Environmental Regulation and Industry Dynamics

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Abstract

We examine the effect of more stringent environmental regulation on the dynamic structure of a *deterministic* competitive industry with endogenous entry and exit where firms invest in reduction of their future compliance cost. The level of regulation is exogenously fixed and constant over time. The compliance cost of a firm at each point of time depends on its current output, its accumulated past investment and the level of regulation. We outline sufficient conditions under which industries with more stringent regulation are associated with higher investment in compliance cost reduction and higher shake-out of firms over time; the opposite may be true under certain circumstances. Our analysis indicates that the effect of a change in regulation on market structure may be lagged over time.

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1 Introduction

In recent decades there is a significant increase in the stringency of environmental regulations imposed on manufacturing industries. These regulations impact the choice of technology, production scale, investment behavior, as well as entry and exit decisions of firms. One significant consequence of regulation is that firms undertake investment in learning, technology adoption and other activities in order to reduce their future costs of compliance. It is important to understand how increasing stringency of regulation affects the incentives of firms to invest in compliance cost reduction and how such investments, in turn, affect the entry and exit decisions of firms and more generally, the *dynamic* structure of the industry. This paper is an attempt to address this question in a simple dynamic competitive framework where an industry with free entry and exit faces an exogenous level of environmental regulation. In particular we study the relationship between the level of environmental regulation and the dynamic equilibrium path of an industry.

The existing literature on environmental regulation and investment has predominantly focused on the so-called Porter Hypothesis (Porter 1991; Porter and van der Linde 1995). According to the hypothesis, more stringent environmental regulation encourages firms to innovate and develop more cost effective methods of achieving regulatory compliance. In the process, firms may also discover new technologies that reduce emissions and production costs. A small body of recent (theoretical and empirical) literature finds limited support for this in their attempt to study the effect of environmental regulation on technological change;[1] however, this literature does not consider the linkage to endogenous changes in market structure. In addition, a growing empirical literature studies the effect of more stringent environmental regulation on the structure of industries (without considering the effect on technological change). Most of these studies indicate that increase in environmental regulation leads to higher exit, entry barriers and market concentration; but some studies do find evidence to the contrary.[2]

The theoretical literature on the links between environmental regulation and endogenous changes in market structure mostly assumes a *static* framework that abstracts from issues of technological change. Assuming a linear demand function and a cost function that is additively separable in outputs and emissions, Katsoulacos and Xepapadeas (1996) find that the equilibrium number of firms in the market is decreasing in emission tax. Shaffer (1995) and Lee (1999) extend this analysis to more general demand and production cost functions, while assuming that emissions are proportional to output and find that the effect of an increase in the emission tax on firm's output is ambiguous, but the impact on the equilibrium number of firms in the market is always negative. More recently, Lahiri and Ono (2007) show that if the inverse demand function is concave, output per firm is unambiguously higher with an increase in the emission tax, implying a decline in the equilibrium

[1] For a survey of the effect of environmental policy on technological change, see Jaffe et al. (2003).

[2] For a recent survey of the existing literature on the effects of environmental regulation on market structure, see Millimet, Roy and Sengupta (2008).

number of firms in the market. However, the converse may be true if the inverse demand function is convex. Requate (1997) finds that a more stringent absolute *emission standard* always reduces the equilibrium number of firms. Farzin (2003) shows that if environmental quality is complementary to the consumption of the industry product then there may exist a positive relationship between the stringency of the standard and the equilibrium number of firms. In models of symmetric monopolistic competition, Lange and Requate (1999) and Requate (2005) find an inverse relationship between emission tax and the number of firms under reasonable parametric restrictions.

Somewhat closer to the spirit of our analysis, is the small body of static models that attempts to link environmental regulation to market structure by explicitly taking into account *how regulation modifies the optimal scale of firms*. In a model where symmetric firms have upward sloping marginal and U-shaped average cost curves, Conrad and Wang (1993) show that an increase in emission tax reduces the optimal scale of firms, increases the effective marginal cost and reduces total output; the net effect of an increase in regulation on the equilibrium number of firms is therefore ambiguous. The equilibrium number of firms declines with an increase in the emission tax if the demand function for the final product is sufficiently elastic. Kohn (1997) argues that if there are sufficient economies of scale in the abatement technology, the optimal scale and output of polluting firms may increase with emission tax and in such situations, the imposition of a (Pigouvian) emission tax is more likely to reduce the number of firms (even if the demand curve for the final product is sufficiently inelastic).

To the best of our knowledge, there is no significant body of work in the existing theoretical literature that systematically links changes in environmental regulation to *dynamic* changes in industry structure that arise *via* their effect on endogenous changes in investment in better abatement and compliance technology. This paper is an attempt to fill this important gap in the literature by explicitly introducing environmental regulation in a model of industry dynamics and technological change.

Over the last few decades, the general literature on theoretical and empirical models of industry dynamics has expanded very sharply.[3] In these models, the scope for technological change through investment in capital formation or learning is a part of the description of the technological environment of the industry; the latter is fixed exogenously and the focus is on characterizing the nature of the dynamic industry path (including technological change). In this paper, the degree of environmental regulation determines the scope for firms to reduce their compliance costs through investment in technological change. Our focus is on how different levels of exogenous regulation lead to differences in the dynamic path of the industry, particularly in the time path of market structure. This differentiates the object of our study from the mainstream literature on industry dynamics.

We introduce environmental regulation in a specific model of technological change and

[3] Seminal papers include Jovanovic (1982), Pakes and Ericson (1998), Hopenhayn (1992a, 1992b, 1993) and Jovanovic and MacDonald (1994).

industry dynamics due to Petrakis and Roy (1999) that generated among other things, increasing size dispersion and endogenous shake-out (early exit) of firms over time in a dynamic competitive industry. In their paper, investment reduces firm-specific future production cost in a deterministic fashion. As in much of the industry dynamics literature, their focus is on characterizing the qualitative properties of the equilibrium path for a given technological environment.[4] In our paper, investment reduces compliance cost and the latter depends on environmental regulation; our focus is the *comparative dynamics of regulation on the equilibrium path of the industry*.

As in Petrakis and Roy (1999), investment in compliance cost reduction generates inter firm heterogeneity and shake-out of firms over the industry equilibrium path, exiting firms have smaller accumulated investment (higher compliance cost). Further, the equilibrium path is socially optimal and shake-out of firms on the time path does not reflect any anti-competitive behavior. The main contribution of the analysis in our paper is the comparison of time paths of entry, exit and investment in the dynamic equilibrium of a more regulated industry to that of a less regulated industry. (It is important to clarify at this stage that we do *not* study the effects of unanticipated changes in regulation along a particular time path; rather we compare the equilibrium paths corresponding to different exogenous regulation levels).

