Signaling Quality Through Prices in an Oligopoly.*

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Abstract

Firms signal high quality through high prices even if the market structure is very competitive and price competition is severe. In a symmetric Bertrand oligopoly where products may differ only in their quality and each firm's product quality is private information (unknown to consumers and to other firms), we show that there exist fully revealing perfect Bayesian equilibria in mixed strategies. High quality firms charge higher prices than low quality firms but lose business to rival firms with higher probability. Some of the revealing equilibria may involve high degree of market power for both low and high quality firms while others are more competitive. Under certain conditions, if the number of firms is large enough, information is revealed in every equilibrium. While the intuitive criterion does not select among equilibria, there is a unique symmetric fully revealing equilibrium satisfying the D1 refinement if the number of firms is above a critical level and below this level, no such equilibrium exists. In the unique D1 fully revealing equilibrium, both low and high quality firms may exhibit persistent market power no matter how large the number of firms.

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1   Introduction

It is a commonplace observation that in many markets where consumers are not fully informed about product quality[1] prior to purchase, goods sold at relatively high prices tend to be associated with high quality. One important economic explanation for this relates to markets where variation in product quality across firms arises, at least in part, from differences in production technology, input quality and other exogenous factors affecting the production process. In such markets, product quality is not fully determined by the seller; however, the seller is likely to be much better informed (relative to potential buyers[2] as well as rival sellers) about the current quality attributes of its own product (for example, through private information about technology, input sourcing or results of quality testing). The prices set by firms may then act as signals of their private information about quality.[3]

Bagwell and Riordan (1991) show that in a market with a single seller (who has private information about exogenously *given* product quality that can be either high or low), high price may act as a signal of high quality. Their main argument is that the low quality seller has a lower marginal cost of production (relative to a high quality seller) and therefore finds it more profitable to sell higher *quantity* at a sufficiently lower price rather than imitate the lower quantity-higher price combination preferred by the high quality seller.[4]

An important question that arises then is whether such signaling can occur in markets with more than one seller. Dissuading low quality sellers from imitating the high price charged by high quality sellers requires that the former earn sufficient rent. However, competition between sellers may dissipate all or most of the rent required for signaling. In oligopolistic markets where each firm's product quality is pure private information (not known to buyers or to other firms), Daughety and Reinganum (2007, 2008) show that if, in addition to unobserved potential differences in product quality, there is sufficient horizontal differentiation between the products of symmetric firms (so that price competition is soft enough), there is a unique symmetric separating equilibrium where the price charged by a firm signals its product quality. This, in turn, raises the question as to whether some degree of horizontal differentiation (or other devi-

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[1] Quality includes attributes such as safety, durability and probability of being "defective".

[2] If firms settle consumer complaints and litigation about realized product attributes associated with lower quality ("defective", "unsafe" etc.) confidentially, then potential buyers are unlikely to learn fully from past purchasers (see, Daughety and Reinganum, 2007a). The same holds if variations in exogenous factors affecting product quality are uncorrelated over time.

[3] If product quality is determined by the seller and lower quality is produced at lower cost, the seller has an incentive to produce the lowest quality. For experience goods, in a dynamic framework, the seller may still provide high quality at high price in order to preserve future reputational rent (quality premium associated with such high price). There is a large literature beginning with Klein and Leffler (1981) and Shapiro (1983). See, among others, Wolinsky (1983), Allen (1984) and Bester (1998). Bergemann and Välimäki (2006) consider dynamic monopoly pricing for a new experience good when buyers have independent private valuations.

[4] See, also, Bagwell (1992) and Daughety and Reinganum (2005).

ation from the perfectly competitive model that creates *ex ante* market power for firms through, say, firm or brand loyalty, search cost etc.) is necessary for signaling to occur through prices. This paper, among other things, provides an answer to this question.

We consider a Bayesian model of price competition in a symmetric oligopoly where the only deviation from the standard homogeneous good Bertrand model is incomplete information about product quality. Quality may be one of two types: high or low. *Ex ante*, product quality is privately known only to the firm supplying the product and is unknown to *all* consumers *as well as* rival firms; this information structure is similar to that in Daughety and Reinganum (2007, 2008) but unlike their model, there is no horizontal differentiation among the products of the firms. In fact, apart from incomplete information, there is no other friction in the market. Production cost is lower in a firm producing low quality output than in a firm that produces high quality. Consumers are identical, have unit demand and value high quality more than low quality.

We show that even in this stark model with severe price competition, signaling occurs through prices. Incomplete information endogenously creates sufficient rent and market power to allow signaling. In particular, we show that there always exist symmetric fully revealing perfect Bayesian equilibria that satisfy the Intuitive Criterion and in every such equilibrium, high price signals high quality. Low quality firms enjoy some monopoly power in states of the world where all other firms produce high quality (and therefore, charge high prices); this stochastic monopoly power allows low quality firms to earn strictly positive expected profit. Further, balancing this monopoly power with the incentive to undercut rivals' prices implies that all fully revealing equilibria involve mixed strategy pricing by low quality firms. Thus, price dispersion occurs as a pure consequence of asymmetric information about quality[5]. Further, even though prices signal quality perfectly, there is significant variation in prices across firms selling identical quality which suggests that a weak empirical relationship between price and quality differences across firms[6] may not necessarily imply that prices do not reveal information about quality.

We characterize fully the class of symmetric fully revealing equilibria where all high quality firms charge a deterministic high price with probability one while every low quality firm chooses a mixed strategy (with a continuous distribution) over an interval of prices that lies entirely below this high quality price.[7] In every such equilibrium, consumers are indifferent between buying low quality at the highest price charged by a low quality firm and buying high quality at the price charged by high quality firms. This implies that the lower the high

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[5] In the existing literature on strategic models of price competition in oligopoly, price dispersion (in the form of mixed strategy equilibria) arises because of capacity constraints (Kreps and Scheinkman, 1983, Osoborne and Pitchik, 1986), search cost (Reinganum, 1979, Stahl, 1989), consumer insenstive to small price changes (Fershtman, 1982), captive market segments (Varian, 1980), trade in information about prices (Baye and Morgan, 2001), uncertainty about the existence of rival firms (Janssen and Rasmusen, 2002) etc.

[6] See, for example, Gerstner (1985).

[7] The equilibrium price distribution actually depends on the prior distribution of types, a somewhat unusual feature (but see also, Daughety and Reinganum, 2007, 2008).

quality price in a signaling equilibrium, the lower the prices charged by and profits earned by low quality firms. Under some conditions, the price charged by high quality firms must be necessarily higher than their marginal cost in order to ensure that low quality firms earn sufficient rent in order to not imitate the high quality firms. However, the set of perfect Bayesian equilibria (satisfying the Intuitive Criterion) may consist of outcomes where high quality firms exercise much higher level of market power than the minimum level required for signaling because of the out-of-equilibrium beliefs of consumers. In fact, under certain conditions, fully revealing equilibria where high quality firms charge their full information monopoly price are sustained no matter how large the number of firms.

As noted above, equilibria where high quality firms charge a high price are also ones where the support of the distribution of low quality prices is high. However, as the number of firms increases, low quality firms always have a higher incentive to undercut rivals as they are not deterred by the out-of-equilibrium beliefs of buyers and therefore, shift probability mass to lower prices. As a result, low quality prices converge in distribution to their deterministic marginal cost as the number of firms becomes infinitely large.

More generally, the fully revealing equilibria (satisfying the Intuitive Criterion) form a rich class of market outcomes and depending on parameters, may range from high degrees of market power to almost competitive outcomes (where prices are close to true marginal cost). This shows that under incomplete information about product quality, there is a wide variety of outcomes that are consistent with Bertrand price competition. In the fully revealing equilibria, low quality firms always earn higher profits than under complete information and high quality firms may be better off too.

Under certain conditions, there are fully revealing equilibria where the total quantity sold in the market is identical for every possible realization of types (and prices) on the equilibrium path. Incentives for signaling are created by differences in expected market share of each firm at different prices - in particular, through the fact that low quality firms sell at lower price but with higher probability than high quality firms. Thus, while in the monopoly model analyzed by Bagwell and Riordan (1991), the downward sloping *market* demand curve for high quality plays an important role in providing incentives for separation of types, in more competitive market structures, market equilibrium can endogenously generate a downward sloping demand curve for an individual firm that can then be used to provide incentives for signaling through prices.

One feature of the fully revealing equilibria of our model is that a high quality product is sold only in the state where all firms are of high quality. In states of nature where both low and high quality products are available, consumers buy the low quality good almost surely. This is a consequence of the assumption that all consumers are identical and, in particular, have identical valuation for the high quality good. We indicate how this feature of the signaling equilibria disappears if we introduce heterogeneity of consumers in their valuation of the high quality good; in that case, there are fully revealing equilibrium outcomes where higher valuation consumers always buy the high quality good, if available.

The possibility of a continuum of fully revealing equilibria that meet the Intuitive Criterion and the important role played by out-of-equilibrium beliefs in sustaining most of these outcomes raises the question whether a stronger refinement criterion can select among the set of revealing equilibria and if so, whether the qualitative properties related to market power of firms survive such refinement. We apply the D1 refinement of perfect Bayesian equilibrium to our model (which, unlike the pure signaling game, involves multiple senders). If the number of firms is below a critical level i.e., the market structure is not sufficiently competitive, there is no symmetric fully revealing equilibrium satisfying the D1 refinement. If the number of firms exceeds this critical level, there is a unique symmetric fully revealing outcome that meets the D1 refinement. This equilibrium may involve market power for both low and high quality types and under certain conditions, market power of high quality firms is bounded away from zero no matter how large the number of firms. This is related to the point made earlier about the fact that, in equilibrium, high quality firms need to charge a somewhat high price in order to allow low quality firms to earn sufficient rent so as not to have any incentive to imitate high quality sellers. Thus, incomplete information not only creates market power that persists no matter how small market concentration is but, in fact, it can do so even when out-of-equilibrium beliefs are consistent with very strong refinement criteria such as the D1.

Finally, we characterize the class of pooling equilibria and the conditions under which they arise. We show that under certain conditions, there is no pooling equilibrium satisfying the Intuitive Criterion if the number of firms is large enough. Thus, not only does a highly competitive market structure lead to fully revealing equilibria that survive a strong refinement criterion such as the D1 but in fact, it ensures that every perfect Bayesian equilibrium involves some revelation of private information. Moreover, when we apply the D1 refinement criterion, all pooling equilibria are eliminated, independent of the number of firms, again underlining the claim of this paper that competition leads to information revelation.

Our model can be applied to analyze oligopolistic markets where products are physically homogenous but production cost varies between firms and consumers have preferences over the production technology (prefer the good produced at higher cost). This preference for the high cost over the low cost product may arise, for example, due to social consciousness of environmental damage or other negative externalities caused during the production process.[8]

The paper is related to some other strands of the literature. In a model where product quality is known to both firms in a duopoly but not to consumers, Hertzendorf and Overgaard (2001) show (in stark contrast to our model) that fully revealing equilibria (satisfying a natural refinement) do not exist. Janssen and Van Reeven (1998) study the role of prices as signals of illegal practices in a model structurally similar to ours and show that prices can convey full information about quality for a subset of the parameter space.

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[8]Spulber (1985) considers a model of Betrand price competition with private information about production cost. But in his model, consumers have no preference for the good produced at higher cost over that produced at lower cost.

Milgrom and Roberts (1986) allow firms to use price and advertising expenditures to signal quality in a dynamic monopoly model where repeat purchases are important and where the quality of a firm's output is sufficiently correlated over time[9]; under certain conditions, *low* prices may signal high quality.[10]

Section 2 outlines the basic model. Section 3 contains existence and characterization results for the set of fully revealing equilibria satisfying the Intuitive Criterion. Section 4 analyzes which of these fully revealing equilibria satisfy the D1 refinement criterion. Pooling equilibria are discussed in Section 5, while Section 6 indicates how our results are modified in the presence of heterogeneity in consumer valuations. Section 7 concludes. All proofs are contained in the appendix.

2   Basic Model

Consider an oligopolistic market with $N > 1$ identical firms that compete in prices. The product of each firm can be of two potential qualities - low (L) and high (H). There is no horizontal differentiation between the products of the firms. Each firm's product quality is given and information about quality is private - only a firm knows the quality of its product (it is unknown to other firms as well as consumers). However, it is common knowledge that the quality of each firm is an independent draw from a probability distribution that assigns probability $\alpha \in (0,1)$ to high quality and probability $1 - \alpha$ to low quality. Each firm produces at constant unit cost that depends on its quality. In particular, for every firm, the unit cost of production is c_L, if its product is of low quality, and c_H, if it is of high quality, where

$$c_H > c_L \geq 0. \tag{1}$$

There is a unit mass of risk-neutral consumers in the market. Consumers have unit demand i.e., each consumer buys at most one unit of the good. All consumers are identical and have identical valuation V_L for a unit of the low quality good and V_H for a unit of the high quality good, where

$$V_H > V_L, \tag{2}$$

and further,

$$V_L > c_L, V_H > c_H. \tag{3}$$

Formally, the oligopoly game is a symmetric $N-$player Bayesian game where the type τ of each firm lies in the type set $\{H, L\}$; nature first draws the type of each firm i independently from a common distribution that assigns probability

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[9]It may be noted here that in markets where variation in quality of a firm's output occurs due to exogenous uncertainty that is intertemporally independent, the quality of a firm's product are not likely to be correlated over time.

