On Sequential and Simultaneous Contributions under Incomplete Information*

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Abstract

Under incomplete information about (independent) private valuations of a public good, we establish sufficient conditions under which, despite the incentive to free ride on future contributors, the expected total amount of voluntary contributions is higher when agents contribute sequentially (observing prior contributions) rather than simultaneously. We establish this in a conventional framework with quasi-linear utility where agents care only about the total provision of the public good (rather than individual contribution levels) and there is *no* non-convexity in provision of the public good. We allow for arbitrary number of agents and fairly general distribution of types.

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1 Introduction

In many situations economic agents contribute sequentially towards a public good or a project of common interest. Often, the order in which they contribute is determined *exogenously.* National governments may commit resources and defence forces to common war effort in some hierarchical order depending on individual stakes in the conflict. R&D investment by private firms with shared research output (as in open source software development or research on mapping of genomes) often follows the scientific order of discoveries with firms specializing in more "basic" research moving earlier (e.g., Bessen and Maskin, 2006).[1] In team production, various members can be deliberately induced to exert their efforts in an exogenous sequential order under full observability of efforts (Winter, 2006). Firm-level investment in providing local public goods for the industry (such as political lobbying for industry-wide benefits) may often be sequenced with incumbents moving earlier than entrants. Contributions to reduction of global environmental damage by various countries may follow a specific order according to perceived leader-follower roles. Finally, contributions to charities by donors during fundraising drives may follow the sequential order in which the donors are contacted.

In many of the examples discussed above, contributions by agents are observed by or revealed to others. Governments announce their own commitments to war efforts or disaster relief. In independent R&D ventures with shared output, leading firms may highlight their investment in the media; the follower firms make investment decisions based on this knowledge. During fundraising campaigns, charities announce donations as they come in. When contributions made by other agents are not observed till the contribution process has ended, the strategic interaction between potential contributors is identical to that in a simultaneous move game. On the other hand, the ability to observe actions taken by prior contributors generates a sequential game of voluntary contributions. A natural question that arises in this context is the effect of observability of contributions on the strategic incentives of voluntary contributors and on the eventual provision of the public good.

Varian (1994) has argued that in sequential games of voluntary contribution to a pure public good where the order of moves is exogenous, the ability of late movers to observe the contributions made by early movers aggravates the free rider

[1] Bessen and Maskin analyze the merits of sequential discoveries with public good features in software developments (due to complementarity between successive innovations).

problem. This is particularly striking if each agent has only one chance to contribute and not able to add to her current contribution later (i.e., early movers can credibly pre-commit to a certain level of contributions); in such situations, the total contribution generated is never greater and can be significantly smaller relative to the game where contributions are not revealed or observed (i.e., a simultaneous contribution game).[2] An important assumption behind Varian's result is *complete information* – agents knew each others' valuations of the public good. Intuition suggests that if agents do not know each others' valuations (the contribution game is one of incomplete information), then early movers who commit to low contributions to free ride on late movers also face the risk that late movers may not value the public good as much and thus, under-provide it relative to what the early movers may consider acceptable. Furthermore, when agents cannot observe others' contributions (as in a simultaneous move game), a contributor with lower valuation has an incentive to contribute much less than her valuation as she gambles on the event that other contributors will be ones with much higher valuations and will contribute generously; this, in turn, means that in states of the world where a large proportion of contributors are actually ones with low valuation, the total contribution is excessively small relative to what these low valuation contributors would have provided under complete information. When contributions are sequentially observed this problem is partially redressed, because later contributors make their donations *knowing* what the earlier ones have contributed and do not need to guess their contributions.

This paper develops the above intuition to show that under incomplete information about individual (independent) private valuations for the public good, a sequential contribution game may actually generate higher total expected contribution than the simultaneous move contribution game.[3] We establish this in a conventional economic framework (as in Varian, 1994) where contributors care only about the total provision of the public good (rather than individual contribution levels of other agents), thus ruling out snob effects, warm glow and more generally, complementarities between individual contributions. This distinguishes our work

[2]Bruce (1990) shows a similar result in the specific context of defence spending.

[3]In a brief section, Varian (1994) discusses the incomplete information game considered by us and argues that incomplete information about the second-mover's utility leads the first-mover to contribute *less than what she would under complete information.* He does not compare sequential and simultaneous contributions under incomplete information.

from Romano and Yildirim (2001) who show that sequential contribution may increase total contributions if utility depends not just on total contributions but also on individual contribution levels. Further, there is no non-convexity in the production technology. This rules out the presence of increasing returns or threshold effect in the production technology; Andreoni (1998) shows that in the presence of such effects, learning about increased contribution of early movers may increase the marginal productivity of followers' contributions, thus creating advantages for a sequential contribution format.[4]

We allow for arbitrary number of agents and fairly general distribution of types. We show that, under certain conditions that include concavity of the marginal utility from the public good, if there is an agent who contributes a strictly positive amount almost surely in an equilibrium of the Bayesian simultaneous move game, then every equilibrium of the sequential contribution game where that agent is placed at the very end of the sequential order of moves would generate higher expected total contribution; in other words, a sequential contribution format is always "better" in terms of generating contributions as long as the last mover is one who does not fully free ride in the simultaneous contribution game, no matter what her realized type. This is a strong result because the superiority of the sequential game is obtained with very little restriction on the order of moves except to specify who should move last. Further, we obtain this result, even though each agent has only one chance to contribute and can therefore pre-commit to contributions;[5] under complete information, early movers have the greatest ability to free ride on late movers precisely when agents have such commitment ability. Our result also

[4] One may view discrete public goods (such as a 0-1 public good that is either provided or not provided) as special cases of such threshold effect in technology. Admati and Perry (1991) and Marx and Matthews (2000) analyze the provision of discrete public goods under complete information and compare the outcomes generated under an alternating offer voluntary contributions format and an unrestricted repeated contributions format. Menezes et al. (2001), and Agastya et al. (2007) analyze simultaneous move contribution to discrete public goods under incomplete information about private valuations.

