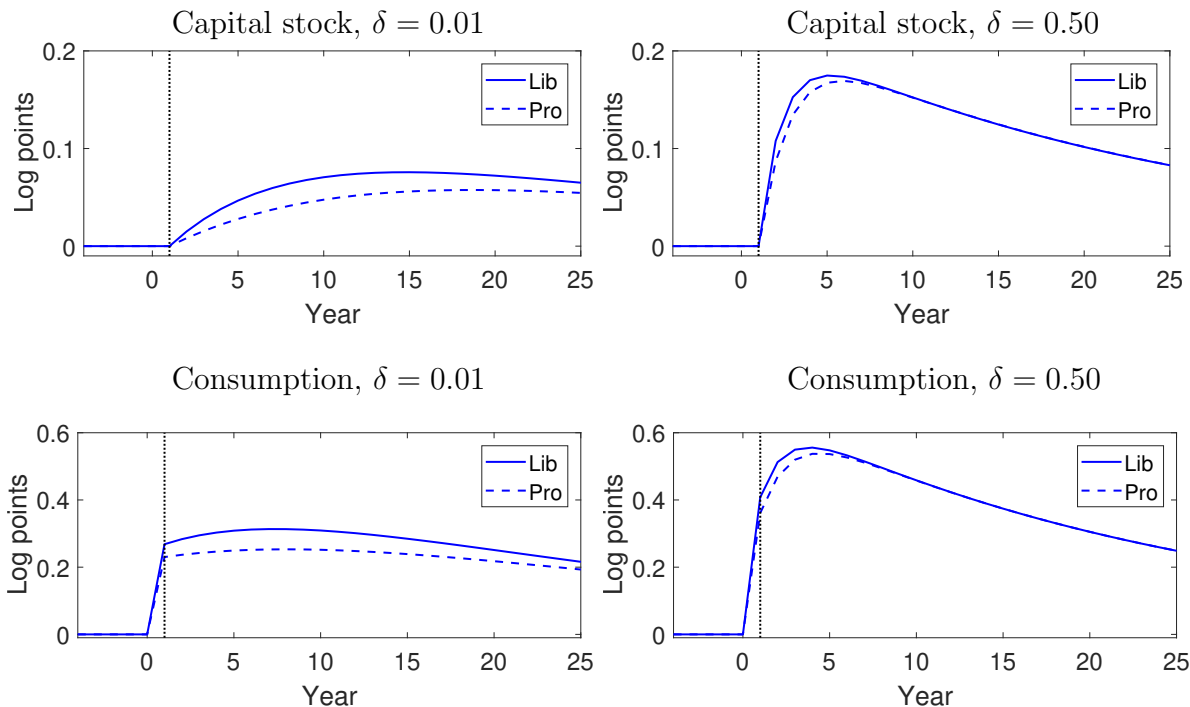
Notes: Discounted log-deviations from initial steady state for Vietnam: (i) measured productivity (home trade share adjusted by the trade elasticity), $(\pi_{n,n,t})^{1/\theta}$, and (ii) capital stock (adjusted by capital’s share in GDP), $(K_{n,t})^\alpha$. “Lib” refers to transition path under trade liberalization following a change from autarky to observed trade costs. “Pro” refers to *negative* of the transition path under protectionism following a change from observed trade costs to autarky. The “shock” occurs in period 1.

change in home trade share, are quantitatively close to being symmetric. However, small asymmetries emerge in the first few periods. In the case of Vietnam, being a small country, its productivity initially jumps after liberalization and then undergoes a slight gradual decline as it accumulates capital more quickly than the rest of the world and grows in relative size, resulting in a decline in its home trade share from period 2 onwards. This results in a positive contribution from its measured productivity.

The short-run boost in the capital stock after liberalization exceeds the corresponding short-run decline after protectionism. The primary reason is that capital is durable, so under protectionism the economy can coast off of previously accumulated capital stock and keep

consumption smooth. Both the size of the asymmetry in any given period, as well as how long the asymmetry persists, depends on the depreciation rate. Figure 9 demonstrates this point. The top-left panel shows the transition paths for capital when the depreciation rate $\delta = 0.01$, while the top-right panel shows these paths when $\delta = 0.5$. With low depreciation, the distance between the liberalization and protectionism paths is greater in every period relative to the distance under high depreciation, and clearly persists for a longer time. These dynamics directly translate into different asymmetries in consumption paths, as shown in the bottom two panels. With $\delta = 0.01$, the total welfare asymmetry is 5.8% for Vietnam, with 2.3% stemming from capital stock, 2.5% from consumption rate, and 1% from measured productivity. With $\delta = 0.50$, the total welfare asymmetry is only 0.7% in Vietnam, with 0.3% stemming from asymmetric paths for the capital stock, 0.2% from the consumption rate, and 0.2% from measured productivity. In sum, asymmetry in the capital stock accounts for about four-tenths of the total welfare asymmetry under various depreciation rates. Meanwhile the contribution from asymmetries in the consumption rate decreases with higher rates of depreciation, while the contribution from asymmetries in measured productivity increases.