[10]Fluet and Garella (2002) extend their analysis to a duopoly (and assume that unlike consumers, firms know each others' product quality).

$\alpha \in (0,1)$ to $H - type$ and probability $1 - \alpha$ to $L - type$ and this move of nature is only observed by firm i. After this, firms simultaneously choose their prices. In particular, the strategy of each firm $i, i = 1, 2, ...N$, is a pair of prices $\{p_L^i, p_H^i\}$ where p_τ^i is the price it chooses if it is of type $\tau, \tau \in \{H, L\}$. We allow for mixed strategies. Consumers observe the prices charged by firms and each consumer decides whether to buy and if so, which firm to buy from. The payoff to a consumer that buys is her expected net surplus (i.e., expected valuation of the product of the firm she buys from net of the price charged by it) and the payoff is zero, if she does not buy. The payoff to each firm is its expected profit.

The basic solution concept used throughout the paper is that of Perfect Bayesian Equilibrium (PBE) satisfying the Intuitive Criterion (Cho and Kreps, 1987). Throughout the text, unless otherwise specified, an equilibrium refers to a PBE satisfying the Intuitive Criterion. In Section 4 and in the last part of Section 5, we consider equilibria that meet the D1 criterion (Cho and Sobel, 1990), which is a significantly stronger than the Intuitive Criterion.

3   Fully Revealing Equilibria

In this section, we establish the existence and the qualitative properties of fully revealing equilibria, where the price charged by a firm reveals all information about its product quality with probability one. In such an equilibrium, the support of the equilibrium price distribution for high and low quality types have null intersection.

We begin with a proposition that outlines some basic qualitative properties that must be satisfied by all fully revealing equilibria.

Proposition 1 *In any fully revealing equilibrium, the following holds:*

(a) The support of the price distribution of a firm when its product quality is high lies strictly above that when its product quality is low i.e., high price signals high quality.

(b) Every firm makes strictly positive expected profit when its product is of low quality.

Further, there is no fully revealing equilibrium in pure strategies.

Part (a) of Proposition 1 states that if prices reveal quality, then high quality is revealed by high price and low quality by low price. Given the assumption that low quality is produced at lower cost, the only way one can provide incentives to both low and high quality sellers to not imitate each other in their pricing is to ensure that high quality producers charge higher price and sell less than low quality producers. Part (b) of Proposition 1 states that a necessary condition for signaling to occur is that low quality firms must earn positive rent - if a low quality seller earns zero profit, it will always have an incentive to imitate the higher price charged by high quality sellers. The last part of Proposition 1 states that a fully revealing equilibrium necessarily involves mixed strategies. If both low and high quality sellers charge deterministic prices, then the incentive

to undercut rivals eliminates all rent for low quality sellers which is necessary for separation of types.

In what follows, we focus on symmetric fully revealing equilibria where all firms choose identical (possibly mixed) price strategies.

It is obvious that no firm can sell at a price higher than V_H. Further, from Proposition 1(b), it is easy to check that in any symmetric fully revealing equilibrium, L-type firms cannot charge prices higher than V_L with positive probability. For any equilibrium where $H-$type firms charge prices higher than V_H with positive probability, one can show that there is a payoff equivalent symmetric fully revealing equilibrium in which they charge price smaller than or equal to V_H with probability one.[11] So, at this stage, we impose a restriction on the strategy set of firms:

$$p_\tau^i \in [0, V_H], \tau = L, H, i = 1, ...N. \tag{4}$$

The next proposition characterizes some properties of symmetric fully revealing equilibria.

Proposition 2 *In any symmetric fully revealing equilibrium, every low quality firm plays a mixed price strategy with a continuous probability distribution whose support is a non-degenerate interval $[\underline{p}_L, \overline{p}_L], c_L < \underline{p}_L < \overline{p}_L \leq V_L$, whose distribution function F is given by:*

$$F(p) = 1 - \frac{\alpha}{1 - \alpha} \left(\sqrt[N-1]{\frac{V_L - c_L - (V_H - \underline{p}_H)}{p - c_L}} - 1 \right), \tag{5}$$

and

$$\underline{p}_L = \alpha^{N-1} \overline{p}_L + (1 - \alpha^{N-1}) c_L. \tag{6}$$

The support of the high quality firms' equilibrium price distribution lies in $[c_H, V_H]$ and its lower bound (infimum) $\underline{p}_H$ satisfies

$$V_H - \underline{p}_H = V_L - \overline{p}_L, \tag{7}$$

i.e., consumers are indifferent between buying from a low quality firm at (its highest) price $\overline{p}_L$ and from a high quality firm at (its lowest) price $\underline{p}_H$. A high quality firm may sell only in the state where all other firms sell high quality; also, at price $\overline{p}_L$, a low quality firm sells only in the state where all other firms are of high quality. Finally, the equilibrium price distribution of high-quality firms necessarily has a mass point.

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[11] To see this, observe that in any such equilibrium, $H-$type firms must make zero expected profit (otherwise, switching probability mass from above V_H to the price at which they sell at strictly positive profit would be gainful). This would imply that H-type firms sell zero quantity and charge prices $\leq V_H$ with probability one. If they concentrate all probability mass at V_H, consumers still find it optimal to not buy from such types; the incentives of L-type firms to deviate and imitate the H-type's price are also not affected.

From Proposition 1, we know that every fully revealing equilibrium must involve mixed strategies and that low quality firms must earn positive rent. Proposition 2 clarifies further that in any *symmetric* fully revealing equilibrium, low quality firms must randomize over prices. The way the market generates rent for low quality firms is by ensuring that such types of firms have some market power in certain states of nature - in particular, when all other firms sell high quality products. This "stochastic market power", generated endogenously by the fact that the equilibrium requires other firms to charge high prices in such states, ensures that the expected profit of low quality types is strictly positive. While this prevents low quality firms from dissipating all rent through price competition, it also implies that these types must mix over a range of prices in order to balance the incentive to charge a high price in the state in which they have market power and the competitive incentive to undercut prices of rival firms when some of them are of low quality type. The upper bound of the low quality firms' price distribution is the maximum price at which they could sell in the state of nature where all other firms are of high quality and the lower bound is the price that is not worth undercutting as it would yield less than the equilibrium payoff even if the firm can sell to the entire market with probability one. The symmetric nature of the equilibrium also ensures that the distribution of prices followed by low type firms has no mass point and, in fact, follows a continuous distribution function. Proposition 2 also points to an asymmetry between the equilibrium behavior of high and low quality firms in that while the low quality firms always randomize over prices according to a continuous distribution, high quality firms choose either a deterministic price for sure or, if they randomize over prices, their price distribution necessarily has a mass point.

The next result, an immediate corollary of Proposition 2, indicates the limiting competitive behavior of low quality firms as the number of firms N becomes indefinitely large and for a given N, as the prior probability of a firm's product being of low quality goes to one. In both cases, the rent earned by low quality firms and their market power converge to zero. In particular, the random price charged by a low quality firm converges in distribution to a degenerate distribution at the marginal cost for low quality.

Corollary 1 *Ceteris paribus, if either $N \to \infty$ or $\alpha \to 0$, the probability distribution of prices followed by a low quality firm in any symmetric fully revealing equilibrium converges to the degenerate distribution $\delta(c_L)$ that charges price equal to its marginal cost c_L with probability one.*

The intuition behind the above result is straightforward. The reason why price competition between low quality firms does not reduce their price to marginal cost is the guarantee of limited monopoly power to every low quality firm in the state where all other firms are of high quality; the probability this state arises goes to zero as $N \to \infty$ or $\alpha \to 0$. A similar result, however, does not necessarily hold for high-quality firms. As we show later, if $V_L \leq c_H$, a fully revealing equilibrium where high quality firms charge their monopoly price V_H with probability one can be sustained in equilibrium even as $\alpha \to 1$ or $N \to \infty$.

8

A high quality firm may *not* undercut its rivals if, at lower prices, the out-of-equilibrium beliefs of buyers perceive the quality to be low with high probability; this dampens price competition even if there are a large number of rivals with high quality product.

Next, we state the main result of this paper, namely that a fully revealing equilibrium always exists. To this end, let Ω denote the set of symmetric fully revealing equilibria where all high quality firms charge the same price p_H with probability one. In such an equilibrium, Proposition 2 implies that the support of prices for low quality sellers is given by $[\underline{p}_L, \overline{p}_L]$, where

$$\overline{p}_L = V_L - (V_H - p_H) \tag{8}$$

$$\underline{p}_L = \alpha^{N-1} \overline{p}_L + (1 - \alpha^{N-1}) c_L. \tag{9}$$

and distribution function F of low quality prices is given by:

$$F(p) = 1 - \frac{\alpha}{1-\alpha} \left(\sqrt[N-1]{\frac{V_L - c_L - (V_H - p_H)}{p - c_L}} - 1 \right), p \in [\underline{p}_L, \overline{p}_L]. \tag{10}$$

Thus, the price distribution for low quality sellers in any equilibrium in Ω is fully determined by the level of high quality price p_H. Also, observe that a higher value of p_H implies that low quality firms charge prices that are (first order) stochastically higher (use (10)) and that, in particular, the interval of support of low quality prices is higher (use (8) and (9)). Thus, the extent of market power in any equilibrium in Ω is fully determined by p_H.

We now state the core result of this paper.

Proposition 3 *A symmetric fully revealing equilibrium always exists. In particular, Ω is non-empty.*

The proof of Proposition 3 follows directly from two important lemmas - that are also of independent interest. The first lemma states the conditions under which there exists an equilibrium in Ω where all consumers buy with probability one.

Let θ_0, θ_1 be defined by:

$$\theta_0 = \max\{c_H, \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L\} \tag{11}$$

$$\theta_1 = \min\{V_H, \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_H\} \tag{12}$$

By definition, $\theta_0 \geq c_H, \theta_1 \leq V_H$.

It is easy to check that

$$\theta_0 \leq \theta_1 \tag{13}$$

if, and only if,

$$N \geq \frac{V_H - c_L}{V_L - c_L}. \tag{14}$$

Lemma 1 *There exists a symmetric fully revealing equilibrium (in Ω) where every $H-$type firm charges a deterministic price p_H and all consumers buy with probability one <u>if, and only if,</u> (14) holds. Further, the set of prices that can be sustained as high quality price p_H in such an equilibrium is the interval $[\theta_0, \theta_1]$.*

Lemma 1 asserts that as long as we can ensure that $\theta_0 \leq \theta_1$, every price in the interval $[\theta_0, \theta_1]$ can be sustained as the (deterministic) high quality price p_H in a symmetric fully revealing equilibrium where the total quantity sold in the market is identical (equal to one) for every possible configuration of the realized qualities of firms' products. As indicated above, the equilibrium price distribution of a low quality firm in this kind of equilibrium is given by (8)-(10).

High quality firms do not find it optimal to deviate to a price higher than p_H as consumers will not buy at such prices, given the equilibrium strategies of other firms. Moreover, they do not find it optimal to deviate to a lower price for the fear of being perceived as selling low quality. As explained in the proof and discussed later in this section, such pessimistic beliefs are consistent with the Intuitive Criterion. Given the pessimistic beliefs of consumers, at any price higher than $\overline{p}_L = V_L - (V_H - p_H)$, a low quality firm can sell with positive probability only if it imitates the high quality price p_H. At price p_H, high quality sellers sell only in the state where all other firms are high quality; condition (14) ensures that the number of firms is large enough so that the expected quantity sold by a high quality firm in equilibrium is relatively small. Further, the lower bound θ_0 on high quality price ensures that *low* quality sellers can charge sufficiently high prices without losing consumers to high quality firms. Put together, these conditions deter imitation of high quality types by low quality types. The upper bound θ_1 on the set of high quality prices sustainable in fully revealing symmetric equilibrium ensures that the *low* quality sellers do not charge too high prices with impunity and therefore, the high quality sellers have no incentive to imitate low quality sellers.

Condition (14) is both necessary and sufficient for the existence of an equilibrium in Ω where all consumers always buy independent of the realization of types. Further, (14) is more likely to hold as the number of firms N increases i.e., as the market structure is more competitive. As N increases, the expected market share of a high quality firm (which sells only in the state where all other firms are of high quality) becomes smaller and this reduces the incentive of the low quality seller to imitate the high price of the high quality seller.

