Dynamic farsighted networks with endogenous opportunities of link formation

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Abstract

I present a three player dynamic network theoretic model where players are farsighted and asymmetric. Unlike the previous literature that imposes an exogenous protocol governing the order of negotiations, I allow the identity of the players who form a link in a given period to depend endogenously on player characteristics. Importantly, I show how this can give different predictions regarding attainment of the complete network relative to models with an exogenous protocol. Regardless of whether the complete network is efficient, a key dynamic trade off drives whether the complete network is attained in my model. A pair of players (insiders) may form a link with each other but, even though link formation is always myopically beneficial, each insider then refuses subsequent link formation with the third player (outsider) because the eventual attainment of the complete network makes each insider worse off relative to the insider–outsider network.

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1   Introduction

As noted recently by Bloch and Dutta (2011, p.762) the coalition and network theory literatures have traditionally used static frameworks to model coalition and network formation.

While some of these frameworks have incorporated notions of farsightedness whereby players anticipate their deviations will induce deviations by other players (e.g. Ray and Vohra (1997), Page et al. (2005) and Herings et al. (2009)), the role of time plays no explicit role in these notions of "introspective farsightedness" (Dutta et al. (2005)). Conversely, even though Watts (2001) and Jackson and Watts (2002) represent early examples of frameworks explicitly modeling the dynamic network formation process, they assume players act myopically: players' decisions are completely determined by the effect of their actions on current period payoffs. Only recently have Konishi and Ray (2003) and Dutta et al. (2005) developed dynamic models of coalition and network formation where players are farsighted in the sense that their actions in any period are determined by the effect of their actions on their discounted stream of payoffs.

Nevertheless, dynamic models of coalition and network formation raise an important issue: what is the order in which players or coalitions have the opportunity to form links or coalitions over time? That is, what is the protocol governing the "order of negotiations"? Dutta et al. (2005), and also Aumann and Myerson (1988) in their extensive form link formation game, impose an exogenous protocol (random in the former, deterministic in the latter) that determines the pair of players who can form a link in the given period. But, naturally, these modeling choices are not without consequence. Indeed, as discussed by Ray and Vohra (1997, p.44) in the context of the Aumann and Myerson (1988) link formation game, the sensitivity of equilibria to the assumed exogenous order of negotiations is well known. This sensitivity is especially problematic because, as stated by Jackson (2008, p.72), "... it is not clear what a natural ordering is".

The main contribution of this paper is the development of a dynamic farsighted model of network formation where there is no exogenous order of negotiations but rather the identity of the pair of players who form a link in a given period depends endogenously on player characteristics. In doing so, a new equilibrium concept emerges and I call this concept a Farsighted Dynamic Network Equilibrium (FDNE). Of course, endogenizing which players form a link in a given period may not be important in and of itself. However, I develop an example illustrating that the complete network will be attained when the pair of players who have an opportunity to form a link in a period is randomly chosen (i.e. exogenous) but the complete network is not attained when the identity of the linking players in a period is endogenous. This difference arises even though link formation is always myopically attractive and the complete network is efficient in the sense that it maximizes the aggregate one period payoff. Thus, endogenizing which players form a link in a given period can have a substantive impact on the predicted equilibrium path of network formation and the attainment of the complete network.

To endogenize which players form a link in a given period, I embed a simultaneous move "announcement game" in each period. In an announcement game, each player announces the link it wants to form and so the order in which links form over time depends endogenously on player characteristics. Moreover, players are farsighted because they base their announcements on continuation payoffs rather than one period payoffs. But, given the complexity added by solving a simultaneous move game in each period of a dynamic game with farsighted players, I only consider a three player game and make two important assumptions to maintain analytical tractability. First, like Dutta et al. (2005), only one link can form in any given period. Second, unlike Dutta et al. (2005) but like Seidmann (2009), agreements formed in previous periods are binding. Given this assumption, the complete network is an absorbing state which permits the use of backward induction to solve the equilibrium path of network formation. I call this path a Farsighted Dynamic Network Equilibrium (FDNE).

Solving the announcement game in each period requires a simultaneous move equilibrium concept. As is well known (e.g. Jackson (2005, pp.26-27)), the dependence of link formation on mutual consent of the linking players makes the use of Nash equilibrium problematic and necessitates some coalitional equilibrium concept. An obvious choice would be Coalition Proof Nash Equilibrium (CPNE; Bernheim et al. (1987)).[1] However, CPNE non–existence arises due to Condorcet paradox situations and these situations arise frequently even in my simple setting which questions the fundamental validity of CPNE to explain strategic network formation in dynamic contexts and naturally leads to similar but stronger concepts. To this end, I use a slight variant of the Equilibrium Binding Agreement solution concept (EBA) originally developed by Ray and Vohra (1997) and suggested by Diamantoudi (2003). This deals with the CPNE non–existence problem because EBA existence only relies on existence of a Nash equilibrium. Moreover, EBA is a desirable concept in itself given Bloch and Dutta (2011, p.761) state that an EBA "... captures "almost" perfectly the intuitive basis of stability in group formation".

The assumption that links formed in previous periods are binding and the use of an EBA to solve the announcement game within a period creates a distinction between the way that coalitions interact contemporaneously versus inter–temporally. When the link to be formed in the current period is still "under negotiation", an EBA allows coalitions to break up into subcoalitions costlessly. Thus, contemporaneous coalition formation is highly fluid. However, the assumption that links formed in previous periods are binding places rigidity on the nature of inter–temporal coalition formation. Of course, the appropriateness of this

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[1]Indeed, Bernheim et al. (1987) define perfectly CPNE as a CPNE in every subgame and, as described in detail later, an FDNE will essentially be an Equilibrium Binding Agreement (Ray and Vohra (1997)) in every subgame.

dichotomy is application dependent. However, I view this dichotomy as a strength rather than a weakness of the model. For example, the dichotomy is well suited to a prominent application of network theory which is international trade agreements. I discuss such an application later in the paper and give examples of how the order in which countries begin bilateral trade agreement *negotiations* does not necessarily translate into the order in which they *form* agreements even if some of these countries already have a bilateral agreements. That is, previously formed agreements do not influence the sequential way that negotiations may commence and/or breakdown within the current period.

A second contribution of the paper is that I show how the complete network may fail to obtain even if link formation is always myopically beneficial (i.e. "link monotonicity" (Dutta et al. (2005)) holds) and the complete network is efficient.[2] This possibility arises because the presence of negative link externalities can mean the one period payoff for players in a one–link network (i.e. "insiders") exceeds the one period payoff under the complete network. That is, insiders have an "insider exclusion incentive" because they want to exclude the third player (i.e. "outsider") from multilateral expansion to the complete network. This insider exclusion incentive plays a key dynamic role in determining whether the complete network obtains.

Attainment of the complete network depends on a dynamic trade off faced by insiders. On one hand, the myopic attractiveness of link formation means that forming an additional link and becoming the "hub" is attractive for an insider. But, on the other hand, an insider anticipates the eventual attainment of the complete network which is unattractive because of the insider exclusion incentive. The complete network is attained when the discount factor falls below a threshold because then the myopic link formation incentive dominates. Here, the "most attractive" insider becomes the "hub" on the path to the complete network. But the complete network is not attained when the discount factor exceeds the threshold because then the insider exclusion incentive dominates. Here, the two "most attractive" players remain insiders. Thus, the insider exclusion incentive drives the possible failure to obtain the complete network in my dynamic setting despite the efficiency of the complete network and despite the myopic attractiveness of link formation.

After presenting the FDNE under the general one period payoff specification just described, I present an application of the three player game. International trade agreements have been a persistent application of network and coalition theory where a bilateral trade agreement between two countries in interpreted as a link and the complete network represents global free trade (see, e.g., Yi (1996), Goyal and Joshi (2006), Furusawa and Konishi (2007), Seidmann (2009), Zhang et al. (2011), Zhang et al. (2013) and Zhang et al. (2014)).

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[2]To be clear, there are no transfers in the model.

Indeed, three country games are commonly used in the trade agreements literature with prominent recent examples including Saggi and Yildiz (2010) and Saggi et al. (2013). I present a three country oligopolistic model of international trade and show that the one period payoffs exhibit link monotonicity, negative link externalities and insider exclusion incentives. Thus, the FDNE characterization described in the main sections of the paper apply. As such, the insider exclusion incentive plays a key role in determining whether global free trade is attained. Moreover, country characteristics endogenously determine the order in which countries form bilateral agreements.

The rest of the paper proceeds as follows. Section 2 develops the network terminology and equilibrium concepts. Sections 3 and 4 characterize the FDNE with symmetric and asymmetric players. Section 5 presents the example illustrating that the equilibrium predictions of the FDNE differ from that if the players who can form a link in a period are chosen randomly. Section 6 gives a simple application to international trade agreements and Section 7 discusses the sensitivity of the model to particular assumptions. Finally, Section 8 concludes. Appendix A collects proofs not given in the text.

2 Network formation games and equilibrium

2.1 Preliminaries

The three player game is an infinite horizon network formation game. N denotes the set of players. Two assumptions make the network formation model tractable. First, at most one link can form in any given period. Second, links formed in previous periods cannot be severed. Obviously, the appropriateness of these assumptions is application specific. However, these assumptions (including restriction to a three player game) fit naturally into a trade agreements application. Such an application is relevant given trade agreements is one area that has seen numerous papers apply network theory. I present such an example in Section 6. I also discuss the (lack of) importance regarding the one link per period assumption in Section 7.

Importantly, the no severance assumption implies the complete network is an absorbing state. Since attention below is restricted to Markov strategies, the network remains unchanged forever once no link forms in period t. This happens after, at most, three periods.