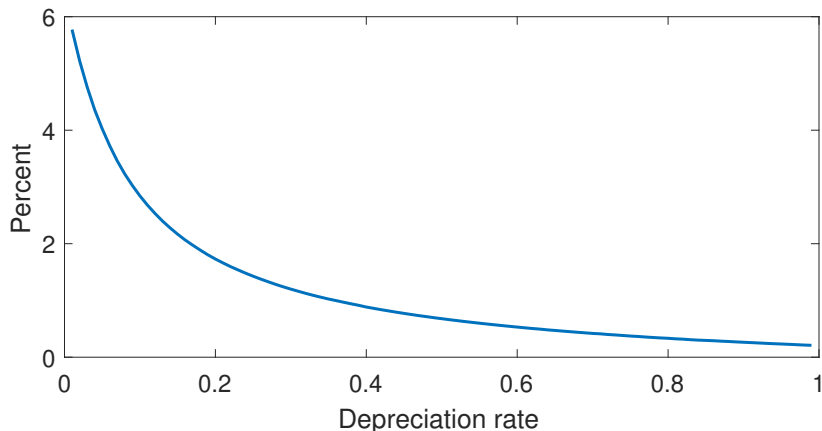
Figure 9: Capital and Consumption Under Low and High Depreciation Rates for Vietnam



Notes: Discounted log-deviations from initial steady state for Vietnam. Consumption, $C_{n,t}$, and capital stock adjusted by capital’s share in GDP, $(K_{n,t})^\alpha$. “Lib” refers to transition path under trade liberalization following a change from autarky to observed trade costs. “Pro” refers to *negative* of the transition path under protectionism after a change from observed trade costs to autarky. The “shock” occurs in period 1.

Depreciation The empirically plausible estimate of the depreciation rate is important for our quantitative results. To see this we compare the gains from liberalization with the losses from protectionism under values for the depreciation rate ranging from 0.01 to 0.99. The asymmetry for Vietnam is plotted in Figure 10. The asymmetry is high when capital depreciates slowly. Past investments yield returns long into the future, so asymmetries persist for a very long time. Conversely, the welfare asymmetry is low when capital depreciates quickly. Intuitively, when capital fully depreciates, it essentially becomes a “nondurable” intermediate input in production, rendering the model akin to a static one, which has no asymmetry.

Figure 10: Welfare Asymmetry for Vietnam Under Various Depreciation Rates



Capital stock and consumption smoothing There are two natural questions at this stage. First, how much does the asymmetry hinge on the economy being able to “eat” into the capital stock following a protectionist policy? That is, if we impose an irreversible-investment constraint that gross investment must be non-negative, then what are the implications for the transitional dynamics under protection? Second, under protectionism, the previously accumulated capital stock helps smooth consumption rendering the cost of protection to be low. Put differently, is the quantitative importance of capital stock in the decomposition of welfare asymmetries in Figure 5 stemming solely from its role in consumption smoothing?

The answer to the first question is, irreversible investment is not important for our quantitative results. In our calibrated model, the gross investment is almost always positive. When the protectionist policy is introduced, while all countries immediately reduce their investment rates, gross investment stays positive. The only exception is Singapore. Even for Singapore, the gross investment is negative only in the initial period. Thus, our results on asymmetry do not hinge on our specification that investment is reversible.

To answer the second question, we disentangle the contributions from the capital stock and consumption rate by considering a specification where the consumption rate is held constant. That is, saving decisions are exogenous as in the Solow growth model. We refer to this specification as the *Solow specification*, and refer to the baseline specification above as the *neoclassical specification*. In the Solow specification, an exogenous fraction, $\bar{\varphi}$, of current income is spent on current consumption in every period:

$$p_{n,t}C_{n,t} = \bar{\varphi} (r_{n,t}K_{n,t} + w_{n,t}L_n). \quad (19)$$

By construction, optimal consumption smoothing using previously accumulated capital stock is not possible in the Solow specification. However, note that the Solow specification admits an identical steady state as the neoclassical specification when the exogenous consumption rate $\bar{\varphi}$ is chosen appropriately as in equation (15). The two specifications differ only in the transitional dynamics.