Lemma 1 also makes a slightly more general point. In our framework, signaling requires that a high quality seller sells less than a low quality seller while charging a higher price. With a single seller, this would require that the total quantity sold in the market be lower at a higher price. When the market structure is more competitive, the incentive to mimic the other quality type can be neutralized even if the total quantity sold is identical for every realization of types (as is true for all equilibria described in Lemma 1). High quality firms that charge a high price are always undercut with a large margin by low quality rivals (in the event that at least one of them is of low quality type) and therefore, sell with lower probability than low quality firms; this lower probability of

sale is generated endogenously through the equilibrium price strategies. In this sense, competition helps information signaling.

The next lemma shows that under certain conditions there exist symmetric fully revealing equilibria where every $H-$type firm charges the full information monopoly price $p^* = V_H$ with probability one and consumers randomize between buying and not buying when all firms charge the high quality price.

Lemma 2 *There exists a symmetric fully revealing equilibrium (in Ω) where every $H-$type firm charges price $p^* = V_H$ (and some consumers do not buy in the state where all firms are of high quality) if, and only if, <u>one of the following</u> holds:*

$$V_L \leq c_H \tag{15}$$

or,

$$N \leq \frac{V_H - c_H}{V_L - c_H}. \tag{16}$$

Lemma 2 outlines conditions under which there is a symmetric fully revealing equilibrium where the high quality firms charge the full information monopoly price that leaves buyers with zero surplus so that they are indifferent between buying and not buying. In equilibrium, in the state where all firms are of H-type, a fraction of consumers may not buy at all and this reduces the incentive of low quality firms to imitate the high quality price. As the incentive of the low quality firm to imitate the high quality firm can be easily taken care of by reducing the fraction of consumers who buy when all firms are of high quality, the only constraint that matters is related to the incentive of the high quality firm to imitate the low quality firm. There is no such incentive if the marginal cost of producing high quality is above the maximum willingness to pay for low quality (condition (15)) or if the number of firms is small enough (condition (16)) so that the market share of the high quality firm is reasonably large.

It can be checked that conditions (14), (15) and (16) cover the entire parameter space (i.e., at least one of them must hold) so that a fully revealing equilibrium in the set Ω always exists and this establishes Proposition 3.

As mentioned earlier, the degree of market power in any equilibrium in the set Ω is captured by the high quality price p_H; equilibria with higher values of p_H are associated with stochastically higher prices for low quality firms. From the previous results, one can immediately see that the degree of market power may vary widely across the set of fully revealing equilibria. Thus, if low quality is socially more valuable than high quality, *i.e.*,

$$V_H - c_H < V_L - c_L, \tag{17}$$

and N is large enough, then (14) holds, $\theta_0 = c_H$ and from Lemma 1, there exists a symmetric fully revealing equilibrium in Ω where $p_H = c_H$ i.e., high quality firms charge prices equal to their true marginal cost for sure; in such an equilibrium, low quality firms continue to exercise some market power but

11

as indicated in Corollary 1, this gradually disappears as the number of firms becomes large.

On the other hand, Lemma 2 indicates that if the valuation for low quality good lies below the unit cost of producing the high quality good, then no matter how large the number of firms, there is always an equilibrium where p_H equals the full information monopoly price V_H. In this equilibrium, low quality firms randomize prices over the interval $[\{\alpha^{N-1}V_L + (1-\alpha^{N-1})c_L\}, V_L]$ so that low quality firms may charge prices close to their full information monopoly price with positive probability. Corollary 1 indicates that the market power exercised by low quality firms must disappear as the number of firms becomes arbitrarily large; however, the market power exercised by high quality firms may continue to be at the monopoly level no matter how competitive the market structure.

The Intuitive Criterion does not select among different fully separating equilibria in Ω. Low quality firms have an incentive to charge a price $p \in (\overline{p}_L, p_H)$ if the probability that they can sell at such price is high enough. Therefore, the Intuitive Criterion cannot be used to rule out the belief that a firm charging these prices are of low type. As for out-of-equilibrium prices other than those in $(\overline{p}_L, p_H)$, consumers never buy at prices above p_H and always buy at prices below $\underline{p}_L$ independent of out-of-equilibrium beliefs, so that the intuitive criterion does not impose any restriction other than requiring that a firm charging a price in $[\overline{p}_L, c_H)$ be perceived as L-type with probability one.

We summarize below the behavior of the symmetric fully revealing equilibrium outcomes in Ω as the number of firms becomes arbitrarily large.

Corollary 2 (*a*) *If* (17) *holds, then for N sufficiently large, there exists a symmetric fully revealing equilibrium outcome in Ω where high quality firms charge price equal to marginal cost. If* (17) *does not hold, then for every N, high quality firms exercise market power in every equilibrium outcome in Ω and the price they charge is uniformly bounded below by* $[V_H - (V_L - c_L)] \geq c_H$.

(*b*) *If* (15) *holds, then for every N, no matter how large, there exists an equilibrium where high quality firms charge the monopoly price V_H. If* (15) *does not hold, then as $N \to \infty$, the price charged by high quality firms in the equilibrium outcome in Ω with the highest degree of market power converges to* $[V_H - (V_L - c_H)]$.

The conditions in Lemma 1 or Lemma 2 for the existence of a fully revealing equilibrium with various degrees of market power (as indexed by p_H) are independent of the prior probability α that a firm is of high quality. At $\alpha = 0$ or $\alpha = 1$, the model degenerates to a complete information homogenous good Bertrand model with zero market power. Yet, high degree of market power (bounded away from zero) may persist for every $\alpha \in (0, 1)$. This indicates a major qualitative difference in the nature of price competition between the complete and incomplete information models; in the latter case, the informational content of the prices charged radically alters the incentives of firms to undercut rivals.

Note that under complete information, firms of quality τ exercise zero market power and earn zero profit unless there is only one firm of quality τ and, in

addition, for this quality $V_\tau - c_\tau > V_{\tau'} - c_{\tau'}, \tau \neq \tau', \tau, \tau' \in \{L, H\}$. Therefore, no matter what the realization of quality is, a low quality firm exercises strictly higher market power and earns strictly higher *ex ante* profit[12] in a symmetric fully revealing equilibrium in Ω compared to that obtained under complete information (with one exception[13] in which case the *ex ante* profits and prices are equal). Further, if $V_H - c_H \leq V_L - c_L$, a high quality firm is weakly better off under incomplete information in an equilibrium in Ω for every realization of types and may be strictly better off in the state where all other firms have high quality products. Even if $V_H - c_H > V_L - c_L$, high quality firms are at least as well off as long as there is more than one high quality firm.[14]

4   D1 Refinement of Fully Revealing Equilibria.

In this section, we examine the extent to which the symmetric fully revealing equilibria analyzed in the previous section satisfy a much stronger refinement criterion - the D1 criterion (Cho and Sobel, 1990).

The D1 criterion examines the reasonableness of out-of-equilibrium beliefs (in perfect Bayesian equilibria) of pure signaling games by considering the set of responses of the receiver for which the sender of a particular type would have an incentive to deviate to some out-of-equilibrium action or signal s. If this set is larger for a certain type θ than for some other type θ', then the D1 criterion requires that the out-of-equilibrium beliefs should assign probability zero to the event that the signal s comes from a sender of type θ'. In evaluating the set of possible responses of the receiver to a signal s, the criterion requires that we ignore responses that could never be a best response of the receiver to s no matter what belief the receiver holds about the distribution of types of the sender.

The D1 refinement criterion was developed in the context of pure signaling games with one sender. The game we consider here is a multiple sender game. In principle, the beliefs of the receivers (buyers) in our model are mappings from the observed vector of prices set by all N firms (senders) to the joint distribution of the types of N firms. However, the out-of-equilibrium beliefs of consumers when two or more firms choose actions outside the support of their equilibrium strategies do not affect the incentives for *unilateral* deviation by any player and hence play no role in supporting a particular equilibrium. To evaluate whether the system of beliefs supporting an equilibrium is reasonable, we confine attention to beliefs of buyers when they observe a single firm charge

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[12]In an equilibrium in Ω, the profit of a low quality firm is equal to $\alpha^{N-1} [V_L - c_L - (V_H - p_H)]$ which is higher than its *ex ante* profit (before information is revealed) in the complete information case, which is equal to $\alpha^{N-1}(\max\{0, (V_L - c_L) - (V_H - c_H)\})$; it is strictly higher if $p_H > c_H$.

[13]There is only one low quality firm, $V_H - c_H < V_L - c_L$ and the equilibrium in Ω is one where $p_H = c_H$.

[14]Daughety and Reinganum (2008) also note a similar result for their model: low quality firms are always better off and, under certain conditions, both low and high quality firms are better off under incomplete information.

an out-of-equilibrium price while others continue to choose prices according to their equilibrium strategy.[15]

In a pure signaling game with one sender, deviation by the sender to an out-of-equilibrium signal generates a set of possible optimal actions or best responses of the receiver. With multiple senders, every best response of the receiver to this deviation is a mapping from the set of signals of other senders to the action set of the receiver. Comparing the incentive to deviate in terms of subsets of mappings can be somewhat complicated. In what follows, we modify and adapt the D1 criterion to our model in a manner that retains tractability.

Consider a firm i that unilaterally deviates to a certain price p that lies outside the support of its equilibrium strategy. Fix p. Let p_{-i} denote the vector of prices charged by firms $j \neq i$. The belief of a buyer associates with each (p, p_{-i}) a probability that firm i is of type H. Consider any profile of beliefs, one for each buyer. For each p_{-i}, the equilibrium strategies of firms $j \neq i$ generates the posterior distribution of the types of firms $j \neq i$. Each buyer can therefore determine a set of optimal purchase decisions given p and p_{-i}. Thus, for any out-of-equilibrium belief of the buyer, her best response to p is a correspondence from the set of all possible p_{-i} to a (possibly mixed) purchase decision. The prior distribution of types of each firm $j \neq i$ and their equilibrium strategies defines a probability distribution F_{-i} over all possible p_{-i} that can arise when firms $j \neq i$ stick to their equilibrium strategy. Given p and using F_{-i}, every measurable selection from the best response correspondence for each buyer (every such selection a function of p_{-i}) determines an expected quantity sold by firm i and doing this for all possible selections, we can generate a set of expected quantities sold by firm i at price p for a given profile of beliefs (where the expectation is taken prior to observing prices set by other firms). If we repeat this for every profile of beliefs that buyers may possibly have about the type of firm i, we generate a set $B_i(p)$ of all possible expected quantities that can be sold by firm i at price p. Each $q_i \in B_i(p) \subset [0, 1]$ is a quantity that firm i can "expect" to sell at price p for some profile of beliefs of buyers about firm $i's$ type and for some configuration of optimal choices of buyers (that depends on realizations of prices charged by other firms) when other firms play according to their equilibrium strategy.

Since the expected profit of firm i at price p is a linear function of the expected quantity sold, the set $B_i(p)$ aggregates the possible rational responses of all buyers in a payoff relevant fashion. In the spirit of the D1 criterion, we compare the subsets of expected quantities in $B_i(p)$ for which it is gainful for different types of firm i to deviate to price p.

More precisely, consider any perfect Bayesian equilibrium where the equilibrium profit of firm i when it is of type τ is given by $\pi_\tau^{i*}, \tau = H, L$. Consider any $p \in [0, V_H]$ outside the support of the equilibrium price strategy of firm i. If for

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[15]Note that unlike signaling games with single sender, for any given profile of actions of the receivers (buyers), the pay-off of a deviating sender or firm depends on the realization of types (and on the realization of the mixed strategies) of other players and therefore, it is not quite obvious how one should evaluate the "expected gain" of an individual sender when multiple senders deviate simultaneously.

$\tau, \tau' \in \{H, L\}$, $\tau' \neq \tau$,

$$\{q_i \in B_i(p) : (p - c_\tau)q_i \geq \pi_\tau^{i*}\} \subset \{q_i \in B_i(p) : (p - c_{\tau'})q_i > \pi_{\tau'}^{i*}\}$$

where "$\subset$" stands for strict inclusion, then the D1 refinement suggests that the out-of-equilibrium beliefs of buyers (upon observing a unilateral deviation by firm i to price p) should assign zero probability to the event that firm i is of type τ and thus (as there are only two types), assign probability one to firm i being of type τ'.