A network g is simply a collection of links. Figure 1 depicts the possible networks and network position terminology. Figure 1 shows that $\emptyset$ and g^c denote, respectively, the empty and complete networks while g_i^H denotes the hub–spoke network with player i as the hub and g_{ij} denotes the insider–outsider network where players i and j are insiders.
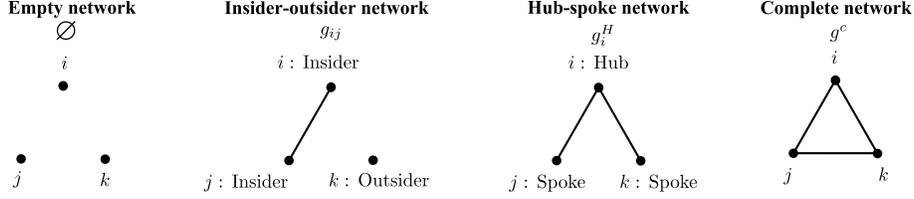
Figure 1: Networks and position terminology

Letting g_{t_0} denote the network at the beginning of the current period t_0, $\mathcal{G}(g_{t_0}) = (g_{t_0+1}, g_{t_0+2}, ...)$ denotes a *path of networks* from the end of the current period onwards. Sometimes it will be convenient to leave g_{t_0} and $g_{t_0+2}, ...$ implicit or unspecified. In these cases, $\langle g_{t_0+1} \rangle$ denotes the path of networks. To avoid the redundant repetition of networks once the network remains unchanged forever, I abuse the network path notation and let the last network in the path indicate the network that remains forever. For example, $\langle g_{ij} \rangle = (g_{ij}, g_i^H, g^c)$ indicates the network path beginning at the insider–outsider network g_{ij} reaches the complete network g^c via the hub–spoke network g_i^H and remains at the complete network forever. Alternatively, $\langle \varnothing \rangle = \varnothing$ indicates the network path that begins at the empty network and remains there forever.

Given a vector of player characteristics $\alpha = (\alpha_i, \alpha_j, \alpha_k)$ and a vector of other payoff relevant parameters τ, $v_i(g; \theta)$ denotes player i's one period payoff from network g where $\theta = (\alpha, \tau)$. Player i's *continuation payoff* from the path of networks $\mathcal{G}(g_{t_0})$ is then $V_i(\mathcal{G}(g_{t_0}); \theta) = \sum_{t=t_0+1}^{\infty} \delta^{t-t_0-1} v_i(g_t; \theta)$ where δ is the discount factor. A coalition $S \subseteq N$ *prefers* a path of networks $\mathcal{G}(g_{t_0})$ over another path $\mathcal{G}'(g_{t_0})$, denoted $\mathcal{G}(g_{t_0}) \succ_S \mathcal{G}'(g_{t_0})$, if $V_i(\mathcal{G}(g_{t_0}); \theta) > V_i(\mathcal{G}'(g_{t_0}); \theta)$ for all $i \in S$.

The notion of a coalition structure will be crucial for the equilibrium concept. A *coalition structure*, denoted P, is a partition on the set of players. Each element of the coalition structure is a coalition. The possible coalition structures are: i) the grand coalition $N = \{i, j, k\}$, ii) the singletons coalition structure $P^* \equiv \{\{i\}, \{j\}, \{k\}\}$, and iii) coalition structures of the form $P_{ij} = P_k \equiv \{\{i, j\}, \{k\}\}$.[3]

2.2 Actions and strategies

Given the assumption of one link per period, each period can be characterized by the network g that exists at the beginning of the period. Given the network at the beginning of a period

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[3] One could think of coalition structures in the following manner. The grand coalition N represents the situation where all players are in a single negotiating room. A coalition structure P_{ij} represents the situation where either k left the initial negotiating room or i and j left the initial negotiating room together. As a result, k is now in one room while i and j are in a second room. The coalition structure P^* represents the situation where either i or j left the room they shared in P_{ij} so that each player is now in a separate room.

is g, players play a simultaneous move "announcement game" to determine which link forms in the period. Like Seidmann (2009), I refer to this announcement game as the "subgame" at network g (perhaps more apt would be "stage game").

For the subgame at network g, player i's action space $A_i(g)$ represents the set of announcements player i can make. For a coalition $S \subseteq N$, $A_S = \Pi_{i \in S} A_S(g)$ has an analogous interpretation. Table 1 shows a player's action space consists of two types of announcements $a_i(g) \in A_i(g)$. First, the player with whom it wants to link with but has not yet done so. Second, no announcement, denoted ϕ. A link forms when both members of the proposed link announce in favor: the link between i and j forms if and only if $a_i(g) = j$ and $a_j(g) = i$.

Network	Player action space		
	$A_i(g)$	$A_j(g)$	$A_k(g)$
$\emptyset$	$\{\phi, j, k\}$	$\{\phi, i, k\}$	$\{\phi, i, j\}$
g_{ij}	$\{\phi, k\}$	$\{\phi, k\}$	$\{\phi, i, j\}$
g_i^H	$\{\phi\}$	$\{\phi, k\}$	$\{\phi, j\}$
g^c	$\{\phi\}$	$\{\phi\}$	$\{\phi\}$

Table 1: Action space for each subgame

A Markov strategy for player i, σ_i, assigns an action $a_i(g)$ for every network g. Notice that a strategy profile $\sigma = (\sigma_i, \sigma_j, \sigma_k)$ and an initial network g_{t_0} at the beginning of period t_0 induce a unique network path $\mathcal{G}(g_{t_0}) = (g_{t_0+1}, g_{t_0+2}, ...)$ from period t_0 onwards. Thus, σ and g_{t_0} induce a continuation payoff for player i of $V_i(\mathcal{G}(g_{t_0}); \theta) = \sum_{t=t_0+1}^{\infty} \delta^{t-t_0-1} v_i(g_t; \theta)$. Given this relationship between a strategy profile and continuation payoffs, players have preferences over action profiles and network outcomes in each subgame.[4] To this end, I refer to a network g' as *most preferred* for a coalition $S \subseteq N$ in a subgame at network g if, for every $i \in S$, $V_i(\langle g' \rangle; \theta) \geq V_i(\langle g'' \rangle; \theta)$ for all possible outcomes $g'' \neq g'$ of the subgame at network g (strictly most preferred if the inequality is strict).[5] That is g' is most preferred for a coalition S in the subgame at network g if, for each member of S, g' induces the highest continuation payoff of all possible outcomes of the subgame.

2.3 Equilibrium

As discussed in the introduction and at the beginning of the previous section, the dynamic game embeds a simultaneous move link announcement game in each period (i.e. each sub-

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[4] Formally, consider two strategy profiles σ and σ' that differ only because the respective action profiles at some network g_{t_0} are $a(g_{t_0})$ and $a'(g_{t_0})$. Then, σ and σ' induce the respective network paths $\mathcal{G}(g_{t_0})$ and $\mathcal{G}'(g_{t_0})$. Thus, player i's respective continuation payoffs from $a(g_{t_0})$ and $a'(g_{t_0})$ are $V_i(\mathcal{G}(g_{t_0}); \theta)$ and $V_i(\mathcal{G}'(g_{t_0}); \theta)$. Hence, player i prefers $a(g_{t_0})$ over $a'(g_{t_0})$ if and only if $\mathcal{G}(g_{t_0}) \succ_i \mathcal{G}'(g_{t_0})$.

[5] Using standard network notation by letting ij denote a link between i and j, the possible outcomes of the subgame at network g are $\{g + ij \mid ij \notin g\}$.

game). The equilibrium concept used to solve the link announcement game in each subgame is a slight variation on Equilibrium Binding Agreement (EBA; Ray and Vohra (1997)) suggested by Diamantoudi (2003).

The key idea of an EBA is that deviating players anticipate their deviation induces reactions by other players. This is formalized by a recursive definition. To define an EBA, let $\beta(P, g)$ denote the set of Nash equilibrium, given network g, where each coalition in the coalition structure P is treated as an individual player.[6] Then, given the subgame at a network g, the definition of an EBA for the three player game proceeds as follows:

1. Define $B(P^*, g) = \beta(P^*, g)$ as the EBAs for P^*.

   That is, the set of EBAs for the singletons coalitions structure P^* is the set of Nash equilibrium.

2. $a(g) \in \beta(P_{ij}, g)$ is an EBA for P_{ij}, denoted $a(g) \in B(P_{ij}, g)$, if there is no self enforcing deviation for i or j. A deviation by, say, i from $a_i(g)$ to $a'_i(g)$ is self enforcing if there exists $a'_{-i}(g)$ such that i) $a'(g) = (a'_i(g), a'_{-i}(g)) \in B(P^*, g)$ and ii) i prefers any such $a'(g)$ to $a(g)$.

   That is, an action profile $a(g)$ is an EBA for the coalition structure P_{ij} if i) it is a Nash equilibrium between $S = ij$ and k and ii) i nor j have a self enforcing deviation.[7] A deviation by, say, i from $a_i(g)$ to $a'_i(g)$ is self enforcing if $a'_i(g)$ is part of an EBA for the induced coalition structure P^* (i.e. part of a Nash equilibrium) and i prefers any such EBA to $a(g)$.

3. $a(g) \in \beta(N, g)$ is an EBA for N, denoted $a(g) \in B(N, g)$, if there is no self enforcing deviation by any coalition $S \subset N$. A deviation by S from $a_S(g)$ to $a'_S(g)$ is self enforcing if there exists $a'_{-S}(g)$ such that i) $a'(g) = (a'_S(g), a'_{-S}(g)) \in B(P_S, g)$ and ii) S prefers any such $a'(g)$ to $a(g)$. Additionally, if $S = i$ and $B(P_S, g)$ is empty then i's deviation from $a_i(g)$ to $a'_i(g)$ is self enforcing if i) there exists $a'_{-i}(g)$ such that $a'(g) = (a'_i(g), a'_{-i}(g)) \in B(P^*, g)$ and ii) i prefers any such $a'(g)$ over $a(g)$.[8]

   That is, an action profile $a(g)$ is an EBA for the grand coalition N if i) it is Pareto optimal and ii) no subcoalition of N has a self enforcing deviation. A deviation by a coalition $S \subset N$ from $a_S(g)$ to $a'_S(g)$ is self enforcing if $a'_S(g)$ is part of an EBA for

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[6]A coalition's preferences are given by the coalition preference relation already described. That is, a coalition has a profitable deviation iff each coalition member prefers the new action profile over the old action profile. Naturally, $\beta(P^*, g)$ is the usual set of Nash equilibrium since each coalition is a singleton, or individual, player.