[5] Winter (2006) examines incentive design in team production problem under complete information with an exogenously given sequential order of tasks where the tasks performed by agents are perfectly complementary. In our model, the participants' contributions are perfect substitutes (rather than complements) and there are no direct, differential incentives for participants except that all participants get equal access to the total public good produced. Under incomplete information, our intuition favoring sequential contributions may carry over to some team production technologies with less than perfect complementarity.

implies that if the simultaneous move game has an "interior" equilibrium where all agents of almost every type make strictly positive contributions, every sequential move game generates higher expected total contribution independent of the order of moves.

In the special case of two agents and two types ('high' and 'low' marginal utilities), we identify a simple sufficient condition that will ensure that at least one agent will always make strictly positive contributions in the simultaneous move game. Further, if the agents are symmetric (i.e., have the same type distribution), the same condition guarantees that both agents will always contribute strictly positive amounts. Finally, it is shown that strictly positive contributions for some agent in the simultaneous move game, for each realization of the agent's type, is not necessary for the sequential game form to dominate the simultaneous move form.

In an earlier paper (Bag and Roy, 2008) we consider a multistage game of contribution to a public good with *all agents having the option to contribute in all stages*. In that paper, it is shown that under incomplete information about the agents' valuations of the public good, the expected total contribution can be higher if the contributions made at each stage are observed before the next stage; the economic reasoning behind this is based on the possibility of revelation of the private preferences of contributors through their actions and the incentive of higher valuation contributors to hide information about their true preferences as it may make them more vulnerable to free riding by other agents in subsequent stages of the game. Such incentives are not important in voluntary contribution processes where agents cannot contribute repeatedly; the current paper focuses on these environments by assuming that agents move sequentially with each agent having only one turn to contribute so that later movers cannot free ride on earlier contributors and contributors have no incentive to either hide or reveal their private information about how much they care about the public good.

There is a substantial literature on voluntary provision of public goods under incomplete information that focus on *inefficiencies* that arise due to incompleteness of information.[6] There is also an extensive literature on mechanism design and public goods that focus on uncertainty caused by private information about

[6] See, among others, Bliss and Nalebuff (1984), Palfrey and Rosenthal (1988), Gradstein (1992), Vega-Redondo (1995).

preferences.[7] Our paper differs from both strands – we do not concern with the efficiency or normative issues, nor do we suggest sophisticated incentive mechanisms; instead, we offer an incomplete information based explanation of why sequential contribution schemes may be better for the total provision of the public good.

The next section presents the model. In section 3 we analyze the simultaneous contribution game, followed by an analysis of the sequential contribution game in section 4. In section 5, sequential and simultaneous move game forms are compared. Section 6 discusses a special case with two-agents and two types.

2 The model

$N > 1$ agents contribute voluntarily to a public good. Each agent $i \in \{1, ..., N\}$ has a budget constraint $w_i > 0$. Agent i's payoff depends on the total contribution of all agents, her own contribution and her own type and is given by

$$u_i(g_i, g_{-i}, \tau_i) = \tau_i V_i(g_i + g_{-i}) + w_i - g_i,$$

where $g_i \geq 0$ is i's contribution, g_{-i} is the total contribution of all other agents $j \neq i$, and τ_i is a private preference parameter of agent i that affects her marginal utility from consumption of the public good, is known only to agent i and is interpreted as the "type" of agent i. It is common knowledge that each agent i's type τ_i is an independent random draw from a probability distribution with distribution function F_i and *compact* support $A_i \subset \mathcal{R}_{++}$. Let $\underline{\tau}_i$ and $\overline{\tau}_i$ be the lowest and highest possible types of agent i defined by

$$\underline{\tau}_i = \min\{\tau : \tau \in A_i\}, \quad \overline{\tau}_i = \max\{\tau : \tau \in A_i\}.$$

Assumption 1. *$\forall i \in \{1, ..., N\}$, $V_i(.)$ is continuously differentiable, concave and non-decreasing on $\mathcal{R}_+$, with $V_i(0) = 0$, $\underline{\tau}_i V_i'(0) > 1$ and $\overline{\tau}_i V_i'(w_i) < 1$.*

Define $\forall i$:

$$z_i = \sup\{x \geq 0 : \ \overline{\tau}_i V_i'(x) = 1\}.$$

Under Assumption 1, $0 < z_i < w_i$. It is easy to check that agent i would never contribute in excess of z_i in any contribution game, whatever be her type. This

[7] See, among others, Groves and Ledyard (1977), Cremer and Riordan (1985), Bagnoli and Lipman (1989), Jackson and Moulin (1992).

allows us to drop w_i and write agent i's payoff function simply as

$$u_i = \tau_i V_i(g_i + g_{-i}) - g_i.$$

Define

$$\overline{G} = \sum_{i=1}^{N} z_i > 0.$$

Assumption 2. *$V_i(.)$ is strictly concave on $[0, \overline{G}]$, $\forall i \in \{1, ..., N\}$.*

Under Assumptions 1 and 2, every agent i of every possible type $\tau_i \in A_i$ has a unique *standalone* contribution $x_i(\tau_i) \in (0, w_i)$ defined by:

$$\begin{aligned} x_i(\tau_i) &= \arg\max_{g_i} u_i(g_i, 0, \tau_i), \\ \text{satisfying} \qquad \tau_i V_i'(x_i(\tau_i)) &= 1; \end{aligned} \tag{1}$$

it puts an upper bound on an agent's contribution, given her type, in *any* contribution game. It is easy to check that $x_i(\tau_i)$ is strictly increasing in τ_i, implying:

$$z_i = x_i(\overline{\tau}_i).$$

The *expected* standalone contribution of agent i, denoted hereafter by θ_i, is given by:

$$\theta_i = \int_{A_i} x_i(\tau_i)\, \mathrm{d}F_i(\tau_i).$$

Also, since $V_i(.)$ is non-decreasing, $V_i'(.) \geq 0$. Assumption 2 therefore implies that $V_i'(G) > 0$ on $[0, \overline{G})$. Finally, we impose:

Assumption 3. *$V_i'(.)$ is concave on $[0, \overline{G}]$.*

Assumption 3 is an important technical restriction that will be useful in comparing expected total contributions under sequential and simultaneous move games.[8]

[8] A simple example that satisfies all of the above assumptions is the situation where all agents have identical utility functions and distribution of types and for all $i = 1, ..., N$, $V_i(.)$ is the quadratic function

$$V_i(G) = \begin{cases} \alpha G - \frac{1}{2}G^2, & 0 \leq G \leq \alpha,\ \alpha > 0 \\ \frac{1}{2}\alpha^2, & G > \alpha, \end{cases}$$

with the additional restrictions that $\underline{\tau}_i = \underline{\tau}$, $\overline{\tau}_i = \overline{\tau}$ satisfy

$$\frac{1}{\alpha\underline{\tau}} < 1, \quad \frac{1}{\alpha\overline{\tau}} \geq 1 - \frac{1}{N}.$$

Note that we do not require that the function $V_i'(.)$ be concave or strictly decreasing on the entire positive real line.