We identify the economic conditions under which more stringent regulation leads to an equilibrium with higher shake-out of firms over time. Often, the latter is associated with higher dispersion in firm size. However, more regulation may also be associated with lower shake-out of firms. More stringent environmental regulation always increases the (minimum) cost of producing any vector of output for the industry and therefore, the equilibrium prices so that the time path of industry output is lower. Whether or not this leads to more shake-out depends on the effect on the (optimal) scale of individual firms. Here, there is a direct and an indirect effect. The direct effect arises from the manner in which change in regulation shifts the intertemporal production cost function (inclusive of compliance cost) for any fixed investment path and, in particular, how it shifts the optimal scale of firms. This is essentially a dynamic version of the effect captured in existing static models. The indirect effect arises from the fact that higher regulation alters optimal investment of firms in compliance cost reduction that, in turn, shifts the cost function and the optimal scale of firms. In our model, investment is complementary to regulation and output i.e., investment reduces the marginal cost of output and higher regulation increases the marginal effectiveness of investment in cost reduction. Therefore, the indirect effect always expands the optimal scale of firms as long as firms invest more with higher regulation. If the direct effect works in the same direction as the indirect effect, higher regulation is likely to lead to an equilibrium path with more shake-out of firms. Even if the direct effect does not expand the optimal scale of firms, if the indirect

[4]See also, Petrakis, Rasmusen and Roy (1997) for a model of cost reduction through learning by doing in a similar framework.

effect generated by cost reducing investment is sufficiently strong and, in particular, the marginal cost of firms fall sharply with investment, larger shake-out of firms can result.

Our analysis indicates that a higher level of regulation may be associated with more initial entry in the market (when increase in regulation makes the initial marginal cost curves significantly steeper). Nonetheless, sufficient shake-out of firms may change the comparison of market structures after some time. In particular, the somewhat mixed empirical evidence on exit of firms in the immediate years following regulation is not surprising and it is, therefore, important to look at delayed effects on turnover to capture the dynamic impact.

Section 2 outlines the basic structure of the model, the definition of industry equilibrium and the basic qualitative properties of the equilibrium path. Section 3 contains the main results of this paper and a set of examples to illustrate some key points. Section 4 concludes.

2 Preliminaries

2.1 Model

Consider a $T(1 < T < \infty)$ *period dynamic* model of a homogenous good industry with a continuum of *ex ante* identical potential entrant firms (each of measure zero) that can enter at any period and after entry, can exit the industry in any period. The model is a direct adaptation of that in Petrakis and Roy (1999) to our specific context. The market demand is stationary over time and given by $D(p)$. We denote the inverse demand function by $P(Q)$ where $P : R_+ \to R_+$ is continuous and strictly decreasing.

In each period t, firm $i's$ production cost depends on its current output $q_t(i) \geq 0$ and it is denoted by $c(q_t(i))$ where $c : R_+ \to R_+$ is continuously differentiable, $c(0) > 0$, $c^{'} > 0$ and $c^{''} > 0$. In other words, firms have upward sloping marginal cost curves and a firm has to incur a positive cost to be active in the industry even if it produces zero output.

Let $\alpha \in R_+$ be the exogenous level of regulation imposed on the industry in order to control the pollution generated by these firms. We assume that α remains constant over time. Higher value of α implies higher level of regulation (say higher tax rate); $\alpha = 0$ indicates no regulation.

In each period t, firm i invests $x_t(i) \geq 0$ in reduction of its own compliance cost. We assume that there are no externalities across firms arising from an individual firm's investment in cost reduction. The stock of capital of firm i in period t is given by $y_t(i) \in R_+$. If firm i enters in period $\underline{\tau}$, then for $t > \underline{\tau}$,

$$y_t(i) = x_{\underline{\tau}}(i) + x_{\underline{\tau}+1}(i) + ... + x_{t-1}(i) \text{ and } y_{\underline{\tau}}(i) = 0.$$

$\gamma(x_t(i))$ is the cost of investment incurred by firm i in period t where $\gamma : R_+ \to R_+$ is continuously differentiable, strictly increasing convex function; $\gamma(0) = 0$, $\gamma^{'}(x) > 0$ and $\gamma^{''}(x) \geq 0 \; \forall \; x > 0$.

Given output q, capital stock y and level of regulation α the cost of compliance of a firm in any time period is $\phi(q, y, \alpha)$, where $\phi : R_+^3 \to R_+$ is twice continuously differentiable in all the arguments. We impose the following assumptions on $\phi(q, y, \alpha)$:

Assumption 1 : $\phi(q, y, 0) = 0$ and $\phi(0, y, \alpha) = 0$.

Assumption 2 : $\phi_q > 0$, $\phi_y \leq 0$ and $\phi_\alpha > 0$.

Assumption 3 : $\gamma'(0) < -\delta\phi_y(q, 0, \alpha)\, \forall q > 0, \alpha > 0$, where $\delta \in (0, 1)$ is the discount factor.[5]

Assumption 4 : $\phi_{qq} > 0$, $\phi_{q\alpha} > 0$, $\phi_{yq} \leq 0$, $\phi_{y\alpha} \leq 0$ and $\phi_{yy} \geq 0$.

Assumption 1 implies that if there is no regulation then a firm does not incur any compliance cost. Further, the cost of compliance is zero if a firm is inactive. Assumption 2 implies that the cost of compliance increases with output, decreases with the stock of capital and increases as the level of regulation increases. Observe that $-\phi_y$ is the marginal reduction in compliance cost due to increase in the stock of capital. Assumption 3 guarantees that if there is a positive regulation then each firm that stays in the industry for more than one period finds it profitable to make strictly positive investment. Assumption 4 says that the marginal (compliance) cost of output increases with output and the level of regulation; marginal return on investment in cost reduction (weakly) increases with output and (weakly) increases with regulation but (weakly) decreases in the level of investment.

The effective production cost function for a firm at any point of time with accumulated investment y and facing regulation level α is therefore given by $c(q) + \phi(q, y, \alpha)$. Let $p_m(y, \alpha) = \min_{q \geq 0} \left[\frac{c(q)+\phi(q,y,\alpha)}{q}\right]$ to be the current minimum average cost and $q_m(y, \alpha)$ the corresponding current minimum efficient scale of a firm with accumulated investment y facing exogenous regulation α.

For all $\alpha > 0$ we assume that

$$\lim_{Q \downarrow 0} P(Q) > p_m(0, \alpha).$$

This ensures the existence of a non-trivial competitive equilibrium. Further, note that the dynamic scale economies created by the possibility of compliance cost reduction are bounded because the effective marginal cost of production $\left(c'(q) + \phi_q(q, y, \alpha)\right)$, the supply curve of an individual firm at any point of time, is bounded below by $c'(q)$ and $c'(q) \to \infty$ as $q \to \infty$.

Observe that the exogenous level of regulation α can be interpreted in terms of different pollution control instruments. Suppose $e(q, y)$ is the net value of emission or pollution when the firm produces output q and possesses stock of capital y. Then

$$\phi(q, y, \alpha) = \alpha e(q, y)$$

[5] Assumption 2 and Assumption 3 are alternative verisons of $(A3)$ and $(A6)$ of Petrakis and Roy (1999).

where α is the unit tax or subsidy or unit emission charge. In case of *marketable permits* (quantity rationing) we can define α as the exogenously given number of marketable permits. Under *liability rules* a producer suffers financial loss of magnitude

$$\phi(q, y, \alpha) = f(e(q, y) - \alpha)$$

if he violates the socially acceptable benchmark α. If there is a *technology standard* α to be met then

$$\phi(q, y, \alpha) = \left[\widehat{C}\left(q, y, \alpha\right) - c(q)\right]$$

where $\widehat{C}\left(q, y, \alpha\right)$ is the cost function under the given technology standard α when a firm produces output q and y is the present stock of capital.