[7]Of course, $S = ij$ is shorthand for $S = \{i, j\}$.

[8]Note, $a_S(g) = a'_S(g)$ is permitted to allow S to break away from the coalition N. Similarly, $a_i(g) = a'_i(g)$ allows i to break away from $S = ij$ in step 2.

the induced coalition structure P_S (or part of an EBA for P^* if $S = i$ and there is no EBA for P_{jk}) and each coalition member of S prefers any such EBA to $a(g)$.

Notice that these steps define EBAs for each coalition structure. So what are *the* EBAs for the subgame at network g? If $B(N, g)$ is non empty, then $B(N, g)$ is the set of EBAs for the subgame at network g. However, if $B(N, g)$ is empty then the set of EBAs for the subgame are the EBAs in the sets $B(P_{ij}, g)$. Finally, if $B(P_{ij}, g)$ is also empty for all ij then the set of EBAs for the subgame is the set of Nash equilibrium $B(P^*, g)$. Intuitively, the set of EBAs for the subgame at network g is the set of EBAs for the coarsest coalition structure such that an EBA exists.[9] Formally, an EBA is an action profile rather a link, but I will often refer to the link induced by an EBA action profile as an EBA itself.

Given the definition of an EBA, Definition 1 presents a new equilibrium concept for dynamic network formation games. I call this equilibrium concept a Farsighted Dynamic Network Equilibrium (FDNE).

Definition 1. A path of networks $\mathcal{G}(\emptyset) = (g_1^*, g_2^*, ...)$ is a Farsighted Dynamic Network Equilibrium (FDNE) if there is a strategy profile σ^* such that: i) given $g = \emptyset$, σ^* induces the path of networks $\mathcal{G}(\emptyset) = (g_1^*, g_2^*, ...)$ and ii) for any subgame at a network g, the action profile $a^*(g)$ is an Equilibrium Binding Agreement.

Intuitively, an FDNE is the equilibrium path of networks that emerges when the action profile in each subgame, on and off the equilibrium path, is an EBA of the associated announcement game. Thus, an FDNE has a strong flavor of subgame perfection. Indeed, since links formed in previous periods cannot be severed, an FDNE can be derived via backward induction because the complete network is an absorbing state. Importantly, in each subgame, player actions are based on continuation payoffs rather than one period payoffs.

The following two (highly stylized) examples illustrate the process of deriving an EBA in a particular subgame and the process of deriving an FDNE using backward induction. Before doing so, notice that, even though deviations are changes in action profiles, deviations are often more conveniently referred to as changes in networks. For example, given $a_i(\emptyset) = j$ and $a_j(\emptyset) = i$, the unilateral deviation by player i to $a_i(\emptyset) = \phi$ yields $g = \emptyset$ rather than $g = g_{ij}$. Thus, i "deviates" from $g = g_{ij}$ to $g = \emptyset$. To this end, let $G(P, g)$ denote the networks induced by the EBAs $B(P, g)$ and let $\gamma(P, g)$ denote the networks induced by

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[9]Notice, the above recursion requires the deviating coalition prefer *any* such EBA a' to the action profile a under consideration. This was proposed by Diamantoudi (2003) and differs from Ray and Vohra (1997) who require a deviating coalition merely prefer *some* EBA a' to a. Essentially, Diamantoudi (2003) assumes deviating coalitions anticipate "pessimistically" while Ray and Vohra (1997) assume they anticipate "optimistically".

$\beta(P, g)$. In a subgame, networks induced by a Nash equilibrium or an EBA are referred to as, respectively, Nash networks and EBA networks.

Example 1. Denoting the players as s (small), m (medium) and l (large), the example derives the EBA network for the subgame at the insider–outsider network $g_0 = g_{ml}$. That is, the network at the beginning of the period is g_0. The three possible outcomes of the subgame are g_{ml}, g_l^H and g_m^H. For illustration, assume the complete network emerges in the following period if a hub–spoke network emerges in the current period. Given the one period payoffs in Table 2, Table 3 computes continuation payoffs assuming $\delta = \frac{4}{5}$ (and normalizing by $(1 - \delta)$).[10]

The example is highly simplified because remaining insiders is strictly most preferred for m and l (even though formation of any link is myopically beneficial for each player in the link). Indeed, given this strictly most preferred outcome of m and l, i) $G(P_{ml}, g_{ml}) = g_{ml}$, ii) the unique Nash network is $\gamma(P^*, g_{ml}) = g_{ml}$ and iii) g_{ml} is Pareto optimal for s and m as well as s and l, i.e. $g_{ml} \in \gamma(P_{sm}, g_{ml})$ and $g_{ml} \in \gamma(P_{sl}, g_{ml})$. Indeed, $g_{ml} \in G(P_{sm}, g_{ml})$ and $g_{ml} \in G(P_{sl}, g_{ml})$ because $\gamma(P^*, g_{ml}) = g_{ml}$ implies s has no self enforcing deviation from g_{ml}. Moreover, $g_{ml} = G(P_{sm}, g_{ml})$ (and $g_{ml} = G(P_{sl}, g_{ml})$) because m (l) has a unilateral self enforcing deviation from $g \neq g_{ml}$ to g_{ml} since $\gamma(P^*, g_{ml}) = g_{ml}$.

Thus, $g_{ml} \in G(N, g_{ml})$ because g_{ml} is Pareto optimal for N and s has no self enforcing deviation given that $g_{ml} = G(P_{ml}, g_{ml})$. Indeed, $g_{ml} = G(N, g_{ml})$ because m and l have a self enforcing deviation from $g \neq g_{ml}$ to $g_{ml} = G(P_{ml}, g_{ml})$. That is, the EBA is that no link forms and the EBA network is that m and l remain insiders.∎

			Network				
$\emptyset$	g_{ml}	g_{sl}	g_{sm}	g_l^H	g_m^H	g_s^H	g^c
$(0,0,0)$	$(-5,15,15)$	$(14,-3,14)$	$(13,13,-1)$	$(7,7,17)$	$(6,18,6)$	$(19,5,5)$	$(11,11,11)$

Table 2: Network dependent one period payoffs $(v_s(g), v_m(g), v_l(g))$

Network at end of current period (g_1)		
g_l^H	g_m^H	g_{ml}
$\left(10\frac{1}{5}, 10\frac{1}{5}, 12\frac{1}{5}\right)$	$\left(10, 12\frac{2}{5}, 10\right)$	$(-5, 15, 15)$

Table 3: Normalized continuation payoffs $(V_s(\cdot), V_m(\cdot), V_l(\cdot))$ given $g_0 = g_{ml}$

Example 1 illustrates that an EBA is similar to a coalition proof Nash equilibrium (CPNE). Each concept restricts attention to profitable coalition deviations that are "self

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[10]For example, l's normalized continuation payoff from becoming the hub, i.e. $g_1 = g_l^H$, is $(1 - \delta)\left(17 + \frac{\delta}{1-\delta}11\right) = 17 - 6\delta = 12\frac{1}{5}$.

enforcing". A deviation is self enforcing if it results in an action profile that is itself an equilibrium. Thus, both concepts require that coalition deviations are robust to subsequent deviation by coalition members. However, the key difference is that deviating players in a CPNE assume other players' actions remain fixed following their deviation while deviating players' in an EBA assume other players actions adjust in response to their own deviation. Intuitively, deviating coalitions in an EBA anticipate the "equilibrium reactions" of other players which is formalized by requiring the outcome of a coalition's deviation is an EBA of the induced coalition structure.

Example 1 also illustrates that insiders remain insiders when doing so is most preferred for each insider. Lemma 1 generalizes this intuition: most preferred networks that can be sustained by coalitions are EBA networks. Following custom, $g + ij$ denotes that the link between i and j is added to the network g.

Lemma 1. *Suppose the network at the beginning of period t is g_t and fix $a(g)$ for all $g \neq g_t$. If $g' = g_t + ij$ or $g' = g_t$ is most preferred for i and j then g' is an EBA network in period t. If g' is strictly most preferred, then g' is the unique EBA network in period t.*

Proof. The only (possibly) profitable deviation from g' is the unilateral deviation by k. However, regardless of $a_k(g_t)$, $a_i(g_t) = j$ and $a_j(g_t) = i$ imply $g' = g_t + ij$ while $a_i(g_t) = a_j(g_t) = \phi$ implies $g' = g_t$. Two implications follow given g' is most preferred for i and j: i) $g' \in \gamma(P_{ij}, g_t)$ and ii) given i nor j have an incentive to deviate, $g' \in G(P_{ij}, g_t)$. Thus $g' \in G(N, g_t)$ because k has no self enforcing unilateral deviation from g' to any $\hat{g} \in G(P_{ij}, g_t)$. Moreover, $g' = G(N, g_t)$ if g' is strictly most preferred by i and j because then i and j have a self enforcing deviation from any $\hat{g} \neq g'$ to $g' \in G(P_{ij}, g_t)$. $\square$

Example 2 illustrates the process of deriving an FDNE by backward induction.

Example 2. Since the complete network is an absorbing state, consider the subgame at the hub–spoke network $g_0 = g_l^H$. That is, the network at the beginning of the period is $g_0 = g_l^H$. The two possibilities for the end of period network, g_1, are g_l^H and g^c. In either case, g_1 is an absorbing state. Thus, using Table 2, s and m have the same continuation payoffs across either outcome: $\frac{7}{1-\delta}$ and $\frac{11}{1-\delta}$ respectively. Hence, $g_1 = g^c$ is strictly most preferred for s and m and Lemma 1 implies the unique EBA network is g^c. That is, $G(N, g_l^H) = g^c$. Given Table 2, the same logic applies for any hub–spoke network meaning any hub–spoke network expands to the complete network; i.e. $G(N, g_i^H) = g^c$ for any i.

Now consider the subgame at an insider–outsider network $g_0 = g_{ij}$. For $g_0 = g_{ml}$, Example 1 illustrates that the EBA network is $G(N, g_{ml}) = g_{ml}$ meaning that m and l remain insiders. Similar calculations to Table 3 reveal that remaining insiders is strictly

most preferred for any pair of insiders given $\delta = \frac{4}{5}$. Thus, Lemma 1 implies the unique EBA network is $G(N, g_{ij}) = g_{ij}$ for any pair of insiders i and j given $\delta = \frac{4}{5}$. That is, any pair of insiders remain insiders.