While our analysis is presented for a continuously variable public good, by setting $V_i(G) = V(G)$ for all i and interpreting $V(G)$ as the *'probability of success'* of a public project with binary outcomes ("success" or "failure") that depends on total investment G, the analysis can be easily applied to a discrete public good setting;[9] in that case, $\tau_i > 0$ is agent i's deterministic utility if the project succeeds, while the utility obtained when the project fails is normalized to zero.

We will compare two game forms – an N-stage *sequential contribution game* and a *simultaneous contribution game* – and the solution concepts are *Perfect Bayesian Equilibrium* and *Bayesian-Nash Equilibrium*, respectively. In the sequential contribution game, the agents contribute in an exogenous order and the contribution amounts become known as and when they are made.[10] Each agent is allowed to contribute only once and is not allowed to add to her contribution at a later stage. In the simultaneous contribution game, each agent contributes without any knowledge of other agents' contributions. We confine attention to pure strategy equilibria.

The contribution games are compared according to the (*ex ante*) expected total contributions made by all N agents i.e., the expected provision of public good.

3 Simultaneous contribution game

First, we analyze the simultaneous contribution game. Let $y_i(\tau_i)$ denote the equilibrium contribution of agent i of type $\tau_i \in A_i$, $i \in \{1, ..., N\}$ and $y_{-i}(\tau_{-i}) = \sum_{j \neq i} y_j(\tau_j)$ where $\tau_{-i} \in \Pi_{j \neq i} A_j$ is the vector of types for agents other than agent i. Then, $y_i(\tau_i)$ is a solution to the following expected utility maximization problem:

$$\max_{y \geq 0} \quad \tau_i E_{\tau_{-i}}[V_i(y + y_{-i}(\tau_{-i}))] - y. \tag{2}$$

In what follows, we denote by $F_{-i}(\tau_{-i})$ the joint distribution of τ_{-i}. We start with a simple observation that follows directly from the definitions of $x_i(\tau_i)$ and $\overline{G}$:

[9] Consider any density function $h(z)$ with support on the positive real line that is weakly decreasing and is, in addition, strictly decreasing and concave over $[0, \overline{G}]$. Then, taking $V(x) = \int_0^x h(z)\,\mathrm{d}z$ as the probability of success of the project satisfies our assumptions.

[10] Bagwell (1995) pointed out the importance of *observability* of agents' actions and its implications for agents' strategies in sequential action games.

Lemma 1. *Consider any Bayesian-Nash equilibrium of the simultaneous move contribution game where $y_i(\tau_i)$ is the contribution made by agent i of type τ_i. Then, $y_i(\tau_i) \leq x_i(\tau_i)$ $\forall \tau_i \in A_i$, $\forall i \in \{1, ..., N\}$, and the total contributions generated*

$$\sum_{i=1}^{N} y_i(\tau_i) \leq \overline{G} \quad \textit{with probability one.}$$

Next, we obtain a much sharper bound for the *expected* total contribution:

Lemma 2. *Consider any Bayesian-Nash equilibrium of the simultaneous move contribution game where for some agent i, $y_i(\tau_i)$, the equilibrium contribution of agent i, is strictly positive τ_i-almost surely i.e.,* $\Pr\{\tau_i : y_i(\tau_i) > 0\} = 1$.

(*i*) *Then, the* expected total contribution *by all agents generated in this equilibrium does not exceed θ_i, the expected standalone contribution of agent i.*

(*ii*) *If, in addition, $V_i'(G)$ is strictly concave on $[0, \overline{G}]$ and there exists some agent $j \neq i$ such that* $\Pr\{\tau_j : y_j(\tau_j) > 0\} > 0$, *then the* expected total contribution *by all agents generated in this equilibrium is strictly lower than θ_i, the expected standalone contribution of agent i.*

Proof. For agent i of type τ_i, the expected marginal utility from contributing $y_i(\tau_i)$ given the equilibrium strategies of other agents is:

$$\begin{aligned} & \tau_i \int V_i'(y_i(\tau_i) + y_{-i}(\tau_{-i})) \, \mathrm{d}F_{-i}(\tau_{-i}) \\ = \; & \tau_i E_{\tau_{-i}}[V_i'(y_i(\tau_i) + y_{-i}(\tau_{-i}))] \\ = \; & 1, \quad \tau_i\text{-almost surely,} \end{aligned} \tag{3}$$

using the first-order condition of the maximization problem (2) faced by agent i of type τ_i as $y_i(\tau_i) > 0$ τ_i-almost surely. First, we establish (*i*). Since (using Assumption 3) $V_i'(.)$ is concave on $[0, \overline{G}]$ and from Lemma 1, $y_i(\tau_i) + y_{-i}(\tau_{-i}) \in [0, \overline{G}]$ almost surely, we have by Jensen's inequality

$$\tau_i V_i'(y_i(\tau_i) + E_{\tau_{-i}}\{y_{-i}(\tau_{-i})\}) \geq 1, \quad \tau_i\text{-almost surely}$$

so that using (Assumption 2) concavity of $V_i(.)$ and (1) it follows that

$$y_i(\tau_i) + E_{\tau_{-i}}\{y_{-i}(\tau_{-i})\} \leq x_i(\tau_i), \quad \tau_i\text{-almost surely}$$

and integrating with respect to the distribution of agent i's type we have:

$$E_{\tau_i}\{y_i(\tau_i)\} + E_{\tau_{-i}}\{y_{-i}(\tau_{-i})\} \leq E_{\tau_i}\{x_i(\tau_i)\} = \theta_i,$$

establishing part (*i*) of the lemma.