Finally, we assume that once a firm exits the industry it loses all its accumulated capital and cannot re-enter on the dynamic equilibrium path.[6]

2.2 Industry equilibrium

In this subsection, we use the analysis in Petrakis and Roy (1999) to define and characterize the properties of industry equilibrium for any given level of environmental regulation α. We will use these results in the subsequent sections to study the effect of change in α.

For any pair of time periods $\underline{\tau}$ and $\overline{\tau}$, where $1 \leq \underline{\tau} \leq \overline{\tau} \leq T$, let $S\left(\underline{\tau}, \overline{\tau}\right)$ be the set of firms and $n\left(\underline{\tau}, \overline{\tau}\right)$ be the measure (the number of firms) of the set $S\left(\underline{\tau}, \overline{\tau}\right)$ of firms that enter in period $\underline{\tau}$ and exit in period $\overline{\tau}$. Firms active between periods $\underline{\tau}$ and $\overline{\tau}$ must incur at least a fixed cost of production $c\left(0\right)$ in every period t. Given price vector $\mathbf{p} = (p_1, ...p_T)$ and the level of regulation α, let $\Pi\left(\mathbf{p}, \alpha, \underline{\tau}, \overline{\tau}\right)$ be the maximum discounted sum of profit (net of investment and compliance cost) that a firm can possibly earn if it enters in period $\underline{\tau}$ and exits in period $\overline{\tau}$:

$$\Pi\left(\mathbf{p}, \alpha, \underline{\tau}, \overline{\tau}\right) = \max_{(q_t, x_t) \geq 0} \sum_{t=\underline{\tau}}^{\overline{\tau}} \delta^{t-\underline{\tau}} \left[p_t q_t - c\left(q_t\right) - \phi\left(q_t, y_t, \alpha\right) - \gamma\left(x_t\right)\right] \tag{1}$$

$$\text{where} \;\; y_t = \sum_{\tau=\underline{\tau}}^{t-1} x_\tau, \qquad t > \underline{\tau}, \quad y_{\underline{\tau}} = 0.$$

Under our assumptions, given price vector $\mathbf{p}$ and regulation level α there exists a solution to the profit maximization problem in the right hand side of (1).

[6] While this assumption may appear to be restrictive note that, in equilibrium, (as we show later) no firm enters after period 1. Therefore, once it exits, no firm can re-enter with its capital and make strictly positive intertemporal profit. This also implies that to the extent this capital is industry-specific, there is no resale value of the accumulated capital of the exiting firm.

Definition Of Industry Equilibrium :Given the level of regulation α, an industry equilibrium consists of (1) measurable sets $S(\underline{\tau}, \overline{\tau})$ of firms that enter in period $\underline{\tau}$ and exit in period $\overline{\tau}$, $1 \leq \underline{\tau} \leq \overline{\tau} \leq T$, (2) output and investment profile $\{(q_t(i), x_t(i)), t = \underline{\tau}, ...\overline{\tau}\}$ $\forall i \in S(\underline{\tau}, \overline{\tau})$ and $\{q_t(i), x_t(i)\}$ integrable on $S(\underline{\tau}, \overline{\tau})$ and (3) price vector $\mathbf{p} = (p_1, ...p_T)$ such that

$$(a)\ \ D(p_t) = Q_t \ \text{ where } \ Q_t = \int_{S_t} q_t(i)di$$

where S_t is the set of all firms that are active in period $t = 1, 2, ...T$, (b) if $n(\underline{\tau}, \overline{\tau}) > 0$, then $\forall i \in S(\underline{\tau}, \overline{\tau})$, the output-investment profile $\{(q_t(i), x_t(i)) \, \forall t = \underline{\tau}, ...\overline{\tau}\}$ solves the maximization problem in the right hand side of (1) and

$$(c)\ \ \Pi(\mathbf{p}, \alpha, \underline{\tau}, \overline{\tau}) \begin{array}{ll} = 0 & \text{if } \ n(\underline{\tau}, \overline{\tau}) > 0 \\ \leq 0 & \quad \text{otherwise.} \end{array}$$

Condition (a) implies that the market clears in every period. Condition (b) states that given the equilibrium price vector **p** and exogenous regulation level α, the output-investment profile for each active firm maximizes the net discounted sum of profits over its lifetime. Condition (c) guarantees that irrespective of the period of entry and exit, all active firms earn exactly zero net intertemporal profit over their lifetime in the industry. Note that no firm can make strictly positive intertemporal profit no matter when it enters or exits the industry. From Proposition 1 of Petrakis and Roy (1999) we have:

Result 1 : For every $\alpha > 0$, there exists an industry equilibrium and it is (restricted) socially optimal i.e., maximizes discounted sum of consumer and producer surplus in the industry over time.

For firm $i \in S(\underline{\tau}, \overline{\tau})$, $1 \leq \underline{\tau} \leq \overline{\tau} \leq T$, the equilibrium output and investment profile $\{(q_t(i), x_t(i)), t = \underline{\tau}, ...\overline{\tau}\}$ satisfies the following first order conditions

$$p_t - c^{'}(q_t(i)) - \phi_q(q_t(i), y_t(i), \alpha) = 0 \qquad \text{if } \ q_t(i) > 0 \tag{2}$$

$$\gamma^{'}(x_t(i)) + \sum_{\tau=t+1}^{\overline{\tau}} \delta^{\tau-t} \phi_y(q_t(i), y_t(i), \alpha) = 0 \qquad \text{if } \ x_t(i) > 0. \tag{3}$$

Equation (2) implies that firm i equates price to its current effective marginal cost when it produces positive output. The effective marginal cost curve of a firm is its individual supply curve in each period. As a firm's stock of capital accumulates, its supply curve shifts to the right whereas with increase in regulation it shifts to the opposite direction. Condition (3) states that the optimal investment for firm i equates the current marginal cost of investment to the future marginal return from investment i.e., the discounted sum of decrease in future compliance costs. It is obvious that $x_{\overline{T}} = 0$ i.e., firms do not invest in their last period in the industry.

Observe that if there is no environmental regulation ($\alpha = 0$) then the cost of compliance is zero (from Assumption 1); in that case, firms have no incentive to invest which implies that the industry supply curve, the market price and the market structure remain stationary over time:

$$p_1 = .. = p_T = p_m(0,0),\ q_1 = .. = q_T = q_m(0,0) \text{ and } n_1 = .. = n_T \tag{4}$$

Even if there is regulation but the marginal compliance cost is independent of investment (which implies that the industry's supply curve does not shift) then again we have stationary equilibrium[7,8] though different from the no-regulation case i.e.,

$$p_1 = .. = p_T = p_m(0,\alpha),\ q_1 = .. = q_T = q_m(0,\alpha) \text{ and } n_1 = .. = n_T. \tag{5}$$

Much of the existing literature on environmental regulation focuses on comparison of the outcomes of these two stationary equilibrium as they do not allow for endogenous changes in compliance cost.

However, if the level of regulation is positive i.e., $\alpha > 0$ and if the effective marginal cost strictly decreases with investment i.e., $\phi_{qy} < 0\ \forall q, y$ then the industry equilibrium path is typically not stationary. In particular, investment changes cost and supply curves of the firms that in turn change the prices over time. Further, it generates the possibility of shake-out (some firms exit earlier than others) and heterogeneity emerges among firms even though they are identical *ex ante*.