The dynamics of the Solow specification satisfy all but one of the conditions from the neoclassical specification. Namely, equation (19) replaces equation (6). The solution boils down to a sequence of static calculations. In each period the capital stock is pre-determined so we apply a standard algorithm to compute the static equilibrium in that period as in Alvarez and Lucas (2007). The capital stock available in the next period is the outcome of the investment made in the current period, which is decided in a non-forward-looking fashion. Detailed steps are outlined in Algorithm A.1, with corresponding equilibrium conditions provided in Table A.1.

The Solow specification yields positive asymmetries, qualitatively in line with those from the neoclassical specification. The median gain from liberalization is 9.2% and the median loss from protectionism is 9.1%, yielding a 0.1% asymmetry for the median country. The countries at the 5th, 50th, and 95th percentiles are Australia, Burkina Faso, and Vietnam, respectively, as in the neoclassical specification. For Vietnam, the asymmetry is 3.1%, a bit lower than the value of 3.7% in the neoclassical specification. In this case the contribution from the consumption rate is nil, by design, meaning that the contribution from capital is not solely a byproduct of the consumption rate dynamics. The contribution from capital (2.0% out of 3.1%) is about twice as large as the contribution from measured productivity (1.1% out of 3.1%). In the neoclassical specification the contribution of capital relative to measured productivity was a factor of 2.8. In both the neoclassical and Solow specifications, capital accumulation accounts for more of the asymmetry in welfare than the consumption rate or measured productivity.

4 Conclusion

We develop a tractable method to decompose welfare changes into contributions from (i) changes in capital stock, (ii) changes in consumption share of income, and (iii) changes in measured productivity, which reflect changes in home trade share through selection. In each of our specifications, we provide algorithms that can efficiently compute the exact transitional dynamics for the global economy in a matter of minutes.

Our results extend to other contexts that involve durable factors of production, such as human capital and intellectual property. For instance, trade liberalization may affect the incentive to invest in skills, as in Auer (2015) and Burstein and Vogel (2017). The stock of skills is likely to persist in spite of the trade regime. Thus, akin to physical capital, human capital may exhibit an asymmetric response to changes in the trade costs. Indeed, other confounding factors, such as sector-specific skills, may affect the usefulness of certain skills under different trade regimes; we leave this interesting issue for future research.

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A Dynamic Equilibria and Computational Algorithms

This section of the Appendix provides equilibrium conditions and algorithms to compute the exact transitional dynamics for each model specification. The equilibrium conditions for the Solow specification are given in Table A.1, and the corresponding methodology to compute the dynamic equilibrium is in Algorithm A.1. The equilibrium conditions for the neoclassical specification are given in Table A.2, and the corresponding methodology to compute the dynamic equilibrium is in Algorithm A.2.

Table A.1: Equilibrium Conditions in Model with Exogenous Consumption Rate (Solow Specification)

| | | |
|-----|---|--------------------|
| 1 | $p_{n,i,t} = \frac{u_{i,t} d_{n,i}}{A_i}$ | $\forall(n, t)$ |
| 2 | $p_{n,t} = \left(\sum_{i=1}^N (p_{n,i,t})^{-\theta} \right)^{-\frac{1}{\theta}}$ | $\forall(n, t)$ |
| 3 | $p_{n,t} C_{n,t} = \bar{\varphi}(r_{n,t} K_{n,t} + w_{n,t} L_n)$ | $\forall(n, t)$ |
| 4 | $p_{n,t} C_{n,t} + p_{n,t} X_{n,t} = r_{n,t} K_{n,t} + w_{n,t} L_n$ | $\forall(n, t)$ |
| 5 | $Q_{n,t} = C_{n,t} + X_{n,t}$ | $\forall(n, t)$ |
| 6 | $q_{n,i,t} = (p_{n,i,t}/p_{n,t})^{-(1+\theta)} Q_{n,t}$ | $\forall(n, i, t)$ |
| 7 | $p_{n,t} y_{n,t} = \sum_{i=1}^N p_{n,i,t} q_{n,i,t}$ | $\forall(n, t)$ |
| 8 | $r_{n,t} k_{n,t} = \alpha p_{n,t} y_{n,t}$ | $\forall(n, t)$ |
| 9 | $w_{n,t} \ell_{n,t} = (1 - \alpha) p_{n,t} y_{n,t}$ | $\forall(n, t)$ |
| 10 | $K_{n,t} = k_{n,t}$ | $\forall(n, t)$ |
| 11 | $L_n = \ell_{n,t}$ | $\forall(n, t)$ |
| 12 | $p_{n,t} Q_{n,t} = p_{n,t} y_{n,t}$ | $\forall(n, t)$ |
| 13 | $K_{n,t+1} = (1 - \delta) K_{n,t} + X_{n,t}$ | $\forall(n, t)$ |
| 13s | $X_n^* = \delta K_n^*$ | $\forall(n, t)$ |