We will now apply this criterion to examine the symmetric fully revealing equilibria in Ω where all high quality firms charge the same price $p_H \in [c_H, V_H]$ with probability one and every low quality firm plays a mixed strategy with a continuous distribution function that has support $[\underline{p}_L, \overline{p}_L]$, where, as indicated in the previous section,

$$\overline{p}_L = V_L - (V_H - p_H) < p_H$$

$$\underline{p}_L = \alpha^{N-1}\overline{p}_L + (1 - \alpha^{N-1})c_L > c_L.$$

Recall the definitions of θ_0, θ_1 in (11) and (12):

$$\theta_0 = \max\{c_H, \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L\}.$$

$$\theta_1 = \min\{V_H, \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_H\}.$$

Note that in any perfect Bayesian equilibrium in Ω where the high quality price is less than V_H, all consumers buy with probability one. From Lemma 1 and Lemma 2, if $\theta_0 \leq \theta_1$ (i.e., (14) holds) and either (15) or 16) holds, then the set of prices that can be sustained as the high quality price in a (perfect Bayesian) equilibrium in Ω is $[\theta_0, \theta_1] \cup \{V_H\}$. If (14) holds but neither (15) nor 16) hold, then the set of prices that can be sustained as the high quality price in an equilibrium in Ω is $[\theta_0, \theta_1]$. Finally, if $\theta_0 > \theta_1$ (i.e., (14) does not hold), then the only price that can be sustained as high quality price in an equilibrium in Ω is $p_H = V_H$. In general, the set of high quality prices sustainable as a perfect Bayesian equilibrium outcome in Ω is bounded below by θ_0.

Our first result indicates that the D1 refinement rules out every equilibrium in Ω where the high quality price exceeds θ_0.

Lemma 3 *No perfect Bayesian equilibrium in Ω where the high quality price $p_H \neq \theta_0$ survives the D1 refinement.*

The basic intuition behind this result is as follows. In equilibrium, the high quality price determines the upper bound of the support of low quality prices and therefore, the equilibrium profit of the low quality seller. If the high quality price is $[\frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L]$, then the low quality seller's profit when he imitates the price charged by a high quality seller is exactly equal to his equilibrium profit. If

the high quality price is higher than $[\frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L]$, then the low quality seller's equilibrium profit is *strictly* higher than what he can get by imitating the price p_H charged by a high quality seller. Now, if the fully revealing equilibrium is one where the high quality price $p_H > \theta_0 \geq [\frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L]$, then (by continuity) low quality sellers would have a strict incentive to not deviate to a price just below p_H unless the (expected) quantity sold jumps up by a significant discontinuous amount (as we move from p_H to a price just below p_H). On the other hand, the high quality seller will have an incentive to deviate to price slightly below p_H if the quantity sold increases slightly (in a continuous fashion). Therefore, the range of buyers' responses for which the high quality seller deviates to a price just below p_H is larger than that for the low quality seller. The D1 refinement then requires that the out-of-equilibrium beliefs assign probability one to the event that a firm that unilaterally deviates to a price just below p_H is a high quality firm. However, with such beliefs, a high quality firm always has an incentive to deviate from its equilibrium strategy and undercut p_H slightly so that it grabs the entire market in the state of the world where all other firms are of type-H.

From Lemma 1, we know that (14) is necessary and sufficient for the existence of a perfect Bayesian equilibrium in Ω where the high quality price equals θ_0. It follows that (using Lemma 3):

Corollary 3 *If* (14) *does not hold, then no equilibrium in Ω meets the D1 refinement.*

However, we can show that if there is an equilibrium in Ω where the high quality price is equal to θ_0, it meets the D1 refinement.

Lemma 4 *Assume* (14) *holds. Then, there is a unique equilibrium in Ω that meets the D1 requirement and in this equilibrium high quality firms charge price $p_H = \theta_0$.*

The above results provide a fairly tight characterization of fully revealing equilibria in Ω that meet the D1 refinement. The equilibria in Ω are ones where every high quality firm chooses a deterministic price. This leaves open the question as to whether there might be symmetric fully revealing equilibria that are not in the set Ω and that meet the D1 refinement. From Proposition 2, it follows that such an equilibrium must be one where high quality firms play mixed strategies. Our next result shows that no such equilibrium exists.

Lemma 5 *No symmetric fully revealing equilibrium where the high quality firms' prices follow a non-degenerate probability distribution meets D1 refinement.*

We can now summarize our results on D1 refinement of fully revealing equilibria in one core proposition:

Proposition 4 *There exists a symmetric fully revealing D1 equilibrium if, and only if, (14) holds i.e.,*

$$N \geq \frac{V_H - c_L}{V_L - c_L}.$$ (18)

Further, when it exists, this D1 equilibrium is unique; in this equilibrium, high quality firms charge a deterministic price equal to

$$\theta_0 = \max\{c_H, \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L\}$$

and all consumers buy with probability one. If (18) holds and $V_H - c_H > V_L - c_L$, then in the unique D1 equilibrium, the price charged by high quality firms is bounded away from true marginal cost c_H for all N and market power persists even as $N \to \infty$.

Proposition 4 implies that the existence of a symmetric fully revealing D1 equilibrium depends on the number of firms i.e., on the competitiveness of the market. Full revelation of information (under this strong concept of equilibrium refinement) occurs if, and only if, the market structure is sufficiently competitive. Further, when it exists, there is a unique symmetric D1 equilibrium and in this equilibrium, high quality firms exercise market power as long as $\frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L > c_H$ i.e., if, either

$$V_H - c_H > V_L - c_L$$ (19)

or ,

$$V_H - c_H \leq V_L - c_L \text{ and } N < \frac{c_H - c_L}{(V_L - c_L) - (V_H - c_H)}.$$

Further, if (19) holds, then the high quality price is bounded below by $V_H - V_L + c_L > c_H$ for all N i.e., market power persists for high quality firms even as $N \to \infty$ (low quality prices must converge in distribution to c_L as $N \to \infty$). The intuition behind this is straightforward; if (19) holds, then if the high quality good is sold at price $p_H = c_H$, then no consumer buys low quality good (even if it is sold at marginal cost) and therefore, there is no rent for low quality sellers. Full revelation of information therefore requires that the high quality price be above (and in fact, bounded away from) c_H. The fact that persistent market power may emerge in the unique symmetric fully revealing equilibrium under a very strong refinement of out-of-equilibrium beliefs indicates the importance of the link between incomplete information, information revelation and market power.

5 Non-revealing Equilibria

In this section, we analyze the nature and possibility of non-revealing pooling equilibria where firms charge the same price for sure, independent of their product quality, so that prices convey no information about quality with probability

one. We first restrict attention to perfect Bayesian equilibria that satisfy the Intuitive Criterion. Later, we apply the D1 refinement criterion.

Proposition 5 *(a) In every pooling equilibrium, all firms charge the same price $\widetilde{p} \geq c_H$ independent of their types, sell strictly positive quantity and earn non-negative profit with probability one; low quality firms earn strictly positive profit.*
(b) A pooling equilibrium exists if, and only if,

$$c_H \leq \min\left\{\alpha V_H + (1-\alpha)V_L, c_L + \frac{\alpha N(V_H - V_L)}{N-1}\right\}. \qquad (20)$$

(c) If (20) holds , the set of prices that can be sustained as the common pooling equilibrium price is the interval $[c_H, \min\{\alpha V_H + (1-\alpha)V_L, c_L + \frac{\alpha N(V_H - V_L)}{N-1}\}]$.

Part (a) of Proposition 5 is intuitive. In a pooling equilibrium, no firm randomizes over prices as both low and high quality types cannot be indifferent between two distinct prices (due to differences in their unit cost of production). Further, all firms must charge the same price in such an equilibrium and sell strictly positive quantity for otherwise, the low quality type of some firm (for instance, one that sells zero) has an incentive to deviate. Finally, if the price is below c_H, then a high quality type has an incentive to deviate. Part(b) of Proposition 5 outlines a necessary and sufficient condition for a pooling equilibrium to exist. It reflects the fact that in order for consumers to buy in equilibrium, the pooling price (which is always bounded below by c_H) cannot exceed the expected valuation of consumers $\alpha V_H + (1-\alpha)V_L$ and further, the pooling price should not be so high that a low quality firm can gain by deviating to a lower price and attracting all consumers (even if consumers believe that the product is of low quality with probability one). Part (c) of the proposition follows immediately and characterizes fully the set of pooling equilibrium outcomes. As can be readily observed, the set of pooling equilibria may involve a wide range of market power.

The next two corollaries (that follow immediately from Proposition 5) provide conditions under which there is no pooling equilibrium (as (20) does not hold) so that in every equilibrium, prices reveal some information about quality. Note that as discussed in Section 3 (Proposition 3), a fully revealing equilibrium always exists.

The first result provides conditions on other parameters such that if the number of firms is large enough, there is no pooling equilibrium. In this sense, a more competitive market structure facilitates an outcome that necessarily involves some revelation of information.

Corollary 4 *Suppose that*

$$\alpha\left(\frac{V_H - V_L}{c_H - c_L}\right) < 1.$$

Then, if the market concentration is sufficiently small and in particular,

$$N \geq \frac{1}{1 - \alpha\left(\frac{V_H - V_L}{c_H - c_L}\right)},$$

no pooling equilibrium exists.

The next result states that a pooling equilibrium does not exist if the prior likelihood of the product being of low quality is high enough - in that case a low quality firm can grab the entire market by undercutting the equilibrium price by a small amount (as the average quality that consumers purchase at the equilibrium price is close enough to the low quality). Thus, paradoxically, the domination of "bad" over "good" product firms in the statistical distribution of types favors revelation of information.

Corollary 5 *No pooling equilibrium exists if*

$$\alpha \leq \frac{c_H - V_L}{V_H - V_L}$$

i.e., the prior likelihood that a firm's product is of low quality is large enough.

From Proposition 3 and Proposition 5, it follows that when (20) holds, both fully non-revealing and fully revealing equilibria coexist with the associated out-of-equilibrium beliefs in both kinds of equilibria satisfying the Intuitive Criterion. To see why the Intuitive Criterion does not rule out the existence of pooling equilibria, note that a low quality type incurs lower production cost and therefore that the maximal deviation pay-off for a high quality firm is higher than the equilibrium pooling pay-off, then this must also be necessarily true for a low quality firm (but the converse need not hold). Therefore, in particular, the Intuitive Criterion does not rule out consumers' beliefs that assign probability one to a deviating firm being of low quality and with such beliefs, consumers refuse to buy if the deviation price is high enough.

Next, we show that no pooling equilibrium satisfies the more demanding D1 equilibrium refinement condition.

Proposition 6 *There is no pooling equilibrium satisfying the D1 refinement.*

The intuition behind this is straightforward. Suppose that for some profile of beliefs and best responses of the consumers and given the equilibrium strategies of the other firms, a low quality firm gains weakly by deviating to a price slightly above the pooling equilibrium price. As the high quality type has a higher marginal cost than the low quality firm, it must necessarily gain (strictly) from a similar deviation. The set of responses of the buyers (in terms of expected quantity sold) for which the high quality type gains by deviating to such a price is larger than that of the low quality type. D1 refinement then implies that buyers should assign probability one to the event that the deviating firm is a high quality firm. But with such beliefs, all consumers buy from the firm that unilaterally deviates to a price slightly above the pooling equilibrium price, (whereas in equilibrium, the firm splits the market with all other rivals). Therefore, the deviation is strictly gainful.

Proposition 6 implies that, independent of the number of firms, if there is a D1 equilibrium, it must necessarily be fully or partially revealing. This result

reinforces the idea that competition leads to revelation of information. However, partially revealing equilibria are not formally characterized in this paper.[16]

6 Heterogeneous Consumers

One feature of the fully revealing equilibria discussed in Section 3 is that a high quality product is sold only in the state where all firms are of high quality. In states of nature where both low and high quality products are available, consumers buy the low quality good i.e., in such states, high prices signal high quality, but nobody buys at the higher price. This is a consequence of the assumption that all consumers are identical and, in particular, have identical valuation for the high quality good. If a low quality seller sells only in the state where at least some rivals sell low quality products, then price competition would reduce its profit to zero and thus create incentive for such a seller to imitate the high quality product. As discussed in Section 3, signaling requires that low quality sellers be able to exercise some market power in some states of nature; a necessary condition for this is that at the equilibrium prices, some consumers should buy from low quality firms even when the high quality product is available. If high quality firms sell with strictly positive probability in states where low quality firms are around, then some low quality firm has an incentive to steal business from high quality firms by shifting probability mass towards a lower price. Therefore, in a fully revealing equilibrium, high quality firms do not sell at all in states when some other firm has low quality product.

This unsavory feature of the revealing equilibrium can, however, be eliminated if consumers differ in their valuation of the high quality good[17] and in that case, signaling of private information through prices is perfectly consistent with a market outcome where some consumers (those with higher valuation for the high quality good) always buy high quality at high price while other (lower valuation) consumers buy low quality (when both types of goods are available in the market).