Now, consider part (*ii*) of the lemma. It is easy to check from the first-order conditions of agent j's maximization problem that given $y_{-j}(\tau_{-j})$, the equilibrium contribution $y_j(\tau_j)$ of agent j is non-decreasing in the type τ_j of agent j. Thus, if $\tau < \tau'$, $\tau, \tau' \in A_j$ and $y_j(\tau) > 0$, then $y_j(\tau') > 0$. Further (from the first-order condition of maximization (3)),

$$\begin{aligned} 1 &= \tau \int V_j'(y_j(\tau) + y_{-j}(\tau_{-j}))\,\mathrm{d}F_{-j}(\tau_{-j}) \\ &< \tau' \int V_j'(y_j(\tau) + y_{-j}(\tau_{-j}))\,\mathrm{d}F_{-j}(\tau_{-j}) \end{aligned}$$

so that $y_j(\tau') > y_j(\tau)$. Since $\Pr\{\tau_j : y_j(\tau_j) > 0\} > 0$ for some $j \neq i$, the total contribution of agents other than agent i, $y_{-i}(\tau_{-i})$, is a non-degenerate random variable. Therefore, using strict concavity of $V_i'(.)$ on $[0, \overline{G}]$ (assumed in part (*ii*) of the lemma), the fact that $y_j(\tau_j) + y_{-j}(\tau_{-j}) \in [0, \overline{G}]$ almost surely (Lemma 1), and Jensen's inequality, we have

$$\tau_i V_i'(y_i(\tau_i) + E_{\tau_{-i}}\{y_{-i}(\tau_{-i})\}) > 1, \quad \tau_i\text{-almost surely}$$

so that using strict concavity of $V_i(.)$ on $[0, \overline{G}]$ (Assumption 2) and (1) it follows that

$$y_i(\tau_i) + E_{\tau_{-i}}\{y_{-i}(\tau_{-i})\} < x_i(\tau_i), \quad \tau_i\text{-almost surely}$$

and integrating with respect to the distribution of agent i's type we have:

$$E_{\tau_i}\{y_i(\tau_i)\} + E_{\tau_{-i}}\{y_{-i}(\tau_{-i})\} < E_{\tau_i}\{x_i(\tau_i)\} = \theta_i,$$

establishing part (*ii*) of the lemma. **Q.E.D.**

The next result follows immediately from Lemma 2:

Corollary 1. *Suppose there is a Bayesian-Nash equilibrium of the simultaneous move contribution game where* <u>*for all*</u> $i \in \{1, ..., N\}$, $y_i(\tau_i)$, *the equilibrium contribution of agent* i *of type* τ_i, *satisfies* $y_i(\tau_i) > 0$, τ_i*-almost surely. Then, the expected total contribution in this equilibrium does not exceed* $\min\{\theta_i : i = 1, ..., N\}$. *If, further,* $V_i'(G)$ *is strictly concave on* $[0, \overline{G}]$ <u>*for all*</u> $i \in \{1, ..., N\}$, *then the expected total contribution in this equilibrium is strictly less than* $\min\{\theta_i : i = 1, ..., N\}$.

4 Sequential contribution game

In this section, we analyze the sequential contribution games where agents contribute in an exogenous order of moves with each agent contributing only once.

Let $P = \{\text{set of all permutations of } (1, ..., N)\}$. For each $p = (p_1, ..., p_N) \in P$, we can define an N-stage sequential contribution game $\Gamma(p)$ where agent p_i contributes (only) in the i-th stage after observing contributions made in every previous stage.

We start with a result on the total contribution in any sequential move game guaranteed by the last mover's type:

Lemma 3. *In any perfect Bayesian equilibrium of* $\Gamma(p)$*, for each possible realization* τ *of the type of the last mover* p_N*, the total contribution generated is at least as large as* $x_{p_N}(\tau)$*, her standalone contribution for type* τ*, and the* expected *total contribution generated in the game is at least as large as* θ_{p_N}*, the* expected *standalone contribution of agent* p_N*.*

Proof. The proof follows from the fact that if $z \geq 0$ is the total contribution of agents in the first $(N-1)$ stages, then in the last stage of the game, the unique optimal action of agent p_N of type τ is to contribute $\max\{0, x_{p_N}(\tau) - z\}$. **Q.E.D.**

Next, we argue that as long as the total contribution generated in the first $(N-1)$ stages is strictly positive, the total expected contribution generated in the sequential game is strictly higher than the expected standalone contribution of the last mover. The main argument here is that earlier contributors know that even if they contribute zero, the last mover will ensure that the total contribution is at least as large as her standalone contribution (depending on her realized true type). If the total contribution on the equilibrium path in the first $(N-1)$ stages is below the standalone level for the lowest type of the contributor in stage N, then the last contributor who contributes strictly positive amount (for some realization of her type) among the first $(N-1)$ movers will always be better off deviating and contributing zero with probability one. Therefore, on an equilibrium path where total contribution in first $(N-1)$ stages is strictly positive (with strictly positive probability), it must exceed the standalone level of the N-th contributor for very low realizations of her type.

Lemma 4. *In any perfect Bayesian equilibrium of* $\Gamma(p)$ *where the total contribution generated in the first* $(N-1)$ *stages is strictly positive with strictly positive* *probability, the expected total contribution is strictly higher than θ_{p_N}, the expected standalone contribution of agent p_N.*

Proof. In view of Lemma 3, it is sufficient to show that for an event (i.e., a set of type profiles for N agents) of strictly positive probability measure, the generated total contributions strictly exceed the standalone contributions of agent p_N (corresponding to her realized types for those type profiles).