Note: Units costs $u_{n,t} = B(r_{n,t})^\alpha (w_{n,t})^{(1-\alpha)}$ with constant $B = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)}$. In steady state, condition 13 collapses to condition 13s.

Algorithm A.1 Model With Exogenous Investment Rate (Solow Specification)

The following steps reference equilibrium conditions in Table A.1.

1. Start in period 1 with the initial capital stock, $K_{n,1}$, as given. In other periods $t > 1$, capital is pre-determined from the equilibrium investment in period $t - 1$ (described below).
 - (a) Guess a vector of wages for period t : \mathbf{w}_t . Normalize this guess to the $(N - 1)$ -simplex:
 $\Delta \equiv \{\mathbf{w} \in \mathbb{R}^N : \sum_{n=1}^N w_n L_n = 1\}$.
 - (b) Use conditions 8–11 to solve for the vector of rental rates, $r_{n,t}(\mathbf{w}_t)$.
 - (c) Compute bilateral prices, $p_{n,i,t}(\mathbf{w}_t)$, using condition 1. This notation exposes the dependency of the computed prices on the guessed factor prices. Then compute country-level prices, $p_{n,t}(\mathbf{w}_t)$, using condition 2.
 - (d) Compute investment, $X_{n,t}(\mathbf{w}_t)$ and consumption $C_{n,t}(\mathbf{w}_t)$, using condition 3 and 4, respectively.
 - (e) Solve for total absorption, $Q_{n,t}(\mathbf{w}_t)$, using condition 5 and bilateral trade, $q_{n,i,t}(\mathbf{w}_t)$, using condition 6.
 - (f) Solve for total output, $y_{n,t}(\mathbf{w}_t)$, and factor demands— $k_{n,t}(\mathbf{w}_t)$ and $\ell_{n,i,t}(\mathbf{w}_t)$ —using conditions 7–9, respectively.
 - (g) Check factor market clearing to see if condition 11 holds. Our preferred metric is the [M]aximum [A]bsolute [E]xcess [D]emand:

$$\text{MAED} = \max_{n=1,\dots,N} \{|\ell_{n,t}(\mathbf{w}_t) - L_n|\}.$$

If MAED is less than some tolerance (in which case, by Walras' Law, condition 12 also holds) then proceed to step 2. Otherwise, update the vector of wages as follows:

$$w_{n,t}^{\text{new}} = \frac{w_{n,t} \ell_{n,t}(\mathbf{w}_t)}{L_n}$$

Normalize this updated wage vector to be in the simplex and return to step 1(b).

2. Compute the capital stock in the next period, $K_{n,t+1}$, using condition 13.
 3. Return to step 1(a) with the next-period capital stock and continue through period T , where T is sufficiently high enough to ensure the economy is “in” the new steady state.
-

Table A.2: Equilibrium Conditions in Model with Reversible Investment (Neoclassical Specification)