Result 2 : Fix $\alpha > 0$. (a) On any industry equilibrium path prices are non-increasing over time; if, further, $\phi_{qy} < 0\ \forall q, y$ then prices are strictly decreasing over a subset of period; in particular $p_1 > p_T$.[9]

(b) No entry occurs after the initial period. Some firms exit before T (shake-out occurs) if

$$\frac{D(p_m(y,\alpha))}{q_m(y,\alpha)} < \frac{D(p_m(0,\alpha))}{q_m(0,\alpha)}, \forall y > 0.$$

(c) Finally, firms that exit earlier on the industry equilibrium path have (weakly and often, strictly) lower accumulated investment, higher compliance cost and smaller size.

[7] This allows for the possibility how investment reduces only the fixed cost of compliance and in which case the profits of the firms may change over time but the outputs, prices and number of firms remain stationary (i.e., no entry-exit).

[8] Here, firms may invest to reduce their fixed cost of compliance so that their average cost as well as profits may change over time. See, Example 4 in Appendix.

[9] For the formal proof of the last part see Appendix.

To understand part (a) of Result 2 note that an increase in accumulated investment per firm reduces the effective marginal cost i.e., supply of the firm and consequently the effective marginal cost curve of the industry declines over time. As a result the competitive equilibrium price is decreasing along the time path of an industry. The intuition behind part (b) of Result 2 is as follows: if a firm enters after period 1 and makes zero intertemporal profit, then by entering and exiting earlier (staying in the industry for the same length of time) it can earn strictly positive discounted sum of profit as it faces a "better" vector of prices (since prices are decreasing over time).

Part (c) of Result 2 provides a sufficient condition for shake-out i.e., for some firms to exit earlier. Recall that $p_m(y, \alpha)$ is the minimum average cost and $q_m(y, \alpha)$ is the corresponding minimum efficient scale of a typical firm with accumulated investment y under the exogenous level of regulation α. The typical profit profile for a firm is that it earns negative profit in initial periods producing below its minimum efficient scale (faces price no larger than its minimum average cost) while in later periods, a mature firm faces prices strictly greater than the minimum average cost and produces more than its minimum efficient scale. Therefore, if the minimum efficient scale expands sufficiently rapidly with investment relative to the expansion of total quantity sold resulting from fall in prices over time, there must be some shake-out of firms. Note that on the equilibrium path, firms that exit earlier as well as those that exit later earn zero intertemporal profit and no firm can do better by altering its exit decision.

Part (d) of Result 2 implies that a firm that finds it profitable to stay in the industry has higher accumulated investment than the firm that exits in the same period; this allows the staying firm to be profitable at lower future prices. The output produced by a firm who stays in the industry is higher than that of the exiting firm.

An important implication of this result for environmental regulation, is that regulation can endogenously create heterogeneity in compliance cost and size dispersion of firms by creating differences in investment and planned survival of firms. Exiting firms are smaller and have higher compliance costs than firms that stay on.

Note that the above mentioned properties are the characteristics of an industry equilibrium path which is socially optimal. One can intuitively justify that on the time path with a given level of regulation, shake-out of firms in an industry is desirable from the social planner's perspective. Initially the social planner may want a large number of firms in the industry to bring down the total industry cost if the marginal cost curve is steep. But over time as firms invest to reduce future compliance cost, the effective marginal cost of an individual firm may become flatter, its efficient scale may expand so that from the social planner's perspective it is no longer necessary to keep large number of firms in the industry and incur the fixed cost.

We present a numerical example to illustrate all the above mentioned properties of an industry equilibrium path.

Example 1 *Let*

$$D(p) = 100 - p, \quad c(q) = 10 + e^{q}, \quad \gamma(x) = 0.5x^{2}$$
$$\phi(q, y, \alpha) = \alpha e(q, y) = \alpha e^{q-\lambda y}$$

where $e^{q-\lambda y}$ *can be interpreted as the emission function,* $\lambda > 0$ *as the efficiency of investment in emission reduction and* α *as the unit emission tax rate. Set* $\delta = 0.5$, $T = 3$. *We describe the equilibrium paths under three different circumstances:*
(i) no regulation i.e., $\alpha = 0$,
(ii) there is a positive regulation $\alpha = 0.03$ *but the cost of compliance does not depend on investment i.e.,* $\lambda = 0$,
(iii) positive environmental regulation $\alpha = 0.03$ *and the compliance cost depends on investment; in particular,* $\lambda = 1$.

Table 1 represents case (i) and case (ii) that illustrate our claim in (4) and (5) :

Table 1: Static equilibrium

Case	t	α	q_t	x_t	p_t	π_t	$D(p_t)$	$n_t = \frac{D(p_t)}{q_t}$	$\frac{n_t - n_{t-1}}{n_{t-1}}$
(i)	–	0	2.1568	0	8.6440	0	91.3560	42.3558	–
(ii)	–	0.03	2.1410	0	8.7637	0	91.2362	42.6125	–

Both cases yield two different static equilibrium with no investment and no shake-out of firms in the industry.

Table 2 depicts case (iii) :

Table 2: Dynamic equilibrium

t	p_t	$D(p_t)$	$n_t = \frac{D(p_t)}{q_t}$	$\frac{n_t - n_{t-1}}{n_{t-1}}$
1	8.7637	91.2362	42.6125	–
2	8.7569	91.2430	42.5474	−0.0015
3	8.7432	91.2567	42.5366	−0.0002

Note that on the industry's equilibrium dynamic path, price is strictly declining over time and firms exit after every period; the last column represents the rate of shake-out of firms over time.

In period 1, all firms produce at the minimum efficient scale (identical across firms in period 1); firms that exit at the end of period 1 earn zero profit whereas other firms earn strictly negative profit as they invest in cost reduction. In period 2, there are two different types of firms; those that exit at the end of period 2 and those that exit at the end of period 3; the former have invested higher amount in period 1 compared to the latter and therefore, have lower effective marginal cost and higher output (though they all face the same market price). A typical firm that enters in period 1 and exits at the end of period 2 has the following profile of output and investment on the industry equilibrium path :

Table 3: Firm that exits at the end of period 2

t	q_t	x_t	π_t
1	2.1410	0.1141	−0.0065
2	2.1434	0	0.0130

For a typical firm that enters the industry in period 1 and leaves at the end of period 3 we get the following output and investment profile for three periods:

Table 4: Firm that exits at the end of period 3

t	q_t	x_t	π_t
1	2.1410	0.1588	−0.0126
2	2.1445	0.0990	0.0180
3	2.1454	0	0.0142

Observe that a typical firm that exits at the end of period 3 invests more in period 1 and produces more in period 2 than a firm that exits at the end of period 2 on the industry equilibrium path (this depicts the part (d) of Result 2).

3 Comparative dynamics in a two period model

In this section, we study the effect of more stringent environmental regulation on the industry equilibrium path with particular focus on the conditions under which increase in regulation leads to a time path with higher shake-out of firms.

For the sake of tractability, we consider a two period model ($T = 2$). We also make the following additional assumption:

Additional assumption : $\frac{D(p_m(x,\alpha))}{q_m(x,\alpha)} < \frac{D(p_m(0,\alpha))}{q_m(0,\alpha)}$ $\forall x > 0, \alpha > 0$.

Using part (c) of Result 2 in the previous section, we can see that this guarantees that for every $\alpha > 0$, the industry equilibrium is one where some firms exit at the end of period 1.