For each $\omega \in \prod_{i=1}^{N-1} A_i$ realization of types of the first $(N-1)$ contributors, let $z(\omega)$ be the total contribution generated in the first $(N-1)$ stages. Observe that the unique optimal action of agent p_N of type τ in stage N is to contribute $\max\{0, x_{p_N}(\tau) - z\}$, if the total contribution in the first $(N-1)$ stages is z. We claim that since $z(\omega) > 0$ with strictly positive probability, it must be the case that

$$z(\omega) > x_{p_N}(\underline{\tau}_{p_N}) \quad \text{with strictly positive probability.} \tag{4}$$

Suppose, to the contrary, that $z(\omega) \leq x_{p_N}(\underline{\tau}_{p_N})$ almost surely. Then, given the optimal strategy of agent p_N, the total contribution generated at the end of the game is exactly identical to that generated if every agent contributes zero with probability one in the first $(N-1)$ stages. In particular, let $k = \max\{1 \leq n \leq N-1 :$ agent p_n makes strictly positive contribution with strictly positive probability$\}$.
By definition, for all n lying strictly between k and N, no contribution occurs (almost surely) on the equilibrium path in stage n. Consider a unilateral deviation where agent p_k contributes zero almost surely and independent of history. The distribution of total contribution generated at the end of the game remains unchanged (as the last mover makes up the difference). Therefore, this deviation is strictly beneficial for agent p_{N-k}. This establishes (4). From (4), it follows that there exists $\epsilon > 0$ small enough such that

$$\Pr\{\omega \in \prod_{i=1}^{N-1} A_i : z(\omega) > x_{p_N}(\underline{\tau}_{p_N}) + \epsilon\} > 0,$$

which also implies[11] that there exists $\widehat{\tau} > \underline{\tau}_{p_N}$ such that

$$\Pr\{\omega \in \prod_{i=1}^{N-1} A_i : z(\omega) > x_{p_N}(\widehat{\tau})\} > 0. \tag{5}$$

[11] Here, we use the continuity of $x_{p_N}(\tau)$ in τ; this follows from the Maximum theorem and the uniqueness of solution to the maximization problem in (1).

Since $\underline{\tau}_{p_N} = \min\{\tau : \tau \in A_{p_N}\}$ and A_{p_N} is the support of the probability distribution of τ_{p_N}, it follows that $F_{p_N}(\widehat{\tau}) > 0$. Let

$$B_1 = \{\omega \in \prod_{i=1}^{N-1} A_i : z(\omega) > x_{p_N}(\widehat{\tau})\}.$$

Let B be the event:

$$B = \{(\tau_{p_1}, ..., \tau_{p_{N-1}}) \in B_1, \tau_{p_N} \leq \widehat{\tau}\}.$$

Then, from (5) and $F_{p_N}(\widehat{\tau}) > 0$, it follows that $\Pr(B) > 0$. Further, for type profiles in the set B, the generated total contributions strictly exceed the standalone contributions of agent p_N (note that $x_{p_N}(\tau_{p_N}) < x_{p_N}(\widehat{\tau})$ for all $\tau_{p_N} < \widehat{\tau}$, as $x_i(\tau_i)$ is strictly increasing in τ_i for all i). The proof is complete. **Q.E.D.**

The next lemma provides a condition under which the total contribution generated in the first $(N-1)$ stages of the sequential move game is strictly positive with strictly positive probability so that the hypothesis and conclusion of Lemma 4 hold.

Lemma 5. *Suppose that, given $p = (p_1, ..., p_N) \in P$,*

$$\exists k \in \{1, ..., N-1\} \text{ such that } V'_{p_N}(G) \leq V'_{p_k}(G), \quad \forall G \in [0, x_{p_N}(\overline{\tau})], \tag{6}$$

and, further,

$$\overline{\tau}_{p_k} E(\frac{1}{\tau_{p_N}}) > 1. \tag{7}$$

Then, in every perfect Bayesian equilibrium of $\Gamma(p)$, the expected total contribution is strictly higher than θ_{p_N}, the expected standalone contribution of agent p_N.

Proof. In view of Lemma 4, it is sufficient to show that in any perfect Bayesian equilibrium of $\Gamma(p)$, the total contribution generated in the first $(N-1)$ stages is strictly positive with strictly positive probability. Suppose, to the contrary, that there exists a perfect Bayesian equilibrium of $\Gamma(p)$ where the total contribution generated in the first $(N-1)$ stages is zero almost surely. In that case, the contribution generated at the end of the game is exactly the standalone contribution $x_{p_N}(\tau)$ of the last mover p_N, depending on her realized type $\tau \in A_{p_N}$. Since for any $k \in \{1, ..., N-1\}$, agent p_k contributes zero almost surely and $\overline{\tau}_{p_k}$ is the upper bound of the support of the distribution of her types, the first-order condition of maximization implies:

$$\overline{\tau}_{p_k} \int_{A_{p_N}} V'_{p_k}(x_{p_N}(\tau))\,\mathrm{d}F_{p_N}(\tau) \leq 1$$

which implies that

$$\begin{aligned} 1 &\geq \overline{\tau}_{p_k} \int_{A_{p_N}} \frac{\tau}{\tau} V'_{p_k}(x_{p_N}(\tau))\,\mathrm{d}F_{p_N}(\tau) \\ &\geq \overline{\tau}_{p_k} \int_{A_{p_N}} \frac{\tau}{\tau} V'_{p_N}(x_{p_N}(\tau))\,\mathrm{d}F_{p_N}(\tau), \quad \text{using (6)} \\ &= \overline{\tau}_{p_k} \int_{A_{p_N}} \frac{1}{\tau}\,\mathrm{d}F_{p_N}(\tau), \quad \text{using (1)} \\ &= \overline{\tau}_{p_k} E(\frac{1}{\tau_{p_N}}) \end{aligned}$$

which violates (7), a contradiction. **Q.E.D.**

The next result indicates that the hypothesis of Lemma 5 is always satisfied in the symmetric case.

Corollary 2. *Consider the symmetric case where* $V_i(G) = V(G)$, $\forall i \in \{1, ..., N\}$, *and all agents have identical distribution of types, i.e.* $F_i = F$, $\forall i \in \{1, ..., N\}$. *For every permutation* $p = (p_1, ..., p_N) \in P$ *and every perfect Bayesian equilibrium of the sequential move game* $\Gamma(p)$*, the expected total contribution generated is strictly higher than* θ_{p_N}*, the expected standalone contribution of agent* p_N*.*

Proof. Follows directly from Lemma 5 and the fact that (6) and (7) are satisfied for every permutation p of $(1, ..., N)$. To see that (7) is always satisfied, use Jensen's inequality, the fact that $f(x) = \frac{1}{x}$ is strictly convex on $\mathcal{R}_{++}$ and $\overline{\tau}_i = \overline{\tau}$, $\forall i \in \{1, ..., N\}$. **Q.E.D.**

5 Comparison of contributions

We now present the paper's main results comparing the expected total contributions generated in the simultaneous and sequential move contribution games. In particular, we provide conditions under which the sequential move contribution game leads to higher expected contributions.