To illustrate the effect of introducing heterogeneity of consumers consider a simple extension of the basic model outlined in Section 2. Assume, as before, that there is a unit mass of consumers, each with unit demand, but now suppose there are two types of consumers - named type 1 (high valuation) and type 2

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[16]For example, under certain conditions, there are symmetric equilibria where H-type firms charge a deterministic price p_H and L-type firms play mixed strategies that assign strictly positive mass $\beta \in (0,1)$ to p_H and probability $(1-\beta)$ to an atomless distribution on an interval $[\underline{p}_L, \overline{p}_L]$. In such an equilibrium, a firm charging a low price (in the interval $[\underline{p}_L, \overline{p}_L]$) reveals itself to be of low quality while a firm that charges a high price (p_H) does not fully reveal its quality - after Bayesian updating, consumers infer that it could be of high quality with probability $\frac{\alpha}{\alpha+(1-\alpha)\beta}$ and of low quality with probability $\frac{(1-\alpha)\beta}{\alpha+(1-\alpha)\beta}$. Consumers are indifferent between buying low quality at price $\overline{p}_L$ and the updated expected quality at price p_H.

[17]It can also be avoided if firms' products are differentiated on some other dimension (for example, horizontal) or there is some brand/firm loyalty. See, Daughety and Reinganum (2007 a,b).

(low valuation). The measure of type 1 consumers is $\lambda \in (0, 1)$ and that of type 2 consumers is $1 - \lambda$. Type 1 consumers have valuation $\overline{V}_H$ and type 2 consumers have valuation $\underline{V}_H$ for quality H. All consumers have identical valuation $V_L > c_L$ for quality L. Assume that:

$$\overline{V}_H > \underline{V}_H > \max\{V_L, c_H\}. \tag{21}$$

All other aspects of the model remain unchanged.

In this extended model, one can show that under reasonable conditions, there are symmetric fully revealing equilibria where all high quality sellers charge a deterministic price $p_H \in [c_H, \overline{V}_H]$, low quality firms randomize over an interval $[\underline{p}_L, \overline{p}_L] \subset [c_L, V_L]$ using a continuous distribution function and as long as there is at least one firm selling the high quality product, all (high valuation) type 1 consumers buy high quality if available, while the (low valuation) type 2 consumers buy low quality except in the state where all firms are of high quality (in which state, type 2 consumers buy high quality if $p_H \leq \underline{V}_H$ and refrain from buying if $p_H \in [\underline{V}_H, \overline{V}_H]$).

As our analysis of the extended model with heterogenous buyers case does not provide any significant additional insight into the possibility of signaling through prices apart from showing that signaling is consistent with both low and high quality goods being sold simultaneously in the market, we do not provide a formal result, but instead illustrate it by providing an example in the appendix (where $N = 2, p_H = \underline{V}_H$ and $\overline{p}_L = V_L$).

7   Conclusion

Competition is not inimical to signaling of private information about product quality by firms. Even when price competition is intense and there are no other market frictions or features that soften competition, the market outcome under incomplete information can generate sufficient incentives for full revelation of information about quality. In such situations, signaling outcomes are associated with endogenously generated price dispersion and stochastic market power. Fully revealing equilibria may be associated with high degree of market power even when market concentration is very small. This remains true even if the out-of-equilibrium beliefs satisfy the strong D1 refinement criterion. Highly competitive market structures may rule out pooling equilibria and make information revelation more likely. Moreover, the D1 refinement criterion rules out all fully non-revealing equilibria and under such refinement, competition necessarily leads to (at least partial) revelation of quality through prices.

Our results have been established in a very simple framework with two types of product quality. Intuitively, it appears that it may be possible to extend our core arguments to a model with a finite number of quality types,[18] but the analysis is likely to be significantly more complicated. Future research in this direction should be of considerable interest.

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[18] Our arguments clearly do not work with a continuum of quality types.

Appendix

Proof of Proposition 1: We first show that (a) holds. Suppose not. Then, there exists a price p_L in the support of the (possibly mixed) equilibrium price strategy of firm i when it is of type L and a price p_H in the support of the equilibrium price strategy of firm i when it is of type H such that $p_H < p_L$. (Note that in a fully separating equilibrium $p_H < p_L$). Let q_L and q_H denote the *expected* quantity sold by firm i when it charges prices p_L and p_H, respectively. Consumers know the type of each firm and since $p_H < p_L$, given the strategies of other firms in equilibrium, it must be true that:

$$q_L \leq q_H. \tag{22}$$

Then, equilibrium requires each type of firm i to have no incentive to imitate the other type's action:

$$(p_L - c_L)q_L \geq (p_H - c_L)q_H \tag{23}$$
$$(p_H - c_H)q_H \geq (p_L - c_H)q_L \tag{24}$$

From (23),

$$
\begin{aligned}
0 &\leq (p_L - c_L)q_L - (p_H - c_L)q_H \\
&= (p_L - c_H)q_L - (p_H - c_H)q_H + (c_H - c_L)(q_L - q_H) \\
&\leq 0 + (c_H - c_L)(q_L - q_H), \text{ using (24),}
\end{aligned}
$$

which implies (as $c_H > c_L$) that

$$q_L \geq q_H. \tag{25}$$

Combining (22) and (25), we have $q_L = q_H$ in which case firm i of H-type has a strict incentive to deviate and charge price $p_L > p_H$, a contradiction.

Next, we show that (b) holds. Suppose to the contrary that type L of firm i makes zero expected profit. Then every other firm must make zero expected profit when it is of type L.[19] Since L type of firm i makes zero expected profit in equilibrium, it must be true that it sells zero with probability one at every price $c_L + \epsilon, \forall \epsilon > 0$ (otherwise it would deviate). Consider the state where all other firms are of type H. Since consumers can get strictly positive surplus by buying at price $c_L + \epsilon$ for $\epsilon > 0$ small enough, they would not buy at such a price only if there is some firm $j \neq i$ of type H that offers higher surplus in equilibrium so that firm j sells strictly positive expected quantity when it is of type H. But in that case there is strict incentive for firm j when it is of type L to imitate type H of firm j and make strictly positive expected profit, a contradiction.

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[19] For if this is not true, there is some firm $j \neq i$ that earns strictly positive profit when it is of type L which means that there is some price $p_L^j > c_L$ at which it sells strictly positive expected quantity. If firm i of type-L charges price $p_L^j - \epsilon$ for $\epsilon > 0$ sufficiently small, the worst that it can be thought of by consumers is that it is of $L-$type with probability one, in which case it can still attract all the consumers away from firm j and hence earn strictly positive expected profit.

Finally, we show that there is no fully revealing equilibrium where firms play pure strategies. Suppose to the contrary that there is an equilibrium where all firms $i = 1, ..N$ charge prices $\{p_L^i, p_H^i\}$ with probability one. From part (a) of the proposition, $p_L^i < p_H^i, i = 1, ..N$. From part (b) of the proposition, $p_L^i > c_L$ and all firms of type L sell strictly positive expected quantity at price p_L^i. Without loss of generality, let us index the firms such that $p_L^1 \leq p_L^2 \leq ... \leq p_L^N$. As type L firms must earn strictly positive rent it must be that $p_L^1 \in (c_L, V_L]$ and every L-type firm sells strictly positive quantity with positive probability. Suppose $p_L^i = p_L^{i+1}$ for some $i = 1, ..N - 1$. We claim that in every state of nature where firms i and $i + 1$ are of type L, they must both sell zero. For if at least one of these two firms, say type L of firm i, sold strictly positive quantity in any such state of nature, then type L of firm $i + 1$ would always have a strict incentive to undercut slightly (upward jump in quantity in the state where i sells and no decline in quantity sold in any other state). Therefore, it must be true that $p_L^1 < p_L^2 < ... < p_L^N$. Firm N of type L can only sell in the state all firms $j = 1, ...N - 1$, are of type H and p_L^N must therefore be the highest price at which firm N can sell in this state so that

$$V_H - \min\{p_H^i : i = 1, ...N - 1\} = V_L - p_L^N. \qquad (26)$$

If in the state where firms $j = 1, ...N - 1$ are of type H, firm N of type H at price p_H^N sells higher quantity than firm N of type L at price p_L^N , then the latter has an incentive to imitate its high type and charge $p_H^N > p_L^N$. Therefore, it must be the true that

$$p_H^N \geq \min\{p_H^i : i = 1, ...N - 1\}. \qquad (27)$$

Firm $N - 1$ of type L can possibly sell only in the event that firms $1, ...N - 2$ are of type H. Further, using (26),(27) and $p_L^{N-1} < p_L^N$,

$$V_L - p_L^{N-1} > V_H - \min\{p_H^i : i = 1, ...N\}.$$

So, the quantity sold by firm $N - 1$ of type L remains unchanged if it charges $p_L^{N-1} + \epsilon$ for $\epsilon > 0$ small enough and thus, increases its expected profit, a contradiction. This completes the proof.

Proof of Proposition 2: Consider any symmetric fully revealing equilibrium. Let $\underline{p}_L$ and $\overline{p}_L$ be, respectively, the lower and upper bounds (inf and sup) of the support of the equilibrium price distribution of L-type firms. Similarly, let $\underline{p}_H$ and $\overline{p}_H$ be the lower and upper bounds (inf and sup) of the support of the equilibrium price distribution of H-type firms. From Proposition 1, $\overline{p}_L < \underline{p}_H$. Let S be the support of the equilibrium price distribution of L-type firms and $F(.)$ be the equilibrium price distribution. From Proposition 1(b), $\underline{p}_L > c_L$ and $L-$type firms sell strictly positive expected quantity at each p in the support of the price distribution which, in particular, implies that $\overline{p}_L \leq V_L$. Thus, S is bounded and hence compact. If F assigns strictly positive probability mass to any price $p \in S$, then an $L-$type firm can earn strictly higher profit at price $p - \epsilon$ for $\epsilon > 0$ small enough, a contradiction. Therefore, the distribution F is

atomless on S. It follows that in the event that at least one other firm is of type L, each $L-$ type firm is undercut with probability one at price $\overline{p}_L$. In other words, in equilibrium, an L-type firm sells at price $\overline{p}_L$ only if all other firms are of type H. We claim that

$$V_L - \overline{p}_L \geq V_H - \underline{p}_H. \tag{28}$$

To see this, suppose to the contrary that:

$$V_L - \overline{p}_L < V_H - \underline{p}_H.$$

Then, at price $\overline{p}_L$, an $L - type$ firm does not sell unless all other firms are of type H and charge price $> \underline{p}_H + \epsilon$ (for some $\epsilon > 0$). If this $L - type$ firm now deviates from price $\overline{p}_L$ to $\underline{p}_H$ (imitates an H type firm), he will not only sell in the event that he would have sold when he charged price $\overline{p}_L$ but in addition, in the event that other firms are either of type H and charge price $\in [\underline{p}_H, \underline{p}_H + \epsilon)$ or of type L and charge price $\in [\overline{p}_L, \overline{p}_L - \eta)$ for some $\eta > 0$. Thus, his expected quantity sold increases as he increases his price from $\overline{p}_L$ to $\underline{p}_H$ and this implies that his expected profit increases, a contradiction. Hence, (28) holds. If inequality (28) holds strictly, then an individual type L firm can charge price $\overline{p}_L + \epsilon$ for $\epsilon > 0$ small enough and still sell the same expected quantity as at price $\overline{p}_L$, which would be a gainful deviation. Therefore, we have in fact that $V_L - \overline{p}_L = V_H - \underline{p}_H$ which yields (7). Thus, a low quality firm sells to all consumers if all other firms are high quality types, no matter what price in the equilibrium price distribution they charge.

The equilibrium expected profit of an L-type firm π_L is given by its profit when it charges $\overline{p}_L$ and as it is undercut with probability one in any state where there is one other L-type firm and only sells in the state in which all other firms are of type H,

$$\pi_L^* = (\overline{p}_L - c_L)\alpha^{N-1} = [\underline{p}_H - (V_H - V_L) - c_L]\alpha^{N-1}.$$

If a low-type firm sets any other price, its profits are given by

$$\left\{ \sum_{i=0}^{N-1} \binom{N-1}{i} \alpha^i [(1-\alpha)(1-F(p))^{N-1-i}] \right\} [p - c_L]$$

$$= (\alpha + (1-\alpha)(1-F(p)))^{N-1} [p - c_L].$$

Equating this to π_L^* gives the expression for F in (5) as well as for $\underline{p}_L$ (using $F(\underline{p}_L) = 0$).

To prove that the high quality price distribution $H(p)$ has a mass point, suppose to the contrary that it does not have a mass point. It then follows that for any price in the support of high quality price distribution, the profit of a high quality firm is $[(\alpha[1-H(p)])^{N-1}(p - c_H)]$. For this to be equal to a constant $\widehat{\pi}_H$, it follows that

$$H(p) = 1 - \frac{1}{\alpha} \sqrt[N-1]{\frac{\widehat{\pi}_H}{p - c_H}}.$$

As consumers will not buy if prices above V_H are charged and as $H(V_H) < 1$ we arrive at a contradiction.