Given (p_1,p_2,α) a firm maximizes the discounted sum of profit over *two* periods:

$$\max_{q_1,q_2,x} p_1 q_1 - c(q_1) - \phi(q_1, 0, \alpha) + \gamma(x) + \delta \left[p_2 q_2 - c(q_2) - \phi(q_2, x, \alpha)\right]. \tag{6}$$

The equilibrium output and investment $\{q_1^*, q_2^*, x^*\}$ profile of each firm in period 1 and 2 satisfies the following first order conditions:

$$p_1 - c^{'}(q_1) - \phi_q(q_1, 0, \alpha) = 0 \tag{7}$$

$$p_2 - c^{'}(q_2) - \phi_q(q_2, x, \alpha) = 0 \tag{8}$$

$$\gamma^{'} + \delta\phi_x (q_2, x, \alpha) = 0.^{10} \quad (9)$$

Firms who do not invest ($x = 0$) immediately exit at the end of period 1 and thus earn zero profit i.e.,

$$p_1 q_1 - c(q_1) - \phi(q_1, 0, \alpha) = 0. \quad (10)$$

A firm who survives till the last period earns negative profit in period 1 but strictly positive profit in period 2; in an equilibrium with shake-out discounted value of this strictly positive profit is equal to the cost of investment incurred by the firm in period 1 i.e.,

$$\gamma(x) - \delta\left[p_2 q_2 - c(q_2) - \phi(q_2, x, \alpha)\right] = 0.^{11} \quad (11)$$

In an industry equilibrium with shake-out (some firms exit at the end of period 1) each firm produces at the minimum efficient scale in period 1 i.e.,

$$p_1^* = p_m (0, \alpha) \quad \text{and} \quad q_1^* = q_m (0, \alpha)$$

(from (7) and (10)). Further to compensate for the negative profit earned in period 1 each firm produces more than the minimum efficient scale in period 2 i.e., $q_2^* \geq q_m (x, \alpha)$, price in period 2 is at least as high as the minimum average cost i.e., $p_2^* \geq p_m (x, \alpha)$ and thus each active firm earns positive profit in period 2. We can conclude that

$$p_m (x, \alpha) \leq p_2^* \leq p_1^* = p_m (0, \alpha)$$

(from part (a) of Result 2) and

$$q_2^* \geq q_m (x, \alpha) \geq q_m (0, \alpha) = q_1^*$$

(from part (d) of Result 2).

We begin with an example that shows that higher environmental regulation does not necessarily generate higher shake-out of firms compared to a path with lower regulation.

Example 2 *Let*

$$D(p) = p^{-1.5}, \qquad c(q) = 1 + q^2, \qquad \gamma(x) = 0.5x^2$$

$$\phi (q, y, \alpha) = \alpha e(q, y), \;\; e(q, y) = q^{1.5}(1 - y)^5$$

where $e(q, y)$ is the emission function and α is an emission tax. We explicitly solve for the two-period industry equilibrium corresponding to four different levels of regulation: $\alpha = 0.03$, $\alpha = 0.05$, $\alpha = 0.07$ and $\alpha = 0.10$. The results are reported in the following

[10] The interpretations of the first order conditions are similar to the T period case.

[11] (10) and (11) can be considered as additional equilibrium conditions to solve for the equilibrium time paths of output and investment when there is shake-out. In lemma 1 (see Appendix) we show that the equilibrium price p_2^* and output q_2^* produced by each firm in period 2 can be obtained by solving (11).

table :

Table 5

t	α	q	x_1	p	$D(p)$	$n = \frac{D(p)}{q}$	$\frac{n_t - n_{t-1}}{n_{t-1}}$
1	0.03	0.9925	0.0585	2.0299	0.3457	0.3483	
2	0.03	0.9961	0	2.0256	0.3468	0.3481	$\boxed{-0.0004}$
1	0.05	0.9876	0.0865	2.0498	0.3407	0.3449	
2	0.05	0.9958	0	2.0392	0.3433	0.3448	$\boxed{-0.0005}$
1	0.07	0.9828	0.1094	2.0696	0.3358	0.3417	
2	0.07	0.9962	0	2.0519	0.3404	0.3416	$\boxed{-0.0001}$
1	0.10	0.9827	0.1358	2.0588	0.3313	0.3371	
2	0.10	0.9827	0	2.0588	0.3313	0.3371	0

The last column indicates the intensity of shake-out of firms. Observe that compared to $\alpha = 0.03$, *the industry equilibrium path with* $\alpha = 0.05$ *is characterized by* ***higher*** *shake-out. However, when we compare between* $\alpha = 0.05$ *and* $\alpha = 0.07$ *the industry equilibrium exhibits* ***lower*** *shake-out of firms on the path more stringent regulation. In fact, if the level of regulation is as high as* $\alpha = 0.10$, *there is no shake-out of firms at all. Also, observe that higher regulation* (α) *is associated with higher investment by firms that survive till period 2.*

The above example illustrates the fact that more stringent regulation does not necessarily lead to higher shake-out in the industry and in particular, it is important to understand the various economic effects that play a role here. In order to do so, we will derive a set of sufficient conditions under which on the path with more stringent environmental regulation, the industry equilibrium exhibits higher shake-out of firms.

First, observe that in an equilibrium with exit in the two period model, the price in period 1 is exactly equal to the minimum average cost of a new entrant i.e., $p_m(0, \alpha)$ and every firm produces at its minimum efficient scale $q_m(0, \alpha)$ earning exactly zero current profit (gross of investment). Therefore, the number of active firms in the market in period 1 is

$$n_1 = \frac{D(p_m(0, \alpha))}{q_m(0, \alpha)}.$$

Lemma 2 An increase in the stringency of environmental regulation (higher α), increases the number of active firms in the industry in period 1, iff $\frac{D(p_m(0,\alpha))}{q_m(0,\alpha)}$ is strictly increasing in α.

Notice that this change in the equilibrium number of firms in period 1 when industry is on a higher regulation path is identical to the effect of increase in the level of regulation under a static framework (Conrad and Wang (1993)).

Next, we compare the equilibrium number of firms in period 2 on time paths corresponding to two different exogenous levels of environmental regulation. There are three different effects of higher regulation on the number of firms:

Effect 1 : For any given profile of investment, higher level of regulation increases the cost structure of the industry that in turn increases the equilibrium price and decreases total industry output sold. This creates a downward pressure on the number of active firms in period 2.

Effect 2 : For any given profile of investment, higher level of regulation shifts both the average cost and the effective marginal cost upward which directly alter the optimal scale of a firm. This may affect the number of firms in either direction depending on the direction and extent of changes in optimal scale.

Effect 3 : Increase in regulation may increase cost reducing investment and if this occurs, there is an expansion in the optimal scale of individual firm which tends to reduce the number of firms.

The first two are direct effects and the last one is the indirect effect of more stringent regulation on the number of firms. The net effect of higher regulation is such that on the industry equilibrium path corresponding to higher level of regulation, the price in period 2 is always higher (see (23) in Appendix), the total industry output sold in period 2 is lower and therefore, the number of active firms in period 2 solely depends how the optimal scale of an individual firm changes (Effect 2 and Effect 3).

In this model we assume that $\phi_{xq} \leq 0$ i.e., investment is more effective in reducing compliance cost at higher levels of output which implies that investment reduces the marginal cost of output. Further, note that the assumption $\phi_{x\alpha} \leq 0$ guarantees that the effectiveness of investment in compliance cost reduction (weakly) increases with regulation i.e., investment is more effective in reducing the future stream of compliance cost at a higher level of regulation. The degree of complementarity between regulation and investment determines the extent to which higher regulation creates incentive for more investment. The extent to which this investment reduces the effective marginal cost determines the expansion in the scale of individual firms.