Now consider the subgame at the empty network $g_0 = \emptyset$. The four possibilities for the end of period network, g_1, are $\emptyset$, g_{ml}, g_{sl}, and g_{sm}. However, given any pair of insiders remain insiders once an insider–outsider network forms, player i's (normalized) continuation payoff from any such g_1 is just the one period payoff $v_i(g_1)$. Thus, Table 2 shows that becoming insiders is strictly most preferred for m and l. In turn, Lemma 1 implies the unique EBA network is $G(N, \emptyset) = g_{ml}$ meaning m and l become insiders.

The FDNE is the equilibrium path of networks that emerges from solving the EBA network in each subgame. Since m and l become insiders in period 1 and remain so forever (given $\delta = \frac{4}{5}$), the unique FDNE is g_{ml}.∎

3 FDNE with symmetric players

In this section, I solve the Farsighted Dynamic Network Equilibrium (FDNE) of the three player dynamic game introduced in the previous section. To do so, Condition 1 imposes some structure on the one period payoffs.

Condition 1. Players are symmetric (i.e $\alpha_i = \alpha_j = \alpha_k$) and

i) $v_i(g + ij) > v_i(g)$ and $v_j(g + ij) > v_j(g)$

ii) $v_i(g) > v_i(g + jk)$ for $g \neq \emptyset$ but $v_i(\emptyset) \gtreqless v_i(\emptyset + jk) = v_i(g_{jk})$

iii) $v_i(g_{ij}) > v_i(g^c) > v_i(\emptyset)$.

Condition 1 is simple. Part i) is the "link monotonicity" property (e.g. Dutta et al. (2005)): bilateral link formation myopically benefits the linking players. Part ii) says that, except at the empty network, link formation between two players imposes negative link externalities on the third player. Bilateral link formation at the empty network may or may not impose negative externalities. Finally, part iii) says that the complete network is more attractive than the empty network but insiders find the insider–outsider network more attractive than the complete network. This possibility arises because even though link monotonicity implies $v_i(g_i^H) > v_i(g_{ij})$, negative link externalities imply $v_i(g_i^H) > v_i(g^c)$. Intuitively, the first inequality in part iii) represents an "insider exclusion incentive": insiders have an incentive to exclude the outsider from direct expansion to the complete network. The insider exclusion incentive plays a central role in the subsequent analysis. Additionally, note that Condition 1 does not impose whether the complete network is the efficient network in the sense of maximizing the aggregate one period payoff.

I also impose the following Condition on continuation payoffs.

Condition 2. $v_i\left(g_{ij}\right) + \delta v_i\left(g_j^H\right) + \frac{\delta^2}{1-\delta} v_i\left(g^c\right) > \frac{1}{1-\delta} v_i\left(\varnothing\right).$

Condition 2 is a type of participation constraint.[11] It says that a player receives a higher continuation payoff from participating in the dynamic game as an insider–turned–spoke on the path to the complete network than it would if no links ever formed. I refer to this as a type of participation constraint because if Condition 2 fails then, given Condition 1, an analogous condition also fails for the outsider in which case the insider–turned–spoke and outsider can, via Lemma 1, ensure the empty network remains forever.

The first step in using backward induction to solve the FDNE is solving the EBA for subgames at hub–spoke networks. However, this task is simple. Given link monotonicity, the spokes have an incentive to form the final link. Thus, Lemma 1 implies the EBA at any hub–spoke network is that spokes form the link leading to the complete network: $G\left(N, g_i^H\right) = g^c$ for any i.

Rolling back to subgames at insider–outsider networks, insiders face a interesting trade off given any hub–spoke network expands to the complete network. Link monotonicity implies $v_i\left(g_i^H\right) > v_i\left(g_{ij}\right)$ so an insider has a myopic incentive to become the hub. However, doing so will then lead to the complete network which is unattractive for an insider given the insider exclusion incentive, i.e. $v_i\left(g_{ij}\right) > v_i\left(g^c\right)$. An insider prefers to become the hub rather than remain an insider forever if and only if $v_i\left(g_i^H\right) + \frac{\delta}{1-\delta} v_i\left(g^c\right) > \frac{1}{1-\delta} v_i\left(g_{ij}\right)$. This reduces to the No Exclusion (NE) condition:

$$\delta < \bar{\delta}^{NE}\left(\theta\right) \equiv \frac{v_i\left(g_i^H\right) - v_i\left(g_{ij}\right)}{v_i\left(g_i^H\right) - v_i\left(g^c\right)}. \tag{1}$$

The No Exclusion condition is intuitive: sufficient impatience, captured by $\delta < \bar{\delta}^{NE}\left(\theta\right)$, places a relatively high weight on the benefit derived from link monotonicity and being the hub but a relatively low weight on the insider exclusion incentive. Lemma 2 formalizes the role of the No Exclusion condition in subgames at insider–outsider networks.

Lemma 2. *Assume Condition 1 holds and consider a subgame at an insider–outsider network* $g = g_{ij}$. *The EBA networks are: i)* g_{ij} *when* $\delta > \bar{\delta}^{NE}\left(\theta\right)$, *but ii) the hub–spoke networks* g_i^H *and* g_j^H *when* $\delta < \bar{\delta}^{NE}\left(\theta\right)$.

Violation of the No Exclusion condition, $\delta > \bar{\delta}^{NE}\left(\theta\right)$, means each insider prefers remaining an insider over becoming the hub on a path to the complete network. Underlying this preference is a strong insider exclusion incentive. By Lemma 1, the EBA is that no link forms

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[11]Condition 2 can be weakened so that the inequality only holds when $\delta \leq \bar{\delta}^{NE}\left(\theta\right)$ (see equation (1)).

and the insiders remain insiders.[12] Conversely, satisfaction of the No Exclusion condition means the insider exclusion incentive is sufficiently weak that each insider wants to become the hub. In this case, multiplicity arises because the outsider is indifferent between forming a link with either insider.

The hub–spoke network g_i^H is an EBA network when $\delta < \bar{\delta}^{NE}(\theta)$, i.e. $g_i^H \in G(N, \emptyset)$, because $g_i^H \in G(P_{ik}, g_{ij})$ implies j has no self enforcing deviation from g_i^H to g_j^H. $g_i^H \in G(P_{ik}, g_{ij})$ follows because g_{ij} is most preferred for i and k. By symmetry, this logic also implies g_j^H is an EBA network. Moreover, g_{ij} is not an EBA network because of the self enforcing deviation by i and k to $g_i^H \in G(P_{ik}, g_{ij})$. Therefore, the EBA networks in the subgame at the insider–outsider network g_{ij} are the hub–spoke networks g_i^H and g_j^H.

Rolling back and solving the EBA in the subgame at empty network reveals the equilibrium path of networks which is the FDNE. Proposition 1 characterizes the FDNE.

Proposition 1. *Assume Conditions 1–2 hold. The FDNE are: i) any insider–outsider network g_{ij} when $\delta > \bar{\delta}^{NE}(\theta)$, ii) any path of bilateral links leading to the complete network when $\delta < \bar{\delta}^{NE}(\theta)$.*

Proposition 1 is depicted in Figure 2 where Ω^{I-O} denotes the set of insider–outsider networks and Ω^c denotes the set of paths where bilateral link formation leads leads to the complete network.

Violation of the No Exclusion condition, $\delta > \bar{\delta}^{NE}(\theta)$, implies each insider holds an insider exclusion incentive and this incentive is sufficiently large that, despite the myopic incentive to become the hub, remaining insiders is strictly most preferred for the insiders. However, any insider–outsider network can emerge because of symmetry (see Lemma 1).
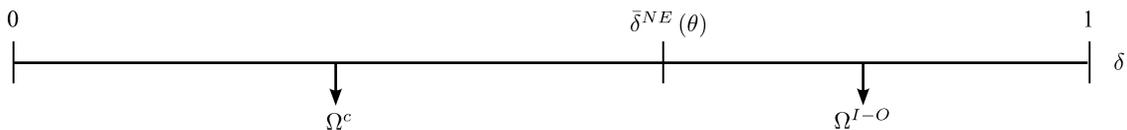

Figure 2: FDNE under symmetry

The logic underlying the FDNE when the No Exclusion condition holds, $\delta < \bar{\delta}^{NE}(\theta)$, is more nuanced. The simple part is that the fear of preference erosion is sufficiently small that the complete network emerges via a hub–spoke network from any insider–outsider network. The nuances arise because of players' preferences over the (equilibrium) path of networks stemming from each insider–outsider network. Given the multiplicity of EBAs at insider–outsider networks, suppose, without loss of generality, that the strategy profile specifies each

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[12]In particular, $a_i(g_{ij}) = a_j(g_{ij}) = \phi$ is an EBA action profile.

player is the hub on some path to the complete network.[13] This creates a Condorcet paradox situation across the insider–outsider networks. An important advantage of using the EBA solution concept is that, unlike other simultaneous move solution concepts (e.g. CPNE), non–existence problems do not arise in this situation. Lemma 3 characterizes the EBA networks in these Condorcet paradox situations.