Proof of Corollary 1: For each $N > 1$, consider any symmetric fully revealing equilibrium and let F_N be the distribution function and $\underline{p}_N, \overline{p}_N$ the endpoints of the interval of support of the random price charged by a low quality firm in such an equilibrium. Note that $\underline{p}_N, \overline{p}_N \in [c_L, V_L]$, and from (6), $\underline{p}_N \to c_L$ as $N \to \infty$. Fix $\epsilon > 0$ small enough. Then, for N large enough, $\underline{p}_N < c_L + \epsilon$, and using (5),

$$\frac{1 - \alpha}{\alpha}[1 - F_N(c_L + \epsilon)] = \sqrt[N-1]{\frac{\overline{p}_N - c_L}{\epsilon}} - 1 \to 0,$$

as $N \to \infty$, so that $F_N(c_L + \epsilon) \to 1$ as $N \to \infty$. Thus, F_N converges to $\delta(c_L)$ as $N \to \infty$. A similar argument holds when $\alpha \to 0$.

Proof of Lemma 1: Consider the following result:

(R.1) *There exists a symmetric fully revealing equilibrium (in Ω) where every $H-$type firm charges price $p_H = p^*$ (with probability one) and all consumers buy with probability one if, and only if,*

$$(1 - \frac{1}{N})(p^* - c_H) \leq V_H - V_L \leq (1 - \frac{1}{N})(p^* - c_L), p^* \in [c_H, V_H] \qquad (29)$$

Using (11) and (12), observe that (29) is equivalent to:

$$\theta_0 = \max\{c_H, \frac{V_H - V_L}{1 - \frac{1}{N}} + c_L\} \leq p^* \leq \min\{\frac{V_H - V_L}{1 - \frac{1}{N}} + c_H, V_H\} = \theta_1$$

so that $\theta_0 \leq \theta_1$ is necessary and sufficient for the existence of a symmetric fully revealing equilibrium (in Ω) where every $H-$type firm charges price $p_H = p^*$ (with probability one) and all consumers buy with probability one. Using the fact that (14) is necessary and sufficient for $\theta_0 \leq \theta_1$, we immediately have Lemma 1. In the rest of the proof, we show that the statement (R.1) holds.

Fix p^* such that (29) holds. Let the equilibrium pricing strategies be as follows: the price charged by an L-type firm follows a continuous distribution function F with support $[\underline{p}_L, \overline{p}_L]$ where

$$\overline{p}_L = p^* - (V_H - V_L) \qquad (30)$$
$$\underline{p}_L = \alpha^{N-1}\overline{p}_L + (1 - \alpha^{N-1})c_L \qquad (31)$$

$$F(p) = 1 - \frac{\alpha}{1 - \alpha}\left(\sqrt[N-1]{\frac{V_L - c_L - (V_H - p^*)}{p - c_L}} - 1\right), p \in [\underline{p}_L, \overline{p}_L]. \qquad (32)$$

Note that from the second inequality in (29), (30) and (31), we have that $p^* > \overline{p}_L > \underline{p}_L > c_L$. We specify the (symmetric) out-of-equilibrium beliefs of consumers as follows. If consumers observe a firm charging a price $p \neq p^*, p \notin [\underline{p}_L,$

25

$\overline{p}_L$], they believe that the firm is of type H with probability $\mu(p) = 0$. At the end of the proof we argue that these beliefs satisfy the Intuitive Criterion.

Next, we specify the consumers equilibrium behavior on the equilibrium path as follows: if all firms charge price p^*, all consumers buy and are equally split between the N firms. If at least one firm charges price $p \in [\underline{p}_L, \overline{p}_L]$, all consumers buy from the firm charging the lowest price and if there are more than one such firm, then they are split equally between such firms. Consumers' behavior when they observe some firm charging a price $p \neq p^*$ or $p \notin [\underline{p}_L, \overline{p}_L]$ is not explicitly specified but required to be rational given their out-of-equilibrium beliefs. The equilibrium expected profit of an L-type firm π_L is given by its profit when it charges $\overline{p}_L$:

$$\pi_L^* = (\overline{p}_L - c_L)\alpha^{N-1} = [p^* - (V_H - V_L) - c_L]\alpha^{N-1}. \tag{33}$$

A $H-$type firm sells only in the state where all other firms are of type H and so its the equilibrium profit is given by:

$$\pi_H^* = \frac{1}{N}(p^* - c_H)\alpha^{N-1}. \tag{34}$$

We now show that the strategies of the firms in the statement of the proposition and that of the consumers as specified above (along with the restriction on their out-of-equilibrium beliefs) constitute a perfect Bayesian equilibrium satisfying the Intuitive Criterion under condition (29). From (30), consumers are indifferent between buying low quality at price $\overline{p}_L$ and high quality at price $p_H = p^*$ (earning positive surplus in both cases). Therefore, consumers' behavior on the equilibrium path is rational (given that prices fully reveal quality). Next, we show that no firm of any type has an incentive to deviate. From the construction of $F(p)$ in the previous proposition it follows that an L-type firm is indifferent between all prices in the interval $[\underline{p}_L, \overline{p}_L]$. We claim that at any $p \in (\overline{p}_L, p^*)$, a firm sells zero quantity so that neither type has an incentive to deviate to such a price. To see this, observe that (given the out-of-equilibrium beliefs) a consumer buys from a firm at price p only if

$$V_L - p \geq \min\{V_H - p^*, V_L - \overline{p}_L\} = V_L - \overline{p}_L,$$

(using (30)) which is never the case for prices $p > \overline{p}_L$. Second, note that neither type has an incentive to charge price $p > p^*$ as at such a price all consumers strictly prefer to buy from some other firm (a high quality firm at price p^* or low quality firm at price $\leq \overline{p}_L$) even if $\mu(p) = 1$. Third, observe that an $L-$type firm will never deviate to a price $p < \underline{p}_L$ as its profit even if it sells to all consumers with probability one is:

$$p - c_L < \underline{p}_L - c_L = [V_L - (V_H - p^*) - c_L]\alpha^{N-1} = \pi_L^*,$$

(using (30) and (31)). Fourth, if an L-type firm deviates and imitates a $H-$type firm charging p^* its expected profit is

$$\frac{1}{N}(p^* - c_L)\alpha^{N-1} \leq [V_L - (V_H - p_H) - c_L]\alpha^{N-1} = \pi_L^*$$

(using the second inequality in (29)) so that the deviation is not gainful. Fifth, suppose a H-type firm deviates to imitate a L−type firm and sets a price $p \in [\underline{p}_L, \overline{p}_L]$. Let $q(p)$ be the expected quantity sold in equilibrium by a L-type firm at price $p \in [\underline{p}_L, \overline{p}_L]$. The expected profit to a H-type firm from charging price p :

$$
\begin{aligned}
(p - c_H)q(p) &= (p - c_L)q(p) - (c_H - c_L)q(p) = \pi_L^* - (c_H - c_L)q(p) \\
&\leq \pi_L^* - (c_H - c_L)q(\overline{p}_L) = (\overline{p}_L - c_L)q(\overline{p}_L) - (c_H - c_L)q(\overline{p}_L) \\
&= (\overline{p}_L - c_H)\alpha^{N-1} = [V_L - (V_H - p^*) - c_H]\alpha^{N-1} \\
&\leq \frac{1}{N}(p^* - c_H)\alpha^{N-1}, \text{ using the first inequality in (29),} \\
&= \pi_H^*
\end{aligned}
$$

and therefore such a deviation cannot be gainful. Finally, if a H-type firm deviates to charge a price $p < \underline{p}_L$, its profit is $p - c_H < \underline{p}_L - c_H$ and the latter is its expected profit at price $\underline{p}_L$ (it sells with probability one at such price) and, as argued above, this is no greater than π_H^*. Thus, we have established that the prescribed strategies constitute an equilibrium. This completes the proof of R.1.

We will now show that the out-of-equilibrium belief $\mu(p)$ satisfies the Intuitive Criterion (IC). In the context of this model, IC requires us to consider the question whether certain prices are equilibrium dominated for certain types in the sense that the maximal pay-off a type possibly could get by deviating is lower than the equilibrium pay-off. It is certainly the case that the Intuitive Criterion implies that if firms charge (out-of-equilibrium) prices below c_H, then consumers infer that these prices are set by low quality types. This follows from the fact that high quality types cannot make positive profit by setting such low prices and thus cannot have an incentive to do so (whatever consumers' reaction). We will next show that the Intuitive Criterion never rules out the possibility that consumers believe the deviating price is set by a low quality firm. To see this, recall that the equilibrium pay-off for low types is equal to $\alpha^{N-1}(\overline{p}_L - c_L)$. Consumers will certainly buy at a price p, with $\overline{p}_L < p < p^*$, if all other firms are of high type, implying that the maximum deviation pay-off for such prices is not smaller than $\alpha^{N-1}(p - c_L)$. Therefore, these prices are not equilibrium dominated for type L and it is consistent with IC to specify $\mu(p) = 0$. For other out-of-equilibrium prices, specifying out-of-equilibrium beliefs is less essential as prices above p^* are never accepted given the equilibrium strategies and, therefore not optimal to charge and prices below $\underline{p}_L$ are so low that it is also not optimal to charge them (see above). That means that these prices are equilibrium dominated for both types and therefore IC does not impose restrictions on out-of-equilibrium beliefs given such prices.

It remains to show that (29) is also necessary for the existence of a symmetric fully revealing equilibrium where H-type firms charge price p^*. It is obvious that $p^* \in [c_H, V_H]$. From Proposition 2, we know that in any such equilibrium, the support of the price distribution of L-type firms must be as indicated in the statement of the proposition and the distribution has no mass points; further,

an H-type firm sells only if all other firms are of H-type. The, equilibrium profit of L and H type firms must be as given in (33) and (34). If the first inequality in (29) is violated, then, $V_L - (V_H - p^*) - c_H > \frac{1}{N}(p^* - c_H)$ so that a H-type firm that deviates and charges $\overline{p}_L$ earns expected profit:

$$(\overline{p}_L - c_H)\alpha^{N-1} = [V_L - (V_H - p^*) - c_H]\alpha^{N-1} > \frac{1}{N}(p^* - c_H)\alpha^{N-1} = \pi_H^*,$$

so that the deviation is strictly gainful. If the second inequality in (29) is violated, then, $V_L - (V_H - p^*) - c_L < \frac{1}{N}(p^* - c_L)$,so that a L-type firm that deviates and charges p^* earns profit

$$\frac{1}{N}(p^* - c_L)\alpha^{N-1} > [V_L - (V_H - p_H) - c_L]\alpha^{N-1} = \pi_L^*,$$

so that the deviation is strictly gainful. This completes the proof of (R.1).

Proof of Lemma 2: First, observe that at least one of the two conditions (15) or (16) hold if and only if there exists $\eta \in [0,1]$ such that

$$(1 - \frac{\eta}{N})(V_H - c_H) \leq V_H - V_L \leq (1 - \frac{\eta}{N})(V_H - c_L). \qquad (35)$$

Consider the following strategies. In equilibrium, high quality firms charge a deterministic price V_H. The price charged by a L-type firm follows a continuous distribution function F with support $[\underline{p}_L, \overline{p}_L]$ where

$$
\begin{align}
\overline{p}_L &= V_L \qquad &(36)\\
\underline{p}_L &= \alpha^{N-1}V_L + (1 - \alpha^{N-1})c_L \qquad &(37)
\end{align}
$$

$$F(p) = 1 - \frac{\alpha}{1-\alpha}\left(\sqrt[N-1]{\frac{V_L - c_L}{p - c_L}} - 1 \right), p \in [\underline{p}_L, \overline{p}_L]. \qquad (38)$$

It is easy to check that $p_H = V_H > \overline{p}_L > \underline{p}_L > c_L$.The (symmetric) out-of-equilibrium beliefs of consumers are as follows. If consumers observe a firm charge a price $p \neq p^*, p \notin [\underline{p}_L, \overline{p}_L]$, they believe that the firm is of type H with probability $\mu(p) = 0$. Along the same lines as in the proof of lemma 1, it can be shown that these beliefs satisfy the Intuitive Criteria. We specify the consumers' behavior as follows: no consumer buys from a firm that charges price $p > V_H$ or $p \in (\overline{p}_L, V_H)$. If all firms charge price V_H, $\eta \in [0,1]$ consumers buy (and are equally split between the N firms in that case) while $1 - \eta$ consumers do not buy.If at least one firm charges price $p \leq \overline{p}_L$, all consumers buy from the firm charging the lowest price and if there are more than one such firm, then they are split equally between such firms. The rest of the construction of this equilibrium consists of showing that the strategies of the firms and consumers as specified in above (along with the restriction on their out-of-equilibrium beliefs) constitute a perfect Bayesian equilibrium under condition (35). This can be verified by proceeding in an almost identical fashion as in the proof of Lemma 1. Finally, following very similar arguments as in the last part of the proof of Lemma 1,

it can be shown that (35) is necessary for the existence of an equilibrium in Ω where $p_H = V_H$ and $\eta \in [0, 1]$ consumers buy when all firms are of type H; since either condition (15) or (16) must hold for (35) to hold, this completes the proof of the lemma.