When the direct effect expands the optimal scale of firms, the cumulative effect of higher level of regulation expands the production scale of individual firms; as higher regulation always leads to higher prices (lower industry output), the industry is more likely to exhibit greater shakeout of firms over time. Even if the direct effect does not expand the scale of firms, if the indirect effect (Effect 3) generated by cost reducing investment is sufficiently strong and, in particular, the marginal cost of firms fall sharply with investment (relative to demand elasticity which determines the contraction of industry output), larger shake-out of firms results.

For the indirect effect (Effect 3) to operate, however, firms need to invest more with increase in regulation. While the higher compliance cost associated with more stringent regulation creates more scope for cost reduction through investment, there is also a disincentive effect on investment that arises because higher regulation is associated with smaller industry output (higher price) so that the quantity a firm produces in the future is also likely to be smaller. Indeed, if regulation is prohibitive, industry shuts down and there is no investment. Of course, at the other extreme, if there is no regulation then once again, firms have no incentive to invest.

Let us define the following elasticities :

$$\widetilde{\phi}_{q,q} = \frac{q}{\phi_q}\phi_{qq}, \quad \widetilde{\phi}_{x,x} = -\frac{x}{\phi_x}\phi_{xx}, \quad \widetilde{\phi}_{x,q} = \frac{q}{\phi_x}\phi_{xq},$$

$$\widetilde{\phi}_{\alpha,q} = \frac{q}{\phi_\alpha}\phi_{\alpha q}, \quad \widetilde{\phi}_{\alpha,x} = -\frac{x}{\phi_\alpha}\phi_{\alpha x}, \quad \varepsilon_{\gamma'} = \frac{\gamma''}{\gamma'}x \text{ and } \varepsilon_{c'} = \frac{c''}{c'}q.$$

Proposition 1 *A marginal increase in the stringency of environmental regulation* <u>*increases*</u> *the investment of all firms* (*that do not exit in period 1*) *if* <u>*at least one*</u> *of the following conditions holds* (*at the current level of regulation*)*:*

$$(1) \quad \widetilde{\phi}_{\alpha,q} \leq 1$$

$$(2) \quad \widetilde{\phi}_{\alpha,x}\left(\varepsilon_{c'}\frac{c'}{\phi_q} + \widetilde{\phi}_{q,q}\right) > \widetilde{\phi}_{q,x}\left(\widetilde{\phi}_{\alpha,q} - 1\right).$$

Proof. See Appendix. ■

If the first condition of proposition 1 is satisfied then the optimal scale of each firm in period 2 is higher on the path with more stringent regulation (see (24) in Appendix) and consequently the first order condition (9) implies that each active firm invests more compared to those on the lower regulation path. The second condition depicts a situation when the disincentive effect on investment of higher regulation (discussed above) is dominated.

The next proposition underlines a set of sufficient conditions for lower number of firms in period 2 on the path with higher level of regulation.

Proposition 2 *On the industry equilibrium path with more stringent regulation* (*marginally higher* α)*, the number of firms in period* 2 *is* <u>*lower*</u> *than the number of firms on a path with lower level of regulation* (*lower* α) *if* <u>*at least one*</u> *of the following conditions holds* (*at the current level of* α)*:*

$$(1) \quad \widetilde{\phi}_{\alpha,q} \leq 1$$

$$(2) \quad \left(\delta\widetilde{\phi}_{x,x} - \varepsilon_{\gamma'}\frac{\gamma'}{\phi_x}\right)\left(\widetilde{\phi}_{\alpha,q} - 1\right) \leq \delta\widetilde{\phi}_{\alpha,x}\widetilde{\phi}_{x,q}$$

Proof. See Appendix. ■

Under both conditions, on the path with higher level of regulation the optimal scale of each active firm in period 2 is higher. Recall the three effects of higher regulation on number of firms described earlier. Condition (1) of proposition 2 implies that higher regulation shifts the effective marginal cost less than the average cost and thus both Effect 2 and Effect 3 work in the same direction i.e., bring down the number of firms. Condition (2) of proposition 2 says though $\widetilde{\phi}_{\alpha,q} > 1$ (i.e., the higher regulation shifts the effective marginal cost more than the average cost) but effective marginal cost is more sensitive to investment than average cost; the indirect effect (Effect 3) of higher level of regulation is sufficiently strong enough to negate the direct effect (Effect 2).

If neither of these conditions is satisfied then higher regulation may not increase the optimal scale. In that case, the number of active firms is less if optimal scale of each firm is decreasing at a lower rate than the fall in total industry output sold in the market. An additional sufficient condition for this is provided in footnote 13 in the Appendix.

Observe that

1. lemma 2 provides a necessary and sufficient condition under which on a higher regulation path the equilibrium number of active firms in period 1 is higher i.e., $\frac{dn_1}{d\alpha} > 0$ and

2. proposition 2 gives a set of sufficient conditions under which on a higher regulation path the equilibrium number of active firms in period 2 is lower i.e., $\frac{dn_2}{d\alpha} < 0$.

Thus, lemma 2 and proposition 2 imply a set of sufficient conditions under which on the equilibrium path with more stringent regulation the rate of shake-out is higher compared to that of a lower regulation path.

We consider the following example to explain the set of conditions given by each proposition in this section.

Example 3 *Let*

$$D(p) = p^{-a},\ D^{'} = -ap^{-a-1}, a > 0 \tag{12}$$

where price elasticity of demand is given by

$$\eta_p = -\frac{D^{'}(p)}{D(p)}p = a,$$

$$c(q) = B + q^b, c^{'} = bq^{b-1} > 0 \text{ and } c^{''} = b(b-1)q^{b-2} > 0 \tag{13}$$

where $c(0) = B > 0$ and elasticity of the production cost is

$$\widetilde{c}_{q,q} = \frac{q}{c^{'}}c^{"} = b > 1,$$

$$\gamma(x) = 0.5Gx^2,\ \gamma' = Gx > 0,^{12}\ \gamma'' = G > 0 \tag{14}$$

$$\text{and}\ \ \phi(q, x, \alpha) = \alpha q^h (A - x)^k \tag{15}$$

where h is the elasticity of marginal compliance cost of regulation with respect to output $\left(\widetilde{\phi}_{\alpha,q}\right)$ and k is the elasticity of marginal compliance cost of investment with respect to regulation $\left(\widetilde{\phi}_{x,\alpha}\right)$. The details of this parametric example are worked out in the Appendix. For the compliance cost function to satisfy Assumption 1-4 we need

$$h > 1 \text{ and } k \geq 1. \tag{16}$$

Observe that $\frac{D(p_m(0,\alpha))}{q_m(0,\alpha)}$ is strictly increasing in α (the necessary and sufficient condition in Lemma 2 holds) if:

$$ah \leq 1 \text{ and } ah\left(\frac{b-1}{h-1}\right) \leq 1 \tag{17}$$

Condition 2 *of proposition* 1 *is satisfied i.e., on the path with higher regulation each active firm invests more if*

$$k + h - 1 \geq (k-1)(h-1)^2. \tag{18}$$

Further, the following always holds:

$$(h-1)\left(k^2 - 1\right) < k^2 h.$$

so that condition 2 *of proposition* 2 *is satisfied i.e., on the equilibrium path with higher regulation, the number of firms in period* 2 *is lower. Therefore, the industry equilibrium path with more stringent environmental regulation generates higher shakeout as long as* (16) *and* (17) *hold.*

Observe that in the above example, on the equilibrium path with more stringent environmental regulation the number of firms in period 1 may be higher whereas the number of firms in period 2 is always lower. Therefore, the effect of more stringent regulation on the market structure is time dependent i.e., though on the industry equilibrium path with higher regulation there may be higher number of firms in the initial periods but greater number of firms exit over time which implies greater rate of shake-out of firms. In particular, the mixed empirical evidence on exit of firms in the immediate years following regulation is not surprising and it is, therefore, important to look at delayed effects on turnover to capture the dynamic impact.