Lemma 3. *Consider the subgame at the empty network $g = \emptyset$. Assume, i) $\langle g_{ij} \rangle \succ_{ij} \langle g_{ik} \rangle \succ_{ik} \langle g_{jk} \rangle \succ_{jk} \langle g_{ij} \rangle$, and ii) $\langle g_{ij} \rangle \succ_i \langle \emptyset \rangle$ for any $i, j \in N$. Then $G(N, \emptyset)$ is empty. However, for any $i, j \in N$, $g_{ij} \in G(P_{ij}, \emptyset)$ but $\emptyset \notin G(P_{ij}, \emptyset)$.*

Proof. To begin, note that $\gamma(P^*, \emptyset) = \{\emptyset, g_{jk}, g_{ij}, g_{ik}\}$. Moreover, $g_{jk} \in G(P_{jk}, \emptyset)$ because i) $g_{jk} \in \gamma(P_{jk}, \emptyset)$, ii) g_{jk} is most preferred for j, and iii) the deviation by k to $a_k(\emptyset) = i$ is not self enforcing given $g_{ij} \in \gamma(P^*, \emptyset)$ when $a_k(\emptyset) = i$. Similar logic establishes $g_{ik} \in G(P_{ik}, \emptyset)$ and $g_{ij} \in G(P_{ij}, \emptyset)$. Thus, there are self enforcing deviations by i) $S = jk$ from $\emptyset$ and g_{ij} to $g_{jk} \in G(P_{jk}, \emptyset)$, ii) $S = ik$ from g_{jk} to $g_{ik} \in G(P_{ik}, \emptyset)$ and iii) $S = ij$ from g_{ik} to $g_{ij} \in G(P_{ij}, \emptyset)$. Hence, $G(N, \emptyset)$ is empty. Finally, $\emptyset \notin G(P_S, \emptyset)$ for any $S = ij$ because $\langle g_{ij} \rangle \succ_S \langle \emptyset \rangle$ implies $\emptyset \notin \gamma(P_S, \emptyset)$. $\square$

The main idea behind Lemma 3 is simple. The insider–turned–spoke and outsider–turned–spoke have a self enforcing deviation where they become insiders with the former subsequently becoming the hub in the following period. This deviation is self enforcing because the fear of being an outsider in a Nash network deters any subsequent deviation. Similarly, any pair of players have a self enforcing deviation from the empty network which installs themselves as insiders. Thus, there is no EBA for the grand coalition yet any insider–outsider network g_{ij}, but not the empty network, is an EBA between $S = ij$ and $N \setminus S = k$ (i.e. $g_{ij} \in G(P_{ij}, \emptyset)$). Hence, the set of EBA networks in the subgame at the empty network is Ω^{I-O} and the FDNE are the set of paths Ω^c where bilateral link formation leads to the complete network.

4   FDNE with asymmetric players

I now move on to solve the FDNE with asymmetric players. Asymmetry brings out interesting features of the equilibrium network formation process that are absent under symmetry. As described in Section 2.1, player i's characteristics are summarized by α_i. I model asym-

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[13]Given there are three paths to the complete network here (one from each of the insider–outsider networks), the other possibility is that one player is the hub on two such paths. It is simple to show that this does not affect the result being discussed.

metry in a simple way: $v_k (g + ik) > v_k (g + jk)$ if and only if $\alpha_i > \alpha_j$.[14] That is, link formation is more attractive with a player who has a higher α.

I also weaken the one period payoff structure imposed under symmetry. Condition 3 summarizes.

Condition 3. Condition 1 holds except that

i) $v_k (g + ik) > v_k (g + jk)$ if and only if $\alpha_i > \alpha_j$

ii) $v_i \left(g_k^H \right) \gtreqless v_i (g_{jk})$ if $\alpha_k = \min \{\alpha_i, \alpha_j, \alpha_k\}$

iii) $v_i (g_{ij}) \gtreqless v_i (g^c)$ if $\alpha_k \neq \min \{\alpha_i, \alpha_j, \alpha_k\}$

iv) $v_i (g^c) \gtreqless v_i (\varnothing)$ for $\alpha_i = \max \{\alpha_i, \alpha_j, \alpha_k\}$

Part i) of Condition 3 captures how asymmetry affects one period payoffs. Part ii) weakens the extent to which link monotonicity holds. An outsider, say player i, may not benefit from link formation with player k if player k is the least attractive player. Part iii) weakens the extent to which insiders hold an insider exclusion incentive. The only insider–outsider network where the insiders necessarily hold an insider exclusion incentive is when the two most attractive players are insiders. Part iv) weakens the extent to which the complete network is attractive relative to the empty network. Now, $v_i (\varnothing) > v_i (g^c)$ is possible if player i is the most attractive player.

Like under symmetry, the first step in using backward induction to solve the FDNE remains simple. Link monotonicity still implies spokes benefit from link formation. Thus, spoke–spoke link formation is the EBA in subgames at hub–spoke networks and, hence, any hub–spoke network expands to the complete network.

However, subgames at insider–outsider networks are richer under asymmetry relative to symmetry. The added richness arises because an insider's exclusion incentive depends on the characteristics of itself and its insider partner. As such, each insider has their own No Exclusion condition. Specifically, player i prefers to become the hub rather than remain a permanent insider with player j if and only if $v_i \left(g_i^H \right) + \frac{\delta}{1-\delta} v_i (g^c) > \frac{1}{1-\delta} v_i (g_{ij})$ which reduces to player i's No Exclusion (NE) condition as an insider with player j:

$$\delta < \bar{\delta}_{i,j}^{NE} (\theta) \equiv \frac{v_i \left(g_i^H \right) - v_i (g_{ij})}{v_i \left(g_i^H \right) - v_i (g^c)}. \tag{2}$$

Lemma 4 extends Lemma 2 to the case of asymmetry.

Lemma 4. *Assume Condition 3 holds and consider a subgame at an insider–outsider network g_{ij} where $\alpha_i > \alpha_j$. The EBA networks are: i) g_{ij} when $\delta > max \left\{ \bar{\delta}_{i,j}^{NE} (\theta), \bar{\delta}_{j,i}^{NE} (\theta) \right\}$, but ii)*

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[14]Thus, the α_i's are either scalars or there is a mapping that reduces the vector α_i to a scalar summary statistic.

g_i^H when $\delta < max \left\{ \bar{\delta}_{i,j}^{NE}\left(\theta\right), \bar{\delta}_{j,i}^{NE}\left(\theta\right) \right\}$.

Like the symmetric case, the insiders remain insiders when the No Exclusion condition of both insiders is violated, $\delta > max \left\{ \bar{\delta}_{i,j}^{NE}\left(\theta\right), \bar{\delta}_{j,i}^{NE}\left(\theta\right) \right\}$, while the complete network is eventually attained when the No Exclusion condition of both insiders is satisfied, $\delta < min \left\{ \bar{\delta}_{i,j}^{NE}\left(\theta\right), \bar{\delta}_{j,i}^{NE}\left(\theta\right) \right\}$. Thus, the role of the insiders exclusion incentive embodied in the No Exclusion condition remains central. However, two new features emerge under asymmetry. First, the larger insider i always becomes the hub in a hub–spoke network. Second, there is an intermediate range, $\delta \in \left(\bar{\delta}_{i,j}^{NE}\left(\theta\right), \bar{\delta}_{j,i}^{NE}\left(\theta\right) \right)$, where only the No Exclusion condition of the least attractive insider, player j, is satisfied.[15]

Interestingly, the larger insider i becomes the hub in the EBA network when $\delta \in \left(\bar{\delta}_{i,j}^{NE}\left(\theta\right), \bar{\delta}_{j,i}^{NE}\left(\theta\right) \right)$ even though only the smaller insider j wants to become the hub. The main logic underlying this outcome emphasizes the distinguishing feature of the EBA solution concept whereby deviating players anticipate the equilibrium reactions of other players. Specifically, g_i^H is the unique EBA network even though, given $\delta > \bar{\delta}_{i,j}^{NE}\left(\theta\right)$ implies i prefers to remain an insider, it is not a Nash network. Nevertheless, i anticipates that backing out of the link with k does not necessarily lead to g_{ij} but rather that j and k will then form their own link. More formally, i anticipates the link between j and k since $g_j^H \in G\left(P_{jk}, g_{ij}\right)$. $g_j^H \in G\left(P_{jk}, g_{ij}\right)$ follows because g_j^H is strictly most preferred for j and the fear of being an outsider in a Nash network deters k's deviation to $a_k\left(g_{ij}\right) = i$. Thus, the anticipation of being a spoke upon backing out of g_i^H deters i's deviation from g_i^H.

While the above logic is the key logic supporting g_i^H as the EBA network, an additional deviation needs to be ruled out. Not only i prefers remaining an insider over becoming the hub, but the insider–turned–spoke j also prefers remaining an insider. Nevertheless, the joint deviation by i and j to g_{ij} is not self enforcing because $g_{ij} \notin G\left(P_{ij}, g_{ij}\right)$ since, given g_i^H is not a Nash network, j will then unilaterally deviate from $a_j\left(g_{ij}\right) = \phi$ to $a_j\left(g_{ij}\right) = k$ anticipating the unique Nash network g_j^H. Thus, g_i^H is an EBA network in a subgame at an insider–outsider network when $\delta \in \left(\bar{\delta}_{i,j}^{NE}\left(\theta\right), \bar{\delta}_{j,i}^{NE}\left(\theta\right) \right)$. Indeed, Lemma 1 implies g_i^H is also an EBA network when $\delta \leq \bar{\delta}_{i,j}^{NE}\left(\theta\right)$ because then g_i^H is most preferred for i and the outsider k. Thus, g_i^H is an EBA network in a subgame at an insider–outsider network when $\delta < max \left\{ \bar{\delta}_{i,j}^{NE}\left(\theta\right), \bar{\delta}_{j,i}^{NE}\left(\theta\right) \right\}$.

It is simple to see that g_i^H is the unique EBA network when $\delta < max \left\{ \bar{\delta}_{i,j}^{NE}\left(\theta\right), \bar{\delta}_{j,i}^{NE}\left(\theta\right) \right\}$. Lemma 1 establishes uniqueness when $\delta < \bar{\delta}_{i,j}^{NE}\left(\theta\right)$ because g_i^H is strictly most preferred for i and k. When $\delta \geq \bar{\delta}_{i,j}^{NE}\left(\theta\right)$, j and k have a self enforcing deviation from g_{ij} to g_j^H given , as discussed above, $g_j^H \in G\left(P_{jk}, g_{ij}\right)$. Moreover i and j have a self enforcing deviation from g_j^H to g_i^H; the deviation is self enforcing, i.e. $g_i^H \in G\left(P_{ik}, g_{ij}\right)$, given g_i^H is most preferred

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[15]This interval may be empty.

for k and the fear of being a spoke in a Nash network deters any subsequent deviation by i. Thus, g_i^H is the unique EBA network in a subgame at an insider–outsider network when $\delta < \max\left\{\bar{\delta}_{i,j}^{NE}(\theta), \bar{\delta}_{j,i}^{NE}(\theta)\right\}$.