First, we provide sufficient conditions under which sequential move games that satisfy a certain restriction on the order of moves (specifically, in terms of who moves in the last stage) generate weakly greater expected total contributions compared to the simultaneous move game.

Proposition 1. *Consider any Bayesian-Nash equilibrium $\mathcal{E}$ of the simultaneous move contribution game where some agent i makes a strictly positive contribution τ_i-almost surely. Then, every perfect Bayesian equilibrium of the N-stage sequential move game $\Gamma(p)$ where $p = (p_1, ..., p_N)$ and $p_N = i$, generates at least as much expected total contribution as in the Bayesian-Nash equilibrium $\mathcal{E}$ of the simultaneous move game.*

Proof. Follows immediately from Lemma 2 and Lemma 3. **Q.E.D.**

Next, we state sufficient conditions under which a sequential move game generates strictly higher expected total contributions compared to the simultaneous move game.

Proposition 2. *Consider any Bayesian-Nash equilibrium $\mathcal{E}$ of the simultaneous move contribution game where some agent i makes a strictly positive contribution τ_i-almost surely. Consider any perfect Bayesian equilibrium $\widehat{\mathcal{E}}$ of the sequential move contribution game $\Gamma(p)$ where $p = (p_1, ..., p_N)$ and $p_N = i$. Suppose, further, that* at least one *of the following hold:*

(a) (6) *and* (7) *are satisfied;*

(b) $V_i'(G)$ *is strictly concave on* $[0, \overline{G}]$ *and there exists some agent* $j \neq i$ *such that in the equilibrium* $\mathcal{E}$ *of the simultaneous move contribution game* $\Pr\{\tau_j : y_j(\tau_j) > 0\} > 0$.

Then, equilibrium $\widehat{\mathcal{E}}$ of the sequential move contribution game $\Gamma(p)$ generates strictly higher expected total contribution than equilibrium $\mathcal{E}$ of the simultaneous move game.

Proof. If (a) holds, then the result follows from Lemma $2(i)$ and Lemma 5. If (b) holds, then the result follows from Lemma $2(ii)$ and Lemma 3. **Q.E.D.**

There is a simple intuition behind the particular choice of the last mover. The agent who does not free ride fully in the simultaneous move game, when placed last in the sequential move game, offers an insurance against total contribution falling too low. This agent would have made strictly positive contributions almost surely in the simultaneous move game in anticipation of other agents' types not reflecting very high valuations with a considerable chance. But at the same time, this agent would have restrained her contribution in the simultaneous move game somewhat in case other agents happen to be of high-value types. So when this particular agent moves last and other agents do not actually contribute much, she ups her contribution beyond her one-shot equilibrium level (corresponding to her type) because she can no longer rely on the possibility of other agents' generous contributions.

The next result follows immediately from Proposition 1 and Corollary 1:

Corollary 3. *Consider any Bayesian-Nash equilibrium $\mathcal{E}$ of the simultaneous move contribution game where every agent makes strictly positive contribution almost surely. Then, for every $p \in P$, every perfect Bayesian equilibrium of the N-stage sequential move game $\Gamma(p)$ generates at least as much expected total contribution as in equilibrium $\mathcal{E}$ of the simultaneous move game.*

Corollary 3 clarifies that if the equilibrium of the simultaneous move game (to which one compares the outcomes of the sequential games) is an "interior equilibrium" where all agents of "almost" all types contribute strictly positive amounts, then there is no need to impose any restriction on the order of moves in the sequential game to ensure that it generates weakly higher expected total contributions.

Similarly, using Proposition 2 and Corollary 1, we have immediately:

Corollary 4. *Consider any Bayesian-Nash equilibrium $\mathcal{E}$ of the simultaneous move contribution game where every agent makes strictly positive contribution almost surely. Suppose, further, that at least one of the following holds:*

(a) (6) and (7) are satisfied, $\forall p \in P$;

(b) $V_i'(G)$ is strictly concave on $[0, \overline{G}]$, $\forall i \in \{1, ..., N\}$.

Then, for every $p \in P$, every perfect Bayesian equilibrium of the N-stage sequential move game $\Gamma(p)$ generates strictly higher expected total contribution than equilibrium $\mathcal{E}$ of the simultaneous move game.

Corollary 4 indicates that if the equilibrium of the simultaneous move game is an "interior equilibrium" then, under certain additional conditions, the sequential move game generates strictly higher expected total contributions independent of the order of moves.

Finally, using Proposition 1, Proposition 2 and Corollary 2, we have:

Corollary 5. *Consider the symmetric version of the model where* $V_i(G) = V(G), \forall i \in \{1, ..., N\}$, *and all agents have identical distribution of types i.e.,* $F_i = F, \forall i \in \{1, ..., N\}$. *Then the following hold.*

(i) *Consider any Bayesian-Nash equilibrium* $\mathcal{E}$ *of the simultaneous move contribution game where some agent* i *makes a strictly positive contribution* τ_i*-almost surely. Then, every perfect Bayesian equilibrium of the* N*-stage sequential move game* $\Gamma(p)$ *where* $p = (p_1, ..., p_N)$ *and* $p_N = i$, *generates strictly higher expected total contribution than in the equilibrium* $\mathcal{E}$ *of the simultaneous move game.*

(ii) *Consider any Bayesian-Nash equilibrium* $\mathcal{E}$ *of the simultaneous move contribution game where every agent makes strictly positive contribution almost surely. Then, for every* $p \in P$, *every perfect Bayesian equilibrium of the* N*-stage sequential move game* $\Gamma(p)$ *generates strictly higher expected total contribution than in equilibrium* $\mathcal{E}$ *of the simultaneous move game.*

6 A special case: two agents, two types

Here the focus will be on a special case of our model with two agents and two potential types. In particular, $N = 2$,

$$V_i(G) = V(G),\ A_i = \{\tau_L, \tau_H\},\ i = 1, 2,$$

with

$$0 < \pi_i = \Pr[\tau_i = \tau_H] < 1.$$

Assumptions 1-3 outlined in section 2 continue to apply. Note that the standalone contribution levels satisfy:

$$x_1(\tau) = x_2(\tau) = x(\tau),\ \tau = \tau_L, \tau_H.$$

In particular, $\overline{G} = 2x(\tau_H)$.