Proof of Proposition 3: If either (15) or (16) hold, then the result follows from Lemma 2. Suppose neither (15) nor (16) holds. Then $V_L > c_H$ and $N > \frac{V_H - c_H}{V_L - c_H}$ which implies that (14) holds. The result follows from Lemma 1.

Proof of Corollary 2: Observe that for N large enough, (14) holds so that from Lemma 1, the set of high quality prices that can be sustained in a symmetric fully revealing equilibrium in Ω includes the interval $[\theta_0, \theta_1]$. To see part (a) observe that if (17) holds and $N \geq \frac{c_H - c_L}{(c_H - c_L) - (V_H - V_L)}$, then $\theta_0 = \max\{c_H, \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L\} = c_H$ and the result follows immediately. If (17) does not hold, then $\theta_0 = \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L > (V_H - V_L) + c_L \geq c_H$ for all N. To see part (b), observe that if (15) holds, then the conclusion follows immediately from Lemma 2. If (15) does not hold, then $c_H < V_L$ and for N large enough, $\theta_1 = \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_H = V_H + [\frac{(V_H - V_L)}{N - 1} - (V_L - c_H)] > V_H$. For N large enough, (16) does not hold so that $\theta_1 = \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_H$ is the highest price sustainable as high quality price in an equilibrium in Ω. Since $\theta_1 \to V_H - V_L + c_H$ as $N \to \infty$, the result follows immediately..

Proof Lemma 3: Consider any symmetric fully revealing equilibrium in Ω where the high quality price $p_H \neq \theta_0$. Every equilibrium in Ω satisfies $p_H \geq \theta_0$ and therefore,

$$p_H > \theta_0 = \max\{c_H, \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L\}. \tag{39}$$

The equilibrium profits of L and H types of every firm in any such equilibrium are given by:

$$\pi_L^* = \alpha^{N-1}(\bar{p}_L - c_L) = \alpha^{N-1}[V_L - (V_H - p_H) - c_L]$$

$$\pi_H^* = \frac{1}{N}\alpha^{N-1}\beta(p_H - c_H)$$

where $\beta = 1$, if $p_H < V_H, \beta \in [0, 1]$, if $p_H = V_H$. Consider a price $p = p_H - \epsilon$ where $\epsilon > 0$ is arbitrarily small so that $p > \theta_0$ and $p \in (\bar{p}_L, p_H)$. We examine the incentives of the high and low quality types of some firm i to unilaterally deviate and charge p. Consider any expected quantity $q_i \in B_i(p)$ that can be sold by firm i at price p (where $B_i(p)$ is as defined in the above discussion on D1 refinement in Section 4) such that ,

$$(p - c_L)q_i \geq \pi_L^* = \alpha^{N-1}(V_L - V_H + p_H - c_L)$$

so that

$$q_i \geq \alpha^{N-1}\frac{V_L - V_H + p_H - c_L}{p - c_L}. \tag{40}$$

The profit of H-type of firm i when it deviates to p and sells expected quantity q_i :

$$
\begin{aligned}
& (p - c_H)q_i \\
\geq\ & (p - c_H)\alpha^{N-1}\frac{V_L - V_H + p_H - c_L}{p - c_L},\ \text{ using (40)} \\
=\ & \frac{\alpha^{N-1}}{N}(p_H - c_H)[N\frac{p - c_H}{p_H - c_H}\frac{V_L - V_H + p_H - c_L}{p - c_L}] \\
\geq\ & \pi_H^*[N\frac{p_H - \epsilon - c_H}{p_H - c_H}\frac{V_L - V_H + p_H - c_L}{p_H - \epsilon - c_L}] \qquad\qquad (41)
\end{aligned}
$$

Using (39),

$$
\frac{V_L - V_H + p_H - c_L}{p_H - c_L} = 1 - \frac{V_H - V_L}{p_H - c_L} > 1 - (1 - \frac{1}{N}) = \frac{1}{N}.
$$

It follows that if $\epsilon > 0$ is small enough $N\frac{p_H - \epsilon - c_H}{p_H - c_H}\frac{V_L - V_H + p_H - c_L}{p_H - \epsilon - c_L}] > 1$so that from (41) we have $(p - c_H)q_i > \pi_H^*$. Thus, we have shown that for $p = p_H - \epsilon$ where $\epsilon > 0$ is small enough,

$$
\{q_i \in B_i(p) : (p - c_L)q_i \geq \pi_L^*\} \subseteq \{q_i \in B_i(p) : (p - c_H)q_i > \pi_H^*\}. \qquad (42)
$$

We now argue that the inclusion in the above relationship is strict. To see this, consider $q_i = \xi\widehat{q}$ where $0 < \xi \leq 1$ and

$$
\widehat{q} = \alpha^{N-1}\frac{V_L - V_H + p_H - c_L}{p - c_L}. \qquad (43)
$$

We will choose $\epsilon > 0$ small enough such that for $p = p_H - \epsilon, \sigma = \frac{V_L - V_H + p_H - c_L}{p - c_L} < 1.$Suppose $(1 - \sigma\xi)$ consumers believe that the firm deviating to price p is of type H with probability 0 and $\sigma\xi$ consumers believe that it is of type H with probability μ where $\mu V_H + (1 - \mu)V_L - p = V_L - \overline{p}_L = V_H - p_H$ then (independent of the realization of types of other firms and their equilibrium strategies) it is a best response for exactly $\sigma\xi$ consumers to buy from the deviating firm i at price p in the state where all other firms charge price p_H (and to not buy from firm i at all in any other state) so that the expected quantity sold by firm i for this profile of beliefs and optimal actions is $\xi\widehat{q} = \sigma\xi\alpha^{N-1}$. Thus, $\xi\widehat{q} \in B_i(p)$ for every ξ in $(0, 1]$. Using (43), for $\xi < 1, (p - c_L)\xi\widehat{q} < \pi_L^* = \alpha^{N-1}(V_L - V_H + p_H - c_L)$ so that L-type of firm i has a strict incentive to *not* deviate to p if the expected quantity sold at that price is $\xi\widehat{q}$. On the other hand, since $(p - c_L)\widehat{q} = \pi_L^* = \alpha^{N-1}(V_L - V_H + p_H - c_L)$and $\widehat{q} \in B_i(p)$, we have from (42), that for $\epsilon > 0$ small enough, $(p - c_H)\widehat{q} > \pi_H^*$ and therefore for all $\xi < 1$ sufficiently close to 1, $(p - c_H)\xi\widehat{q} > \pi_H^*$.i.e., $\xi\widehat{q} \in \{q_i \in B_i(p) : (p - c_H)q_i > \pi_H^*\}, \xi\widehat{q} \notin \{q_i \in B_i(p) : (p - c_L)q_i \geq \pi_L^*\}$. Therefore,

$$
\{q_i \in B_i(p) : (p - c_L)q_i \geq \pi_L^*\} \subset \{q_i \in B_i(p) : (p - c_H)q_i > \pi_H^*\}
$$

where $\subset$ stands for strict inclusion. The D1 refinement therefore suggests that upon observing a unilateral deviation by firm i to a price $p = p_H - \epsilon$ where $\epsilon > 0$ is small enough, the out-of-equilibrium beliefs should assign probability one to the event that the firm is of type H. However, in that case a H-type firm i deviating to price p will sell to all consumers in the state where all other firms are of type H charging price p_H and therefore earns a deviation profit at least as large as $(p_H - \epsilon - c_H)\alpha^{N-1} > (p_H - c_H)\frac{\alpha^{N-1}}{N} = \pi_H^*$ for $\epsilon > 0$ sufficiently small. Thus, deviation by H-type firm i would be strictly gainful. QED.

Proof of Lemma 4. From Lemma 1 we know that there is a perfect Bayesian equilibrium in Ω where all high quality firms charge $p_H = \theta_0$ and all consumers buy with probability one. We will show that the out-of-equilibrium beliefs supporting this equilibrium as outlined in the proof of Lemma 1 satisfy the D1 refinement. First, note that for any out-of-equilibrium price $p < c_H$, the beliefs assign probability one to the event that the firm charging such a price is L-type. This is perfectly consistent with D1 refinement as H-type firm could never gain strictly by deviating to such a price. Second, for any out-of-equilibrium price $p > p_H$, the D1 refinement imposes no restriction on out-of-equilibrium beliefs. This is because if a firm unilaterally deviates to such a price, it sells zero (given the equilibrium strategies of other firms) no matter what buyers believe about its type. Therefore, neither type can gain strictly by unilateral deviation to such a price. Finally, consider out-of-equilibrium prices $p \in (c_H, \theta_0)$. Since, $\theta_0 = \max\{c_H, \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L\}$ this is relevant only if $\theta_0 = \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L > c_H$. We will show that for $p \in (c_H, \theta_0)$, the D1 refinement is consistent with (i.e., does not rule out) assigning probability 1 to the event that a firm unilaterally deviating to p is of type L (this is the out-of-equilibrium belief specified in the proof of Lemma 1). In order to show this, it is sufficient to establish that

$$\{q_i \in B_i(p) : (p - c_H)q_i \geq \pi_H^*\} \subseteq \{q_i \in B_i(p) : (p - c_L)q_i > \pi_L^*\}.$$

for any $q_i \in B_i(p)$. Consider any $q_i \in B_i(p)$ such that $(p - c_H)q_i \geq \pi_H^*$. Then,

$$
\begin{aligned}
q_i &\geq \frac{\alpha^{N-1}}{N}\frac{(p_H - c_H)}{p - c_H} = \frac{\alpha^{N-1}}{N}(\frac{\theta_0 - c_H}{p - c_H}) = \frac{\alpha^{N-1}}{p - c_L}\frac{p - c_L}{p - c_H}(\frac{\theta_0 - c_H}{N}) \\
&> \frac{\alpha^{N-1}}{p - c_L}(\frac{\theta_0 - c_L}{\theta_0 - c_H})(\frac{\theta_0 - c_H}{N}), \text{ as } (\frac{p - c_L}{p - c_H}) \text{ is strictly decreasing in } p \text{ on } (c_H, \theta_0) \\
&= \frac{\alpha^{N-1}}{p - c_L}\frac{\theta_0 - c_L}{N} = \frac{\alpha^{N-1}}{p - c_L}\frac{1}{N}\frac{(V_H - V_L)}{(1 - \frac{1}{N})} = \frac{\alpha^{N-1}}{p - c_L}(V_L - V_H + \frac{V_H - V_L}{1 - \frac{1}{N}}) \\
&= \frac{\alpha^{N-1}}{p - c_L}(V_L - V_H + \theta_0 - c_L) = \frac{\alpha^{N-1}}{p - c_L}(V_L - V_H + p_H - c_L) = \frac{\pi_L^*}{p - c_L}
\end{aligned}
$$

so that $(p - c_L)q_i > \pi_L^*$. QED.

Proof of Lemma 5: Suppose, to the contrary, that there is such a perfect Bayesian symmetric fully revealing equilibrium that meets D1 refinement. Let

$\underline{p}_H, \overline{p}_H$ be the lower and upper bounds of the support of the non-degenerate probability distribution of prices followed by every high type firm. Then, $\underline{p}_H < \overline{p}_H$. From Proposition 2 it follows that

$$V_H - \underline{p}_H = V_L - \overline{p}_L \tag{44}$$

so that in equilibrium high quality firms sell only if all other firms sell high quality. It follows that, high quality firms must charge $\overline{p}_H$ with strictly positive probability for otherwise a firm charging $\overline{p}_H$ sells zero with probability 1 resulting in zero profit. Moreover, it must be the case that prices just below $\overline{p}_H$ are not part of the support of high quality firms as the expected profit at $\overline{p}_H$ and at prices just below it cannot be equal. Next, we argue that $\underline{p}_H$ must also be charged with strictly positive probability. The reason is that the low quality firm could otherwise deviate to a price equal to $\underline{p}_H$ yielding a profit equal to $\alpha^{N-1}(\underline{p}_H - c_L)$, which is larger than the equilibrium profits $\pi_L^* = \alpha^{N-1}(\overline{p}_L - c_L)$. We claim that $\overline{p}_H > \theta_0 = \max\{c_H, \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L\}$. It is obvious that $c_H \leq \underline{p}_H < \overline{p}_H$. Further, since $\overline{p}_H$ is charged with probability less than one,

$$\pi_H^* < \frac{\alpha^{N-1}}{N}(\overline{p}_H - c_H)$$

Incentive compatibility requires

$$
\begin{aligned}
\pi_H^* &\geq (\overline{p}_L - c_H)\alpha^{N-1} = (V_H - V_L - \underline{p}_H - c_H)\alpha^{N-1} \\
&> (V_H - V_L - \overline{p}_H - c_H)\alpha^{N-1}.
\end{aligned}
$$

Thus, $\frac{\alpha^{N-1}}{N}(\overline{p}_H - c_H) < (V_H - V_L - \overline{p}_H - c_H)\alpha^{N-1}$ which implies $\overline{p}_H > \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L$. Thus, $\overline{p}_H > \theta_0$. Now, consider a deviation by the high quality firm to a price $p = \overline{p}_H - \epsilon$ for $\epsilon > 0$ small enough. For ϵ small enough, such a price p must lie outside the support of the high quality firm's equilibrium price strategy and exceeds θ_0. The rest of the proof follows in a very similar fashion as the proof of lemma 3.