[12] This ensures that optimal investment will never reach the upper bound as marginal benefit from investing the maximum amount possible is strictly less than the marginal cost of investment i.e., $\phi_x|_{x=A} = 0 < Gx$.

4 Conclusion

We examine the effect of increasing stringency of environmental regulation on the dynamic structure of a *deterministic* perfectly competitive industry with endogenous entry and exit. The level of regulation is exogenously fixed and constant over time. The compliance cost of a firm at each point of time depends on its current output, its accumulated past investment in firm-specific compliance cost reduction and the level of regulation. Exiting firms are smaller and have higher compliance cost. We identify sufficient conditions under which more stringent regulation leads to higher shake-out of firms; the effect may be the opposite under certain circumstances. Our analysis indicates that the effect of a change in regulation on market structure may be lagged over time.

5 Appendix

Example 4 *Let*

$$D(p) = 100 + p^{-1}, \qquad c(q) = 1 + q^2, \qquad \gamma(x) = 0.5x^2$$

$$\phi(y, \alpha) = \alpha F(1 - y)^3$$

where αF is the initial fixed cost of complying with regulation α and this can reduced by investment. We consider two alternative levels of regulation: $\alpha = 0.05$ and 0.10. We set $F = 10$, $\delta = 0.5$. The following are the equilibrium price, output per firm, investment by each firm and number of firms in the industry for $\alpha = 0.05$ and 0.10 respectively :

Table 6

α	$p_1 = p_2$	$q_1 = q_2$	x	$n_1 = n_2$
0.05	2.3830	1.1915	0.3333	84.2776
0.10	2.6755	2.6755	0.4514	72.7513

Therefore, if the environmental regulation is such that it does not affect the effective marginal cost of production then on the dynamic equilibrium path the price, output produced by each firm and number of firms do not change.

Proof for the last part of Result 2(a): Since from the first part of Result $2\,(a)$ we already know that prices are non-increasing over time therefore it is sufficient to show that if $\phi_{qy} < 0$ then $p_1 \neq p_2$. Suppose this is not true i.e., $p_1 = p_2$. If some firms exit at the end of period 1 then equilibrium price in period 1 is exactly equal to the minimum average cost of a firm with zero accumulated investment i.e., $p_1 = p_m(0, \alpha) = p_2$ and the firms produce at the minimum scale in period 1 i.e., $q_1 = q_m(0, \alpha)$. Then because of Assumption 3 i.e., $\gamma'(0) < -\delta\phi_y(q, 0, \alpha)\,\forall q > 0, \alpha > 0$, with $\varepsilon > 0$ investment a firm can make strictly positive intertemporal profit if it continues to produce the same output in period 2 i.e., $q_2 = q_1 = q_m(0, \alpha)$. Q.E.D.

Lemma 1 Define $f(p_2) = \max_{q_2,x}\left[\delta\{p_2q_2 - c(q_2) - \phi(q_2,x,\alpha)\} - \gamma(x)\right]$. There exists a unique p_2, say $\widehat{p_2}$, such that $f(\widehat{p_2}) = 0$. Further, $q_2(\widehat{p_2}) = q_2^*$ and $x\left(\widehat{p_2}\right) = x^*$ where q_2^* and x^* are the output produced and investment incurred by each firm in period 2 on the industry equilibrium path..

Proof. Observe that, $f(p_2)$ is continuous in p_2 by the theorem of the maximum. Now, $x = 0$ cannot be a solution to this maximization problem as we have assumed $\gamma^{'}(0) + \delta\phi_y(q,0,\alpha) < 0\forall q,\alpha$ (Assumption 3). Therefore, for any $x > 0$, at $p_2 = p_m(0,\alpha)$ $f(p_m(0,\alpha)) > 0$ and at $p_2 = 0$ $f(0) < 0$.Thus we can conclude that $f(p_2)$ is strictly increasing in p_2 and from intermediate theorem we can say that there exists a unique $p_2 = \widehat{p_2}$ such that $f(\widehat{p_2}) = 0$.From equilibrium condition given by (11) it is obvious that $p_2^* = \widehat{p_2}$ and thus $q_2^* = q_2(\widehat{p_2})$ and $x^* = x\left(\widehat{p_2}\right)$. ■

Proof of Proposition 1 : To determine the sign of $\frac{dn_2}{d\alpha}$ we take total differential of $(11), (8), (9)$, and market clearing condition for period 2 i.e., $n_2q_2 = D\left(p_2\right)$ w.r.t. α. respectively:

$$\delta\left[p_2 - c^{'} - \phi_q\right]\frac{dq_2}{d\alpha} - \left[\gamma^{'} + \delta\phi_x\right]\frac{dx}{d\alpha} + \delta\left[q_2\frac{dp_2}{d\alpha} - \phi_\alpha\right] = 0 \tag{19}$$

$$(c^{''} + \phi_{qq})\frac{dq_2}{d\alpha} + \phi_{qx}\frac{dx}{d\alpha} - \frac{dp_2}{d\alpha} + \phi_{q\alpha} = 0 \tag{20}$$

$$\delta\phi_{xq}\frac{dq_2}{d\alpha} + \left(\gamma^{''} + \delta\phi_{xx}\right)\frac{dx}{d\alpha} + \delta\phi_{x\alpha} = 0 \tag{21}$$

$$n_2\frac{dq_2}{d\alpha} + q_2\frac{dn_2}{d\alpha} - D'\frac{dp_2}{d\alpha} = 0 \tag{22}$$

Substituting (8) and (9) in (19) we get

$$\frac{dp_2}{d\alpha} = \frac{\phi_\alpha}{q_2} > 0 \tag{23}$$

Further, solving (20) and (21) we derive the following:

$$\frac{dq_2}{d\alpha} = \frac{\left(\gamma^{''} + \delta\phi_{xx}\right)\overbrace{\left(\frac{\phi_\alpha}{q_2} - \phi_{q\alpha}\right)} + \delta\phi_{x\alpha}\phi_{qx}}{\left[\left(c^{''} + \phi_{qq}\right)\left(\gamma^{''} + \delta\phi_{xx}\right) - \delta\phi_{xq}\phi_{qx}\right]} \tag{24}$$

$$\frac{dx}{d\alpha} = \frac{-\delta\phi_{x\alpha}\left(c^{''} + \phi_{qq}\right) - \delta\phi_{xq}\overbrace{\left(\frac{\phi_\alpha}{q_2} - \phi_{q\alpha}\right)}}{\left[\left(c^{''} + \phi_{qq}\right)\left(\gamma^{''} + \delta\phi_{xx}\right) - \delta\phi_{xq}\phi_{qx}\right]} \tag{25}$$