Proposition 1 shows that there is a sequential game that generates weakly higher expected total contribution than the simultaneous move game as long as there is at least one agent who contributes strictly positive amount in the simultaneous move game for (almost) every realization of her type. If, in addition, $\pi_1 = \pi_2$ (the symmetric case, see Corollary 5) or the requirements of Proposition 2 are satisfied, then the sequential game generates strictly higher expected total contributions. To elaborate on the content of these results, we now show that for the special case of two agents and two types described above, under certain conditions, there is always an agent who contributes strictly positive amount for all realizations of her type.

Proposition 3. *Suppose that*

$$V'(0) > \frac{1}{1-\overline{\pi}}\left[\frac{1}{\tau_L} - \overline{\pi}\cdot\frac{1}{\tau_H}\right], \tag{8}$$

where $\overline{\pi} = \max\{\pi_1, \pi_2\}$. *Then, in every Bayesian-Nash equilibrium of the simultaneous contribution game, at least one agent contributes strictly positive amount for both realizations of her type.*

Proof. Let $y_1(\tau), y_2(\tau), \tau = \tau_H, \tau_L$, denote the equilibrium contributions of the two agents in a Bayesian-Nash equilibrium of the simultaneous move game. The first-order condition of maximization for agent i of type τ is:

$$\begin{aligned} \pi_j \tau V'(y_i(\tau) + y_j(\tau_H)) + (1-\pi_j)\tau V'(y_i(\tau) + y_j(\tau_L)) &= 1, \text{ if } y_i(\tau) > 0 \\ &\leq 1, \text{ if } y_i(\tau) = 0 \end{aligned}$$

where $i, j = 1, 2,\ j \neq i$, $\tau = \tau_H, \tau_L$. Using strict concavity of $V(.)$ on $[0, \overline{G}]$, it is easy to check that $y_i(\tau_H) \geq y_i(\tau_L)$ and, further, $y_i(\tau_L) > 0$ implies $y_i(\tau_H) > y_i(\tau_L)$.

Suppose now that the proposition does not hold. Then, $y_1(\tau_L) = y_2(\tau_L) = 0$ so that for $i, j = 1, 2,\ j \neq i$,

$$\pi_j \tau_L V'(y_j(\tau_H)) + (1-\pi_j)\tau_L V'(0) \leq 1. \tag{9}$$

Using Assumption 2,

$$\pi_j \tau_L V'(x(\tau_L)) + (1-\pi_j)\tau_L V'(0) > 1, \tag{10}$$

which implies that (comparing (9) and (10))

$$y_j(\tau_H) > x(\tau_L) > 0,\ j = 1, 2.$$

From the first-order condition we have again for $i, j = 1, 2,\ j \neq i$,

$$\pi_j \tau_H V'(y_i(\tau_H) + y_j(\tau_H)) + (1-\pi_j)\tau_H V'(y_i(\tau_H)) = 1 \tag{11}$$

so that $\tau_H V'(y_i(\tau_H)) > 1$, $i = 1, 2$. As $\overline{\pi} = \max\{\pi_1, \pi_2\}$, $V'(0) > V'(y_j(\tau_H))$, from (9), we have for $j = 1, 2$,

$$\begin{aligned}
1 &\geq \pi_j \tau_L V'(y_j(\tau_H)) + (1 - \pi_j)\tau_L V'(0) \\
&\geq \overline{\pi}\tau_L V'(y_j(\tau_H)) + (1 - \overline{\pi})\tau_L V'(0) \\
&= \tau_L \left[\frac{1}{\tau_H} \overline{\pi}\tau_H V'(y_j(\tau_H)) + (1 - \overline{\pi}) V'(0) \right] \\
&> \tau_L \left[\frac{1}{\tau_H} \overline{\pi} + (1 - \overline{\pi}) V'(0) \right], \qquad \text{using (11)},
\end{aligned}$$

which violates (8). **Q.E.D.**

An immediate consequence of this result is that in the symmetric case where $\pi_1 = \pi_2 = \pi$, under condition (8), the *symmetric* Bayesian-Nash equilibrium must be one where both types of both agents contribute strictly positive amounts. This illustrates a concrete situation where the antecedents of Corollary 3, Corollary 4 and Corollary 5(*ii*) that require almost sure interiority of equilibrium contributions (of all agents) in the simultaneous move game can be satisfied (so that the sequential move game may generate more of expected contributions than the simultaneous move game independent of the order of moves).

If the simultaneous move game is such that every agent contributes zero with positive probability (low realizations of τ_i), then the results outlined in the previous section no longer apply. For the special case considered in this section, we can show that under certain conditions, the sequential contribution game generates higher expected total contributions even though the equilibrium in the simultaneous move game is one where both agents contribute zero when their realized type is τ_L. Thus, the interiority of equilibrium contributions (for some agent) is not necessary for higher contributions under the sequential form of the contribution game.

Proposition 4. *Suppose that $\pi_i \leq \pi_j$ and*

$$\frac{x(\tau_H)}{x(\tau_L)} < \min\{\frac{1}{\pi_i}, 2\pi_j + \frac{1 - \pi_j}{\pi_i}\}. \tag{12}$$

Then, for $p = (i, j)$, the expected total contribution generated in the unique perfect Bayesian equilibrium of the sequential game $\Gamma(p)$ is strictly greater than that generated in any Bayesian-Nash equilibrium of the simultaneous contribution game where both agents contribute zero when their realized type is τ_L.