Rolling back to the subgame at the empty network and solving for the EBA reveals the equilibrium path of network formation. Before proceeding to this characterization, the following assumption and condition help simplify its presentation. It will also be useful to distinguish the players as s, m and l where $\alpha_l > \alpha_m > \alpha_s$. These players can be interpreted as the small (s), medium (m) and large (l) players.

Assumption 1. *i)* $\delta \leq \max\left\{\bar{\delta}_{m,l}^{NE}(\theta), \bar{\delta}_{l,m}^{NE}(\theta)\right\}$ *implies* $\langle g_{ml}\rangle = \left(g_{ml}, g_l^H, g^c\right) \succ_m \langle\varnothing\rangle = \varnothing$, $\langle g_{sm}\rangle = \left(g_{sm}, g_m^H, g^c\right) \succ_s \langle\varnothing\rangle = \varnothing$ *and* $\langle g_{sl}\rangle = \left(g_{sl}, g_l^H, g^c\right) \succ_l \langle\varnothing\rangle = \varnothing$.
ii) $\bar{\delta}_{l,s}^{NE}(\theta) < \bar{\delta}_{m,s}^{NE}(\theta)$ *implies* $\bar{\delta}_{m,l}^{NE}(\theta) < \bar{\delta}_{s,l}^{NE}(\theta)$.
iii) If $\bar{\delta}_{m,l}^{NE}(\theta) < \bar{\delta}_{l,m}^{NE}(\theta)$, *then* $\langle g_{sm}\rangle = g_{sm} \succ_m \langle g_{ml}\rangle = \left(g_{ml}, g_l^H, g^c\right)$ *implies* $\langle g_{sm}\rangle = g_{sm} \succ_s \langle g_{sl}\rangle = \left(g_{sl}, g_l^H, g^c\right)$.

Assumption 1 places restrictions on continuation payoffs. Part i) says that if the insider–outsider network g_{ml} leads to the complete network then the discount factor is low enough that i) Condition 2 holds for m as an insider with l, ii) Condition 2 holds for s as an insider with m (and hence with l as well) and iii) the possibility of $v_l(\varnothing) > v_l(g^c)$ cannot outweigh the effect of link monotonicity for l in being an insider with s and then the hub. Parts ii) and iii) of Assumption 1 rule out "cross player preference reversals". Part ii) says that if a "larger" insider l has a slacker No Exclusion condition compared with a "smaller" insider m when the fixed insider partner is s, then the same is true when the "larger" insider is m, the "smaller" insider is s and the fixed insider partner is l. Part iii) is more tedious. When m and l are insiders, m's No Exclusion condition could be slacker than l's. Intuitively, this can arise when the link monotonicity effect is stronger for more attractive players. Part iii) says that if the more attractive player m prefers remaining a permanent insider with s rather than being an insider with l and a spoke on the path to the complete network, then the less attractive player s holds an analogous preference. Condition 4 now follows.

Condition 4. $\delta < \max\left\{\bar{\delta}_{s,m}^{NE}(\theta), \bar{\delta}_{m,s}^{NE}(\theta)\right\}$

Letting $\hat{\Omega}^c$ denote the set of paths where bilateral link formation leads to the complete network via a hub–spoke network with the larger insider as the hub and defining $\bar{\delta}^m(\theta)$ such that $\langle g_{ml}\rangle = \left(g_{ml}, g_l^H, g^c\right) \succ_m \langle g_{sm}\rangle$ if and only if $\delta < \bar{\delta}^m(\theta)$, Proposition 2 characterizes the FDNE.[16]

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[16]If $\langle g_{sm}\rangle = \left(g_{sm}, g_m^H, g^c\right)$ then the definition of $\bar{\delta}^m(\theta)$ implies $\bar{\delta}^m(\theta) \equiv \frac{v_m(g_{ml}) - v_m(g_{sm})}{v_m(g_m^H) - v_m(g_l^H)}$. If $\langle g_{sm}\rangle = g_{sm}$ the definition of $\bar{\delta}^m(\theta)$ implies $\bar{\delta}^m(\theta)$ is defined such that $f(\delta) = [v_m(g_{ml}) - v_m(g_{sm})] + \delta\left[v_m(g_l^H) - v_m(g_{ml})\right] + \delta^2\left[v_m(g^c) - v_m(g_l^H)\right] > 0$ iff $\delta < \bar{\delta}^m(\theta)$.

Proposition 2. *Assume Condition 3 and Assumption 1 hold. The FDNE is g_{ml} when $\delta > max\left\{\bar{\delta}_{m,l}^{NE}\left(\theta\right), \bar{\delta}_{l,m}^{NE}\left(\theta\right)\right\}$. Now suppose $\delta < max\left\{\bar{\delta}_{m,l}^{NE}\left(\theta\right), \bar{\delta}_{l,m}^{NE}\left(\theta\right)\right\}$. If $\delta < \bar{\delta}^m\left(\theta\right)$ the FDNE is $\left(g_{ml}, g_l^H, g^c\right)$. If $\delta > \bar{\delta}^m\left(\theta\right)$ the FDNE are $\hat{\Omega}^c$ if Condition 4 holds but g_{sm} if Condition 4 fails.*

Figure 3 illustrates the FDNE where, for illustrative purposes, Condition 4 holds when $\delta < \max\left\{\bar{\delta}_{m,l}^{NE}\left(\theta\right), \bar{\delta}_{l,m}^{NE}\left(\theta\right)\right\}$.

When $\delta > \max\left\{\bar{\delta}_{m,l}^{NE}\left(\theta\right), \bar{\delta}_{l,m}^{NE}\left(\theta\right)\right\}$, m and l prefer to remain insiders because they have sufficiently strong insider exclusion incentives. Thus, Lemma 1 implies this is the unique FDNE.

Whether multiple equilibria arise when $\delta \leq \max\left\{\bar{\delta}_{l,m}^{NE}\left(\theta\right), \bar{\delta}_{m,l}^{NE}\left(\theta\right)\right\}$ depends on whether m prefers the path $\langle g_{ml}\rangle = \left(g_{ml}, g_l^H, g^c\right)$ over $\langle g_{sm}\rangle$. Myopically, m benefits from being an insider with the larger player l. However, m may prefer to be an insider with the smaller player s so that either i) it becomes the hub or ii) it can remain an insider and exploit its No Exclusion condition when $\max\left\{\bar{\delta}_{m,s}^{NE}\left(\theta\right), \bar{\delta}_{s,m}^{NE}\left(\theta\right), \bar{\delta}_{m,l}^{NE}\left(\theta\right)\right\} < \bar{\delta}_{l,m}^{NE}\left(\theta\right)$.[17] In either case, m prefers to be an insider with l rather than s if and only if $\langle g_{ml}\rangle = \left(g_{ml}, g_l^H, g^c\right) \succ_m \langle g_{sm}\rangle$ which reduces to $\delta < \bar{\delta}^m\left(\theta\right)$. Thus, g_{ml} is strictly most preferred for m and l and, hence $\left(g_{ml}, g_l^H, g^c\right)$ is the unique FDNE, when $\delta < \bar{\delta}^m\left(\theta\right)$.
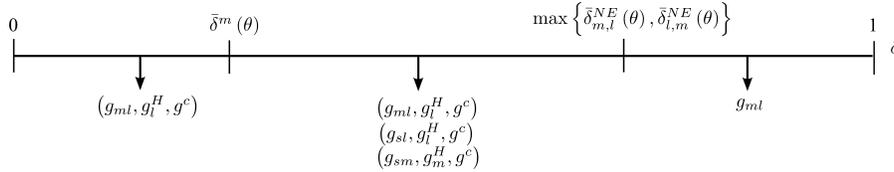

Figure 3: FDNE under asymmetry

However, multiple equilibria can arise when $\delta \in \left(\bar{\delta}^m\left(\theta\right), max\left\{\bar{\delta}_{l,m}^{NE}\left(\theta\right), \bar{\delta}_{m,l}^{NE}\left(\theta\right)\right\}\right]$ and do arise if Condition 4 holds. Then, like the symmetric case, multiplicity stems from a Condorcet paradox situation across the insider–outsider networks: there is always a pair of players who benefit from the self enforcing deviation to install themselves as insiders. Moreover, Lemma 3 applies because part i) of Assumption 1 implies part iii) of Lemma 3 holds. Thus, like Proposition 1, there is a self enforcing deviation from any action profile (i.e. $G\left(N, \emptyset\right)$ is empty), but the self enforcing deviations to the various insider–outsider networks imply Ω^{I-O} are the EBA networks for the subgame at the empty network. Thus, $\hat{\Omega}^c$ are the FDNE.

The final case to consider for bilateral beliefs is when $\delta \in \left(\bar{\delta}^m\left(\theta\right), max\left\{\bar{\delta}_{l,m}^{NE}\left(\theta\right), \bar{\delta}_{m,l}^{NE}\left(\theta\right)\right\}\right]$ but Condition 4 fails. In this case, s and m both hold a No Exclusion incentive as insider

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[17]The inequality does not hold in Figure 3. But when it does hold then, by application of Lemmas 1 and 4, g^c is eventually obtained from g_{ml} but g_{sm} remains permanently once attained.

partners and remain insiders forever upon becoming insiders. Note that Condition 3 implies Condition 4 can only fail if $\bar{\delta}_{m,l}^{NE}(\theta) < \bar{\delta}_{l,m}^{NE}(\theta)$. Thus, given part iii) of Assumption 1, g_{sm} is strictly most preferred for s and m so, by Lemma 1, g_{sm} is the unique FDNE. This may seem like a counter–intuitive result: the least attractive players remain insiders. However, the logic is simple. If larger players enjoy a greater myopic benefit from becoming the hub, the larger insider's No Exclusion condition can be slacker than the smaller insider's meaning $\bar{\delta}_{m,l}^{NE}(\theta) < \bar{\delta}_{l,m}^{NE}(\theta)$ could hold. Thus, as an insider with l, s and/or m may then become a spoke even though it prefers remaining an insider with l. As such, despite the myopic appeal of being an insider with l, s and m may prefer to form their own link to exploit the failure of their No Exclusion condition and remain permanent insiders.