| | | |
|----|---|--------------------|
| 1 | $p_{n,i,t} = \frac{u_{i,t} d_{n,i}}{A_i}$ | $\forall(n, t)$ |
| 2 | $p_{n,t} = \left(\sum_{i=1}^N (p_{n,i,t})^{-\theta} \right)^{-1/\theta}$ | $\forall(n, t)$ |
| 3 | $p_{n,t} C_{n,t} + p_{n,t} X_{n,t} = r_{n,t} K_{n,t} + w_{n,t} L_n$ | $\forall(n, t)$ |
| 4 | $K_{n,t+1} = (1 - \delta) K_{n,t} + X_{n,t}$ | $\forall(n, t)$ |
| 5 | $C_{n,t} = (1 + \mathcal{R}_{n,t}) \left(\beta^t / \sum_{t'=1}^T \beta^{t'} \right) \mathcal{W}_n$ | $\forall(n, t)$ |
| 6 | $\mathcal{W}_n = \sum_{t=1}^T \frac{w_{n,t} L}{P_{n,t}(1 + \mathcal{R}_{n,t})} + K_{n,1} - \frac{K_{n,T+1}}{1 + \mathcal{R}_{n,T}}$ | $\forall(n)$ |
| 7 | $1 + \mathcal{R}_{n,t} \equiv \prod_{t'=1}^t \left(\frac{r_{n,t'}}{p_{n,t'}} + 1 - \delta \right)$ | $\forall(n, t)$ |
| 8 | $Q_{n,t} = C_{n,t} + X_{n,t}$ | $\forall(n, t)$ |
| 9 | $q_{n,i,t} = (p_{n,i,t}/p_{n,t})^{-(1+\theta)} Q_{n,t}$ | $\forall(n, i, t)$ |
| 10 | $p_{n,t} y_{n,t} = \sum_{i=1}^N p_{n,i,t} q_{n,i,t}$ | $\forall(n, t)$ |
| 11 | $r_{n,t} k_{n,t} = \alpha p_{n,t} y_{n,t}$ | $\forall(n, t)$ |
| 12 | $w_{n,t} \ell_{n,t} = (1 - \alpha) p_{n,t} y_{n,t}$ | $\forall(n, t)$ |
| 13 | $K_{n,t} = k_{n,t}$ | $\forall(n, t)$ |
| 14 | $L_n = \ell_{n,t}$ | $\forall(n, t)$ |
| 15 | $p_{n,t} Q_{n,t} = p_{n,t} y_{n,t}$ | $\forall(n, t)$ |
| 4s | $X_n^* = \delta K_n^*$ | $\forall(n)$ |
| 5s | $r_n^* = \left(\frac{1}{\beta} - (1 - \delta) \right) P_n^*$ | $\forall(n)$ |

Note: Units costs $u_{n,t} = B (r_{n,t})^\alpha (w_{n,t})^{(1-\alpha)}$ with constant $B = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)}$. In steady state, conditions 4 and 5 collapse to conditions 4s and 5s, respectively.

Algorithm A.2 Model With endogenous, Reversible Investment (Neoclassical Specification)

The following steps reference equilibrium conditions in Table A.2, given initial and terminal (steady state) capital stocks $(\mathbf{K}_1, \mathbf{K}_{T+1})$.

1. Guess matrices of factor prices $\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T$. Normalize this guess to the $(N - 1) \times N \times T$ -simplex: $\Delta \equiv \left\{ \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right) \in \mathbb{R}^{2NT} : \sum_{n=1}^N w_{n,t} L_n = 1, \forall(t) \right\}$.
2. Compute bilateral prices, $p_{n,i,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)$, using condition 1. Then compute country-level prices, $p_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)$, using condition 2.
3. First, construct the compounded return to capital $\mathcal{R}_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)$ using condition 7, then compute the present value of lifetime wealth $\mathcal{W}_n \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)$ using condition 8. Finally, compute the path for consumption $C_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)$, using condition 5.
4. Compute the paths for investment, $X_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)$, and capital, $K_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)$, using conditions 3 and 4, respectively.
5. Solve for total absorption, $Q_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)$ and bilateral trade, $q_{n,i,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)$, using conditions 8 and 9, respectively.
6. Solve for total output, $y_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)$, and factor demands— $k_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)$ and $\ell_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)$ —using conditions 10–12, respectively.
7. Check factor market clearing to see if conditions 13 and 14 hold:

$$\text{MAED} = \max_{\substack{n=1, \dots, N \\ t=1, \dots, T}} \left\{ \left| k_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right) - K_{n,t} \right| + \left| \ell_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right) - L_n \right| \right\}.$$

If MAED is higher than some tolerance, then update the matrices of factor prices:

$$w_{n,t}^{\text{new}} = \frac{w_{n,t} \ell_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)}{L_n} \quad r_{n,t}^{\text{new}} = \frac{r_{n,t} k_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)}{K_{n,t} \left(\{\mathbf{w}_t, \mathbf{r}_t\}_{t=1}^T \right)}.$$

Normalize these updated factor prices to be in the simplex and return to step 2. If MAED is higher than some tolerance, then condition 15 will automatically hold by Walras' law.