From the social planner problem it can be shown that

$$\left[\left(c^{''} + \phi_{qq}\right)\left(\gamma^{''} + \delta\phi_{xx}\right) - \delta\phi_{xq}\phi_{qx}\right] > 0. \tag{26}$$

Therefore, from (25) note that

$$\boxed{\tfrac{dx}{d\alpha} > 0 \ \text{ if } \ \widetilde{\phi}_{\alpha,q} \leq 1} \ \text{ and}$$

$$\boxed{\tfrac{dx}{d\alpha} > 0 \text{ if } \ \widetilde{\phi}_{\alpha,x}\varepsilon_{c'}\tfrac{c'}{\phi_q} + \widetilde{\phi}_{\alpha,x}\widetilde{\phi}_{q,q} > \widetilde{\phi}_{q,x}(\widetilde{\phi}_{\alpha,q} - 1)}.$$

Proof of Proposition 2 : Substituting (23) and (24) in (22) we get

$$\frac{dn_2}{d\alpha} = \frac{1}{q_2}\left[D'\frac{dp_2}{d\alpha} - n_2\frac{dq_2}{d\alpha}\right] \tag{27}$$

$$= \frac{\overleftrightarrow{D'\phi_\alpha\left[\left(c'' + \phi_{qq}\right)\left(\gamma'' + \delta\phi_{xx}\right) - \delta\phi_{xq}\phi_{qx}\right] - \delta D(p_2)\phi_{x\alpha}\phi_{qx}}}{q_2^2\left[\left(c'' + \phi_{qq}\right)\left(\gamma'' + \delta\phi_{xx}\right) - \delta\phi_{xq}\phi_{qx}\right]}$$

$$-\frac{D(p_2)\left(\gamma'' + \delta\phi_{xx}\right)\overbrace{\left(\frac{\phi_\alpha}{q_2} - \phi_{q\alpha}\right)}}{q_2^2\left[\left(c'' + \phi_{qq}\right)\left(\gamma'' + \delta\phi_{xx}\right) - \delta\phi_{xq}\phi_{qx}\right]} \tag{28}$$

Observe that, $\frac{dn_2}{d\alpha} < 0$ if $\frac{dq_2}{d\alpha} > 0$ (from (27)) and from (24) $\frac{dq_2}{d\alpha} > 0$ if either of these holds

$$\boxed{(1)\ \widetilde{\phi}_{\alpha,q} \leq 1}$$

$$\boxed{(2)\ \left(\delta\widetilde{\phi}_{x,x} - \varepsilon_{\gamma'}\tfrac{\gamma'}{\phi_x}\right)(\widetilde{\phi}_{\alpha,q} - 1) \leq \delta\widetilde{\phi}_{\alpha,x}\widetilde{\phi}_{x,q}}.$$

The proof is complete.[13]

Calculations for the example 3 :

$$\phi\left(q, x, \alpha\right) = \alpha q^h (A - x)^k$$

satisfies **Assumption 1 − 4** stated in section 2 i.e.,

Assumption 1: $\phi\left(q, x, 0\right) = 0$ and $\phi\left(0, x, \alpha\right) = 0$.

Assumption 2: $\phi_q = \alpha h q^{h-1}(A - x)^k > 0 \Rightarrow h > 0,\ \phi_x = -\alpha k q^h (A - x)^{k-1} \leq 0 \Rightarrow k \geq 0$ and $\phi_\alpha = q^h (A - x)^k > 0$.

Assumption 3: $\gamma'\left(0\right) + \delta\phi_x\left(q, 0, \alpha\right) = -\delta\alpha k q^h A^{k-1} < 0$.

[13] From (28) and (26) $\frac{dn_2}{d\alpha} < 0$ if $\varepsilon_p\left[\left(\varepsilon_q\frac{c'}{\phi_q} + \widetilde{\phi}_{q,q}\right)\left(\delta\widetilde{\phi}_{x,x} - \varepsilon_{\gamma'}\frac{\gamma'}{\phi_x}\right) - \delta\widetilde{\phi}_{q,x}\widetilde{\phi}_{x,q}\right]$ $> \frac{p_2}{\phi_q}\left[\left(\delta\widetilde{\phi}_{x,x} - \varepsilon_{\gamma'}\frac{\gamma'}{\phi_x}\right)(\widetilde{\phi}_{\alpha,q} - 1) - \delta\widetilde{\phi}_{\alpha,x}\widetilde{\phi}_{x,q}\right]$

Assumption 4: $\phi_{qq} = \alpha h\,(h-1)\,q^{h-2}(A-x)^k > 0 \Rightarrow h > 1,\ \phi_{qx} = -\alpha k h q^{h-1}(A-x)^{k-1} \leq 0,\ \ \phi_{q\alpha} = h q^{h-1}(A-x)^k > 0,\ \ \phi_{xx} = \alpha k\,(k-1)\,q^h(A-x)^{k-2} \geq 0 \Rightarrow k \geq 1$ and $\phi_{x\alpha} = -kq^h(A-x)^{k-1} \leq 0$.

In order to illustrate lemma 2 we calculate the following

$$\begin{aligned}
\frac{dn_1}{d\alpha} &= \frac{d\left(\frac{D(p_m(0,\alpha))}{q_m(0,\alpha)}\right)}{d\alpha} \\
&= \frac{D(p_m\,(0,\alpha))}{\alpha q_m\,(0,\alpha)}\left[\frac{D'\,(p_m\,(0,\alpha))}{D(p_m\,(0,\alpha))}p_m\,(0,\alpha)\,\frac{\alpha}{p_m\,(0,\alpha)}\frac{dp_m\,(0,\alpha)}{d\alpha} - \frac{\alpha}{q_m\,(0,\alpha)}\frac{dq_m\,(0,\alpha)}{d\alpha}\right] \\
&= \frac{D(p_m\,(0,\alpha))}{\alpha q_m\,(0,\alpha)}\left[-\frac{ah\alpha A^k}{bq^{b-h}+\alpha h A^k} + \frac{\alpha A^k}{b\left(\frac{b-1}{h-1}\right)q^{b-h}+\alpha h A^k}\right] \qquad (29)
\end{aligned}$$

Observe that on a higher regulation path the rate of fall of total output sold is captured by the first term in parenthesis where the change in equilibrium price is induced by the introduction of a higher level of regulation whereas the rate of decline of the minimum efficient scale in period 1 is given by the second term.

$$\frac{dn_1}{d\alpha} > 0 \text{ if } \ bq^{b-h}\left[1 - ah\left(\frac{b-1}{h-1}\right)\right] + \alpha h A^k[1-ah] > 0.$$

One of the conditions on the parameters under which this is possible is

$$ah \leq 1 \text{ and } ah\left(\frac{b-1}{h-1}\right) \leq 1.$$

the first condition of proposition 2 is not satisfied as

$$\widetilde{\phi}_{\alpha,q} = h > 1.$$

Whereas condition 2 of proposition 2 i.e.,

$$\delta\alpha\,(h-1)\,k\,(k-1)\,(A-x)^{k-2}\,q^h - G(h-1) \leq \delta\alpha h k^2\,(A-x)^{k-2}\,q^h$$

is always true since

$$(h-1)\left(k^2-1\right) < k^2 h.$$

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