5   The protocol for link formation opportunities matters

A novel feature of the dynamic network formation model in the previous sections is there is no exogenous protocol governing who has the opportunity to form a link in a given period. Rather, who forms a link in a given period is determined endogenously by a simultaneous move announcement game. This contrasts with, for example, Dutta et al. (2005) who assume a random pair of players, called the active pair and denoted by η, have the opportunity to form a link in a period.

Maintaining the assumption that links formed in previous periods cannot be severed (which Dutta et al. (2005) do not impose), I will now present an example where the complete network is not attained when the order of link formation is endogenous but is attained when the active pair is randomly chosen in each period. Defining a state as (g, η), i.e. a network and an active pair, Dutta et al. (2005, p.152) loosely describe their equilibrium concept by stating "... an equilibrium process of network formation is a strategy profile with the property that there is no active pair at any state ... which can benefit—either unilaterally or bilaterally—by departing from [that state] ...". Given players cannot sever links formed in previous periods, a member of the active pair has a very simple action space: announce it wants to form the link with the other member of the active pair or announce that it does not want to do so. A link forms if and only if each member of the active pair announces in favor of link formation. An equilibrium strategy profile is denoted by $\mu^*(g, \eta)$ since it must specify the actions of each active pair at each network.

The intuition of the example is simple. The two largest players, m and l, have a strong enough insider exclusion incentive that they want to become insiders and remain so forever. However, when either m or l is an insider with s then their insider exclusion incentive is weak enough that the link monotonicity effect dominates and the complete network is attained via

a hub–spoke network. With an endogenous order of link formation, m and l become insiders immediately and remain so permanently. But, in the example, m and l are not prepared to wait for the opportunity to be the active pair and so they immediately form a link with s if the opportunity presents itself. As such, the complete network can obtain when the active pair is random and does obtain if m and l are not the first active pair chosen.

For this example, I use the one period payoffs in Table 2. These payoffs satisfy link monotonicity, negative link externalities and each pair of insiders has an insider exclusion incentive. Moreover, the complete network is the efficient network in the sense it delivers the maximum aggregate payoff. Note that Example 2 showed the unique FDNE is that m and l become insiders and remain so permanently. In proceeding, I assume the active pair is randomly chosen from the set of links yet to form. Thus, for example, given the insider–outsider network $g = g_{ij}$, the active pair, denoted η, is randomly selected from $\{\{i, k\}, \{j, k\}\}$.

To begin the backward induction, consider any subgame at a hub-spoke network $g = g_i^H$. Given link monotonicity, the equilibrium strategy profile $\mu^* (g, \eta)$ must specify the spokes form the final link that leads to the complete network.

Now roll back to subgames at insider–outsider networks $g = g_{ij}$ remembering that any hub–spoke network will expand to the complete network in the following period. First, consider $g = g_{ml}$. Given Table 2, remaining insiders is strictly most preferred for m and l if $\delta > \max \left\{ \bar{\delta}_{l,m}^{NE} (\theta), \bar{\delta}_{m,l}^{NE} (\theta) \right\}$. Using Table 2, $\bar{\delta}_{l,m}^{NE} (\theta) = \frac{1}{3} < \bar{\delta}_{m,l}^{NE} (\theta) = \frac{3}{7}$. Thus, let $\delta > \frac{3}{7}$. Given m and l's most preferred outcome is g_{ml} regardless of η, $\mu^* (g, \eta)$ must specify that m and l refuse link formation when either is a member of an active pair at the insider–outsider network $g = g_{ml}$. Then, by construction, there is no profitable joint deviation by an active pair nor a profitable unilateral deviation by a member of an active pair at $g = g_{ml}$.

Second, consider the subgame at the insider–outsider network $g = g_{ij} \neq g_{ml}$. Suppose $\delta < \min \left\{ \bar{\delta}_{i,j}^{NE} (\theta), \bar{\delta}_{j,i}^{NE} (\theta) \right\}$ so that, conditional on being in the active pair, an insider prefers becoming the hub rather than remaining an insider. Does an insider, say i, have an incentive to delay link formation as a member of the active pair? The continuation payoff of link formation is $V_i \left(\langle g_i^H \rangle \right) = v_i \left(g_i^H \right) + \frac{\delta}{1-\delta} v_i (g^c)$. Given $V_i \left(\langle g_i^H \rangle \right) > V_i \left(\langle g_j^H \rangle \right)$, delaying link formation is not optimal if $V_i \left(\langle g_i^H \rangle \right) > v_i (g_{ij}) + \delta V_i \left(\langle g_i^H \rangle \right)$ which holds when $\delta < \bar{\delta}_{i,j}^{NE} (\theta)$. Given Table 2, $\frac{1}{2} = \bar{\delta}_{l,s}^{NE} (\theta) < \bar{\delta}_{s,l}^{NE} (\theta) < \bar{\delta}_{m,s}^{NE} (\theta) < \bar{\delta}_{s,m}^{NE} (\theta)$. So, given the above restriction of $\delta > \frac{3}{7}$, I now restrict attention to $\delta \in \left(\frac{3}{7}, \frac{1}{2} \right)$. Does the outsider have an incentive to delay link formation? Given link monotonicity, the answer can only be yes if the outsider waits to be in the active pair with a larger player. However, letting $\alpha_i > \alpha_j$, waiting is not optimal for k if $v_k \left(g_j^H \right) + \frac{\delta}{1-\delta} v_k (g^c) > \delta \left(v_k \left(g_i^H \right) + \frac{\delta}{1-\delta} v_k (g^c) \right)$ which holds for $k = m, l$ and $j = s$ given Table 2 and $\delta \in \left(\frac{3}{7}, \frac{1}{2} \right)$. Thus, $\mu^* (g, \eta)$ must specify the players in the active pair at

an insider–outsider network $g = g_{ij} \neq g_{ml}$ form the link.

Finally, roll back to the empty network $g = \varnothing$. Given $\mu^* (g, \eta)$, the insider–outsider networks $g = g_{sm}$ and $g = g_{sl}$ will expand to the complete network via a hub–spoke network (with equal probability of either insider becoming the hub) but the insider–outsider network $g = g_{ml}$ will remain forever. Indeed, becoming and remaining insiders forever is strictly most preferred for m and l conditional on $\eta = \{m, l\}$. Thus, at the empty network $g = \varnothing$, $\mu^* (g, \eta)$ must specify that m and l become insiders if $\eta = \{m, l\}$. However, m (or l) faces a trade off when $\eta = \{s, m\}$ (or $\eta = \{s, l\}$). Ideally, m (or l) wants to form a link with l (or m) but it has to wait until $\eta = \{m, l\}$ for that to happen. Waiting is not optimal for m if

$$v_m (g_{sm}) + \delta \left(\frac{1}{2} v_m \left(g_m^H \right) + \frac{1}{2} v_m \left(g_s^H \right) \right) + \frac{\delta^2}{1 - \delta} v_m (g^c) > \delta V_m^{wait} \tag{3}$$

where, given $v_m (\varnothing) = 0$, an upper bound on m's continuation payoff of waiting is

$$\begin{aligned} V_m^{wait} &\equiv \left[\frac{1}{3} \frac{1}{1 - \delta} v_m (g_{ml}) + \frac{2}{3} v_m (\varnothing) \right] \left[1 + \delta \frac{2}{3} + \delta^2 \left(\frac{2}{3} \right)^2 + ... \right] \\ &= \left[\frac{1}{3} \frac{1}{1 - \delta} v_m (g_{ml}) \right] \left[\frac{1}{1 - \frac{2}{3} \delta} \right] . \end{aligned}$$

Using Table 2 and $\delta \in \left(\frac{3}{7}, \frac{1}{2} \right)$, (3) holds. That is, if m is in the active pair with s then m prefers to form the link with s rather than wait for the opportunity to form a link with l even though forming the link with l would be the ideal outcome for m. Using Table 2 reveals an analogous condition holds for l and thus l will also form the link with s when it has the opportunity rather than wait to form the link with m.

Not only do the larger players m and l have a potential incentive to delay link formation when either of them is in the active pair with s. s may also have an incentive to delay link formation. But, given s's ideal outcome is to form a link with l, this can only be true when $\eta = \{s, m\}$. Nevertheless, waiting cannot be optimal for s if

$$v_s (g_{sm}) + \delta \left(\frac{1}{2} v_s \left(g_s^H \right) + \frac{1}{2} v_s \left(g_m^H \right) \right) + \frac{\delta^2}{1 - \delta} v_s (g^c) > \delta V_s^{wait} \tag{4}$$

where, given $v_s (\varnothing) = 0$, an upper bound on s's continuation payoff of waiting is

$$\begin{aligned} V_s^{wait} &\equiv \left[\frac{1}{3} \frac{1}{1 - \delta} v_s (g_{ml}) + \frac{1}{3} V_{sl} + \frac{1}{3} v_s (\varnothing) \right] \left[1 + \delta \frac{1}{3} + \delta^2 \left(\frac{1}{3} \right)^2 + ... \right] \\ &= \left[\frac{1}{3} \frac{1}{1 - \delta} v_s (g_{ml}) + \frac{1}{3} V_{sl} \right] \left[\frac{1}{1 - \frac{1}{3} \delta} \right] \end{aligned}$$

and $V_{sl} \equiv v_s \left(g_{sl} \right) + \delta \left(\frac{1}{2} v_s \left(g_s^H \right) + \frac{1}{2} v_s \left(g_l^H \right) \right) + \frac{\delta^2}{1-\delta} v_s \left(g^c \right)$ is s's continuation payoff conditional on being in the active pair with l. However, (4) holds given Table 2 and $\delta \in \left(\frac{3}{7}, \frac{1}{2} \right)$. Thus, as a member of the active pair with m, s prefers to form the link with m rather than wait for the opportunity to form the link with l even though link formation with l is the ideal outcome for s. Hence, $\mu^* \left(g, \eta \right)$ must specify that link formation occurs at the empty network between members of the active pair when $\eta = \{s, m\}$ or $\eta = \{s, l\}$.