Proof. Let $y_1(\tau), y_2(\tau), \tau = \tau_H, \tau_L$, denote the equilibrium contributions of the two agents in the simultaneous move game where $y_1(\tau_L) = y_2(\tau_L) = 0$. Using the same arguments as in the proof of Proposition 3, it is easy to check that $y_k(\tau_H) > x(\tau_L) > 0,\ \tau_H V'(y_k(\tau_H)) > 1,\ k = 1, 2$ so that

$$x(\tau_L) < y_k(\tau_H) < x(\tau_H), \quad k = 1, 2. \tag{13}$$

The expected total contribution generated in this game is $[\pi_i y_i(\tau_H) + \pi_j y_j(\tau_H)]$. Further, from the first-order conditions:

$$\pi_j V'(y_i(\tau_H) + y_j(\tau_H)) + (1 - \pi_j) V'(y_i(\tau_H)) = \frac{1}{\tau_H} = V'(x(\tau_H)) \tag{14}$$

$$\pi_i V'(y_i(\tau_H) + y_j(\tau_H)) + (1 - \pi_i) V'(y_j(\tau_H)) = \frac{1}{\tau_H} = V'(x(\tau_H)) \tag{15}$$

and using the concavity of $V'(.)$ on $[0, \overline{G}]$ and Jensen's inequality we have:

$$\pi_j\, y_j(\tau_H) + y_i(\tau_H) \ \leq\ x(\tau_H) \tag{16}$$

$$\pi_i\, y_i(\tau_H) + y_j(\tau_H) \ \leq\ x(\tau_H). \tag{17}$$

Also, from (14) and (15), $V'(y_i(\tau_H) + y_j(\tau_H)) < V'(x(\tau_H))$ so that

$$y_i(\tau_H) + y_j(\tau_H) > x(\tau_H). \tag{18}$$

From (13) and (18),

$$y_i(\tau_H) + y_j(\tau_H) > \max\{x(\tau_H), 2x(\tau_L)\}. \tag{19}$$

Multiply (16) by π_i and (17) by π_j and add to obtain $\pi_i \pi_j\, y_j(\tau_H) + \pi_i\, y_i(\tau_H) + \pi_i \pi_j\, y_i(\tau_H) + \pi_j\, y_j(\tau_H) \ \leq\ (\pi_i + \pi_j) x(\tau_H)$, implying

$$\begin{aligned}
& \pi_i\, y_i(\tau_H) + \pi_j\, y_j(\tau_H) \\
\leq\ & (\pi_j + \pi_i) x(\tau_H) - \pi_i \pi_j (y_i(\tau_H) + y_j(\tau_H)) \\
=\ & [\pi_j x(\tau_H) + (1 - \pi_j) x(\tau_L)] + [\pi_i (x(\tau_H) - \pi_j (y_i(\tau_H) + y_j(\tau_H)))] - (1 - \pi_j) x(\tau_L) \\
\underbrace{<}_{\text{using (19)}}\ & \pi_j x(\tau_H) + (1 - \pi_j) x(\tau_L) + [\pi_i (x(\tau_H) - \pi_j (\max\{x(\tau_H), 2x(\tau_L)\}))] - (1 - \pi_j) x(\tau_L) \\
<\ & \pi_j x(\tau_H) + (1 - \pi_j) x(\tau_L). \qquad \text{(using (12))}
\end{aligned}$$

Therefore, the expected total contribution in this equilibrium of the simultaneous move game $\pi_i\, y_i(\tau_H) + \pi_j\, y_j(\tau_H) < \pi_j x(\tau_H) + (1 - \pi_j) x(\tau_L)$, the expected standalone contribution of agent j. Finally, from Lemma 3, we know that in the sequential move game $\Gamma(p)$ where $p = (i, j)$ i.e., agent j is the last mover, the expected total contribution generated is at least as large as the expected standalone contribution $[\pi_j x(\tau_H) + (1 - \pi_j)x(\tau_L)]$ of agent j. The proposition follows. **Q.E.D.**

Finally, we would like to point out that even if the sequential move game is such that the agent (or agents) who contribute strictly positive amount for all realization of types in the simultaneous move game move earlier than other agents, the sequential move game may still generate higher expected total contribution than the simultaneous move game. We illustrate this in the example (summarized) below where within the framework of two agents and two types considered in this section, we choose a specific quadratic functional form for the $V(.)$ function.

An Example. Let

$$V(G) = \begin{cases} [1 - (1 - G)^2], & 0 \leq G \leq 1 \\ 1, & G > 1. \end{cases}$$

Here, $x(\tau) = 1 - 1/(2\tau)$, $\tau = \tau_H, \tau_L$. Note that $V'(.)$ is linear (hence, concave) and strictly decreasing ($V(.)$ is strictly concave) on $[0, 1]$. Here, $\overline{G} = 2 - \frac{1}{\tau_H}$. Assumptions 1-3 are satisfied as long as $\tau_L > \frac{1}{2}$, $\tau_H < 1$.

Suppose $\pi_j > \pi_i$. Consider the sequential move game $\Gamma(p)$ where $p = (j, i)$ i.e., player j moves first and player i moves next. It can be checked that if

$$\pi_i \leq \max\left\{0, 1 - \frac{\tau_L}{\tau_H}\left[2\tau_L + \sqrt{4\tau_L^2 - 1}\right]\right\}, \qquad (20)$$

then the first-mover (player j) of τ_H-type will make a strictly positive contribution in the (unique perfect Bayesian equilibrium) sequential move game and the expected total contribution generated is:

$$\tilde{z} = (\pi_i + \pi_j)x(\tau_H) + (1 - \pi_i)(1 - \pi_j)x(\tau_L) - \pi_i\pi_j.$$

In the simultaneous move game, it can be shown that there is a unique Bayesian-Nash equilibrium and in this equilibrium, agent j makes a strictly positive contribution for both realization of types, while agent i makes a strictly positive contribution only if it is of τ_H-type. The expected total contribution in the simultaneous move game $\tilde{y}$ is exactly equal to the expected standalone contribution of agent j i.e., $\tilde{y} = \pi_j x(\tau_H) + (1 - \pi_j)x(\tau_L)$. Observe that $\tilde{z} - \tilde{y} = [\frac{1-\pi_j}{\tau_L} - \frac{1}{\tau_H}][\frac{\pi_i}{2}] > 0$ if

$$\pi_j < 1 - \frac{\tau_L}{\tau_H}. \qquad (21)$$

Thus, even though the agent who has strictly higher probability of being τ_H-type and contributes strictly positive amount almost surely in the simultaneous move game moves first in the sequential move game and the agent who does not contribute strictly positive amount almost surely in the simultaneous move game is the last mover in the sequential move game, the sequential game generates (strictly) higher expected total contribution relative to the simultaneous move game if (20) and (21) hold. ||

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