Proof of Proposition 4. The first two parts of the proposition follow immediately from Lemmas 3,4 and 5. If $V_H - c_H > V_L - c_L$ and (18) holds,

$$\theta_0 = \frac{(V_H - V_L)}{(1 - \frac{1}{N})} + c_L > (V_H - V_L) + c_L > c_H, \forall N.$$

The proof is complete.

Proof of Proposition 5: We first show that in a pooling equilibrium, all firms choose pure strategies. Suppose not. Then there are two prices, p^1 and p^2, with $p^1 \neq p^2$, such that both high and low quality types of some firm i are indifferent between charging these two prices, implying that

$$\Pr(p^1)(p^1 - c_L) = \Pr(p^2)(p^2 - c_L)$$

32

and

$$\Pr(p^1)(p^1 - c_H) = \Pr(p^2)(p^2 - c_H),$$

where $\Pr(p^i)$ is the probability of selling at price $p^i, i = 1, 2$. From these two equations it follows that:

$$\frac{p^1 - c_H}{p^1 - c_L} = \frac{p^2 - c_H}{p^2 - c_L}.$$

As $\frac{p - c_H}{p - c_L}$ is strictly increasing in p, we have a contradiction to $p^1 \neq p^2$. The rest of part (a) follows from the fact that if firms do not sell at price $\widetilde{p}$, low quality firms have a strict incentive to deviate and set a price $p \in (c_L, V_L)$ as all consumers will buy from the deviating at such low prices even if they believe that the product is of low quality for sure.

We now derive the necessary conditions for a pooling equilibrium. In a pooling equilibrium both types of firms set the same price $\widetilde{p}$, with $\widetilde{p} \geq c_H$ as both types of firms must make non-negative profits. Given part (a), consumers' equilibrium pay-off $\alpha V_H + (1-\alpha)V_L - \widetilde{p}$ must be larger than or equal to 0. Taken together, this implies that it is necessary that $c_H \leq \alpha V_H + (1 - \alpha)V_L$ holds. The profits firms make are equal to $\pi_L = (\widetilde{p} - c_L)/N$ and $\pi_H = (\widetilde{p} - c_H)/N$, respectively. To create the most favorable conditions for this pricing behavior to be part of an equilibrium, consider out-of-equilibrium beliefs that are the most pessimistic: $\mu(p) = 0$ for all $p \neq \widetilde{p}$, i.e., if any price other than $\widetilde{p}$ is observed, consumers believe that this price comes from a low-quality producing firm. These beliefs are consistent with the Intuitive Criterion (IC). To see this, note that for both types, the maximal possible deviation pay-off from setting a price $p \neq \widetilde{p}$ is equal to $p - c_i, i = L, H$. Therefore, setting such a price is equilibrium dominated for type i, if and only if, $p < \frac{1}{N}\widetilde{p} + (1 - \frac{1}{N})c_i$. Thus, if it is not profitable for type L to deviate to price p, then it is certainly not profitable for type H to deviate to p. Therefore, for every price out of equilibrium price p, $\frac{1}{N}\widetilde{p} + (1 - \frac{1}{N})c_L < p < \frac{1}{N}\widetilde{p} + (1 - \frac{1}{N})c_H$, IC requires us to set $\mu(p) = 0$; for other prices, IC does not impose restrictions, so that $\mu(p) = 0$ is consistent with IC.

Given this specification of the out-of-equilibrium beliefs, consumers will buy at a price other than $\widetilde{p}$, if and only if, $\alpha V_H + (1 - \alpha)V_L - \widetilde{p} < V_L - p$, or in other words, if and only if, $p < \widetilde{p} - \alpha(V_H - V_L)$. As the low-quality firms have more incentives to deviate and set a low price than high-quality firms, the above behavior is an equilibrium if deviating to the highest price at which consumers will buy is not profitable, i.e., if and only if, $(\widetilde{p} - c_L)/N \geq \widetilde{p} - c_L - \alpha(V_H - V_L)$, or equivalently,

$$\left(1 - \frac{1}{N}\right)(\widetilde{p} - c_L) \leq \alpha(v_H - v_L).$$

So, we can conclude that for given values of the parameters we can find out-of-equilibrium beliefs satisfying the Intuitive Criterion that are such that any price $\widetilde{p} \in \left[c_H, \min\left\{\alpha v_H + (1 - \alpha)v_L, c_L + \frac{\alpha N(v_H - v_L)}{N-1}\right\}\right]$ can be sustained in equilibrium. This interval is non-empty if, and only if, the condition under (b) is met. Part (c) then follows immediately from the above argument.

Proofs of Corollary 4 and Corollary 5: Follow immediately from Proposition 5.

Proof of Proposition 6: Consider a pooling (perfect Bayesian) equilibrium where both types set a price p^*. From Proposition 5, it follows that $p^* < V_H$. Equilibrium profits for type τ are given by $\pi_\tau^* = \frac{(p^* - c_\tau)}{N}, \tau = L, H$. Consider a firm i of type τ that unilaterally deviates to a price $p = p^* + \epsilon$ where $\epsilon > 0$ is sufficiently small. This deviation is gainful if, and only if, the expected quantity q_i sold by the firm at price p (given the equilibrium strategies of others firms) satisfies $q_i > \frac{(p^* - c_\tau)}{N(p - c_\tau)}$. It is straightforward to check that for $p > p^*$,

$$\frac{(p^* - c_H)}{N(p - c_H)} < \frac{(p^* - c_L)}{N(p - c_L)}$$

and this can be used to easily verify that for $p = p^* + \epsilon$ and ϵ small enough,

$$\{q_i \in B_i(p) : (p - c_L)q_i \geq \pi_L^*\} \subset \{q_i \in B_i(p) : (p - c_H)q_i > \pi_H^*\}$$

(where " $\subset$ " stands for strict inclusion) and therefore the D1 criterion requires that the beliefs assign probability one to the event that a firm unilaterally deviating to price p is actually a high quality firm almost surely. It follows that with this kind of belief, all consumers will strictly prefer to buy from the firm deviating to price $p = p^* + \epsilon$ in which case, independent of its type, firm i earns strictly higher profit by deviating to price p.

Extended Model with heterogeneous consumers: An Example.
Consider the extended model with two types of consumers described in Section 5. Let $N = 2$. We construct a fully revealing equilibrium where all high quality sellers charge a deterministic price $p_H = \underline{V}_H$, low quality firms randomize over an interval $[\underline{p}_L, \overline{p}_L]$ where $\overline{p}_L = V_L$, using a continuous distribution function, all type 1 consumers buy high quality if available and all type 2 consumers buy low quality except in the state where all firms are of high quality. In such an equilibrium, the equilibrium profits of high and low quality types are given by

$$\pi_H^* = \lambda(1 - \alpha)(\underline{V}_H - c_H) + \frac{\alpha}{2}(\underline{V}_H - c_H); \quad \pi_L^* = (1 - \lambda)\alpha(V_L - c_L).$$

For the behavior of high-valuation consumers to be optimal, we require that even if the lowest price in the price distribution of low quality types is charged, these consumers prefer to buy the high quality, *i.e.,*

$$(\overline{V}_H - \underline{V}_H) > V_L - \underline{p}_L = \left[1 - \frac{\alpha(1 - \lambda)}{\lambda(1 - \alpha) + 1 - \lambda}\right](V_L - c_L).^{20} \quad (45)$$

---

[20] Note that a firm charging $\underline{p}_L$ will attract all (low and high valuation) consumers if the other firm is a low-quality firm as well. Its profits are therefore given by $[(1 - \alpha)\lambda + (1 - \lambda)](\underline{p}_L - c_L)$. Equating this with π_L^* gives the expression for $\underline{p}_L$.

Given that the surplus of low-valuation consumers buying high quality is 0, these consumers always prefer to buy low quality if available.

The main question then is whether we can find parametric restrictions and out-of-equilibrium beliefs satisfying the Intuitive Criterion such that firms do not have an incentive to deviate. If low-quality types mimic the pricing behavior of high quality types, their payoff is $\lambda(1-\alpha)(\underline{V}_H - c_L) + \frac{\alpha}{2}(\underline{V}_H - c_L)$ and incentive compatibility requires that

$$\pi_L^* = (1-\lambda)\alpha(V_L - c_L) \geq \lambda(1-\alpha)(\underline{V}_H - c_L) + \frac{\alpha}{2}(\underline{V}_H - c_L). \qquad (46)$$

Condition (46) also ensures that the incentive compatibility condition for the high quality type is satisfied. If consumers have pessimistic out-of-equilibrium beliefs, i.e., $\mu(p) = 0$, it is easy to see that no type of firm has an incentive to charge out-of-equilibrium prices. Low-quality firms may also want to deviate to a price equal to $V_L - (\overline{V}_H - \underline{V}_H)$ in an attempt to attract all consumers. To make this deviation unprofitable, we have to require that

$$V_L - (\overline{V}_H - \underline{V}_H) \leq (1-\lambda)\alpha(V_L - c_L). \qquad (47)$$

It remains to check that the pessimistic out-of-equilibrium beliefs outlined above are consistent with the Intuitive Criterion. The maximum possible deviation pay-off at price $p > \underline{V}_H$ is equal to $\lambda(1-\alpha)(p - c_i)$, $i = L, H$. This means that charging a price $p > \underline{V}_H$ is equilibrium dominated for H types if, and only if,

$$p \leq \underline{V}_H + \frac{\alpha}{2\lambda(1-\alpha)}(\underline{V}_H - c_H). \qquad (48)$$

Thus, pessimistic beliefs given prices above $\underline{V}_H$ are certainly consistent with the Intuitive Criterion if p is equilibrium dominated for the H type firm for all prices $\underline{V}_H < p < \overline{V}_H$, i.e., if the RHS of (48) is larger than $\overline{V}_H$. This requires that

$$\overline{V}_H - \underline{V}_H < \frac{\alpha}{2\lambda(1-\alpha)}(\underline{V}_H - c_H). \qquad (49)$$

This condition can always be satisfied if λ is small enough.

The maximum possible deviation pay-off for prices $V_L < p < \underline{V}_H$ is equal to $\{\alpha + (1-\alpha)(\lambda + (1-\lambda)q(p)\}(p - c_i)$, $i = L, H$, where $q(p)$ is the probability that low-valuation consumers buy at price p given that the other firm sells low quality. This means that charging a price p with $V_L < p < \underline{V}_H$ is equilibrium dominated for H types if, and only if,

$$\{\alpha + (1-\alpha)(\lambda + (1-\lambda)q(p))\}(p - c_H) \leq \lambda(1-\alpha)(\underline{V}_H - c_H) + \frac{\alpha}{2}(\underline{V}_H - c_H),$$

and for L types if and only if,

$$\{\alpha + (1-\alpha)(\lambda + (1-\lambda)q(p))\}(p - c_L) \leq \alpha(1-\lambda)(V_L - c_L),$$

respectively. Pessimistic beliefs given prices $V_L < p < \underline{V}_H$ are consistent with the Intuitive Criterion if p is equilibrium dominated for $L - type$ only if it is

35

equilibrium dominated also for $H - type$ i.e.,

$$\{\alpha + (1-\alpha)(\lambda + (1-\lambda)q(p))\} p$$
$$\leq \quad \alpha(1-\lambda)(V_L - c_L) + \lambda(1-\alpha)c_L + \{\alpha + (1-\alpha)(1-\lambda)q(p)\} c_L.$$

implies

$$\{\alpha + (1-\alpha)(\lambda + (1-\lambda)q(p))\} p$$
$$\leq \quad \lambda(1-\alpha)\underline{V}_H + \frac{\alpha}{2}(\underline{V}_H - c_H) + \{\alpha + (1-\alpha)(1-\lambda)q(p)\} c_H.$$

This holds if the RHS of the second inequality is larger than that of the first i.e.,

$$(1-\lambda)\alpha(V_L - c_L) \tag{50}$$
$$\leq \quad \lambda(1-\alpha)(\underline{V}_H - c_L) + \frac{\alpha}{2}(\underline{V}_H - c_L) + \left\{\frac{\alpha}{2} + (1-\alpha)(1-\lambda)q(p)\right\}(c_H - c_L).$$

Conditions (45), (46), (47),(49) and (50) together constitute the full set of equilibrium conditions. One can check that these conditions can be satisfied simultaneously, for example by the following parameter values $\lambda = 1/8, \alpha = 3/4, c_L = 10, c_H = 15, V_L = 18, \underline{V}_H = 20$ and $\overline{V}_H = 35$.

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