Given $\mu^* \left(g, \eta \right)$, what is the equilibrium path of networks? With a probability of $\frac{1}{3}$, g_{ml} forms and remains forever. Additionally, each of the following four paths of networks arise in equilibrium with a probability of $\frac{1}{6}$: $\left(g_{sm}, g_m^H, g^c \right)$, $\left(g_{sm}, g_s^H, g^c \right)$, $\left(g_{sl}, g_l^H, g^c \right)$, $\left(g_{sl}, g_s^H, g^c \right)$. Thus, the complete network eventually obtains with a probability of $\frac{2}{3}$. Conversely, Example 2 showed the FDNE is g_{ml}. This shows how randomly selecting the active pair leads to the prediction that the complete network will emerge yet endogenously determining the opportunity for link formation predicts the complete network will not emerge.

6   Application to Free Trade Agreements

I now present a very simple model of international trade that has been used frequently in the trade agreements literature and generates a payoff structure satisfying Conditions 1–3 and, except for a very small range of the parameter space, Assumption 1. To this end, a bilateral link between countries i and j is interpreted as a Free Trade Agreement (FTA) between countries i and j. According to WTO rules, FTA formation between countries i and j imposes two requirements: i) removal of tariffs on trade between i and j and ii) i and j do not raise tariffs on other countries. Thus, the complete network g^c can be interpreted as global free trade because all tariffs have been eliminated.

Before moving on, it is important to note that the trade agreements application fits nicely into the dynamic network formation model described above. First, many authors (e.g. Ornelas (2008, p.218) and Ornelas and Liu (2012, p.13)) have argued the binding nature of trade agreements is not only pervasive in the trade agreements literature but entirely realistic. Second, international trade agreements typically take many years to form and thus a period can reasonably be interpreted as the amount of time necessary to complete negotiation of an agreement.[18] Third, as discussed in the introduction, previously formed agreements do not necessarily influence the sequential way that negotiations may commence and/or breakdown within the current period. For example, consider US–Canada–Colombia FTA negotiations. After the 1987 formation of the Canada–US FTA, Canada and the US became insiders. Pre 2002, Colombia was the outsider but Canada began negotiations with

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[18]For example, NAFTA was signed in 1992 despite negotiations dating back to 1986.

Colombia in 2002. Even though US–Colombia negotiations did not begin until 2004, the US–Colombia FTA was signed in 2006 prior to the 2008 signing of the Canada–Colombia FTA. Thus, the trade agreements application fits into a framework where links formed in previous periods are binding but do not affect the way in which coalitions can break into subcoalitions when the single FTA to be formed in the current period is still under negotiation.

The trade model is a three country oligopolistic intra industry model. Krishna (1998), Ornelas (2005b, 2008), Goyal and Joshi (2006), Mukunoki and Tachi (2006) and Saggi and Yildiz (2011) use slight variations of this. Apart from Goyal and Joshi (2006), all of these models are three country models which is typical in the trade agreements literature. Indeed, the dynamic farsighted models of trade agreement formation by Mukunoki and Tachi (2006) and Seidmann (2009) are three country models.

$N = \{i, j, k\}$ denotes the set of countries and a single firm exists in each country. Given the network of FTAs g, $q_{ij}(g)$ denotes country i's exports to country j and $q_{jj}(g)$ denotes the quantity produced by country j for its domestic market. Country j's inverse demand function is $P_j(g) = \alpha_j - Q_j(g)$ where α_j is a measure of country j's market size (the only form of asymmetry) and $P_j(g)$ and $Q_j(g) = \sum_{i \in N} q_{ij}(g)$ denote price and aggregate quantity in country j. For simplicity, a common non prohibitive tariff, $\tau_{ji}(g) = \tau \leq \bar{\tau}$, is implemented by country j on country i if i and j do not have an FTA, i.e. $ij \notin g$.[19] Otherwise $\tau_{ji}(g) = 0$ and, naturally, $\tau_{jj}(g) = 0$.

The common non prohibitive tariff removes the possibility of "tariff complementarity". Tariff complementarity is a common feature of trade agreements models (including the oligopolistic model, e.g. Ornelas (2005a)) whereby FTA formation induces members to lower their tariff on the non-member. Importantly, consideration of this effect can mean an FTA leading to the insider–outsider network imposes a positive link externality rather than a negative link externality. Nevertheless, Conditions 1 and 3 allow for this possibility meaning the general payoff specification underlying Propositions 1 and 2 permit tariff complementarity.

Taking tariffs as given and assuming segmented international markets and a common and constant marginal cost (normalized to zero) then, firm i maximizes profits in country j by solving the following optimization problem (which has the standard form of the aforementioned papers):

$$\max_{q_{ij}} \left[(\alpha_j - Q_j(g)) - \tau_{ji}(g) \right] q_{ij}.$$

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[19]$\bar{\tau}$ is the lowest tariff such that some firm makes (weakly) negative profits by exporting to some country in *some* network structure. Given the optimal quantity formula $q_{ij}^*(g)$ below, the lower bound on $q_{ij}^*(g)$ occurs for $g = g_{jk}$. In this case $q_{ij}^*(g) = \frac{1}{4}[\alpha_j - 3\tau]$ and, thus, $\bar{\tau} = \frac{\alpha_s}{3}$ where $\alpha_s \equiv \min\{\alpha_i, \alpha_j, \alpha_k\}$. Note, $\tau \in \left[\bar{\tau}, \frac{\alpha_s}{2}\right]$ ensures non negative profits in the empty network so each firm makes strictly positive profits in each market in the empty network when $\tau = \bar{\tau}$.

Solving the first order conditions reveals the equilibrium quantity produced by country i and sold in country j's market is

$$q_{ij}^*(g) = \frac{1}{4}\left[\alpha_j + (3 - \eta_j(g))\,\bar{\tau}_j(g) - 4\tau_{ji}(g)\right]$$

where $\bar{\tau}_j(g)$ is the tariff faced by countries who do not have a PTA with country j and $\eta_j(g)$ is the number of countries that face a zero tariff in country j (including country j itself). Note that WTO rules stipulate a country cannot set tariffs that discriminate between non-members (this is the most favored national principle) which implies $\tau_{ji}(g) = \bar{\tau}_j(g)$ for all i such that $ij \notin g$. Country i's equilibrium export profits in country j are $v_{ij}(g) = \left(q_{ij}^*(g)\right)^2$ and the total profits earned by country i from exporting and domestic production are $v_i(g) = \sum_{j \in N} v_{ij}(g)$.

The one period payoffs have simple closed form solutions. Letting $\tilde{\alpha}^2 \equiv \sum_{i \in N} \alpha_i^2$, country i's payoff in the empty network is $v_i(\emptyset) = \frac{1}{16}\left[\tilde{\alpha}^2 + 4\tau\alpha_i - 4\tau(\alpha_j + \alpha_k) + 12\tau^2\right]$ and their payoff in the complete network is $v_i(g^c) = \frac{1}{16}\tilde{\alpha}^2$. For $g = g_{ij}$, an insider's payoff, say i, is $v_i(g_{ij}) = \frac{1}{16}\left[\tilde{\alpha}^2 + 2\tau(\alpha_i + \alpha_j) - 4\tau\alpha_k + 6\tau^2\right]$ and the outsider's payoff is $v_k(g_{ij}) = \frac{1}{16}\left[\tilde{\alpha}^2 + 4\tau\alpha_k - 6\tau(\alpha_i + \alpha_j) + 22\tau^2\right]$. At the hub–spoke network $g = g_i^H$, the hub's payoff is $v_i\left(g_i^H\right) = \frac{1}{16}\left[\tilde{\alpha}^2 + 2\tau(\alpha_j + \alpha_k) + 2\tau^2\right]$, and a spoke's payoff is $v_k\left(g_i^H\right) = \frac{1}{16}\left[\tilde{\alpha}^2 + 2\tau\alpha_k - 6\tau\alpha_j + 10\tau^2\right]$. These payoffs yield significant analytical tractability because the difference between any pair of one period payoffs is independent of $\tilde{\alpha}^2$ and has a common factor of τ.

It is easy to use these one period payoffs to check Conditions 1–3 and Assumption 1. Conditions 1–2 follow immediately and, under the restriction $\alpha_l < \min\{3\alpha_s - 5\tau, 3\alpha_m - 6\tau\}$, Condition 3 holds.[20,21] Additionally, $\bar{\delta}_{l,m}^{NE}(\alpha) < \bar{\delta}_{m,l}^{NE}(\alpha) < \bar{\delta}_{s,l}^{NE}(\alpha)$ ensure parts ii) and iii) of Assumption 1. However, a very minor additional restriction on the parameter space is needed to ensure part i) of Assumption 1.[22]

Given the oligopolistic model of international trade satisfies Conditions 1–3 and Assumption 1, then Propositions 1 and 2 characterize the equilibrium path of FTAs. In particular, these propositions describe how the attainment of global free trade depends crucially on the

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[20] Given $q_{ij}^*(g)$ and $\tau \le \bar{\tau}$, one can verify that the three country joint optimal tariff is zero and thus the complete network is efficient. To be clear, the restriction $\tau \le \bar{\tau}$ together with the non–monotonicity of the world profit function drive this result. Given $v_i(\emptyset, \tau) = \frac{1}{16}\left[\sum_{j \in N}(\alpha_j)^2 + 4\tau(\alpha_i - \alpha_j - \alpha_k) + 12\tau^2\right]$, world profits are $v_W(\emptyset, \tau) \equiv \frac{1}{16}\left[3\sum_{j \in N}(\alpha_j)^2 - 4\tau\sum_{j \in N}\alpha_j + 36\tau^2\right]$ which is non–monotonic in τ: $\frac{\partial v_W(\emptyset, \tau)}{\partial \tau} \gtrless 0$ when $\tau \gtrless \underline{\tau} \equiv \frac{\sum_{j \in N}\alpha_j}{18}$. Thus, $v_W(\emptyset, \tau)$ is minimized at $\underline{\tau}$. Letting $\alpha_s \equiv \min\{\alpha_i, \alpha_j, \alpha_k\}$, $v_W(\emptyset, 0) \ge v_W(\emptyset, \bar{\tau})$ reduces to $\sum_{j \in N}\alpha_j \ge 3\alpha_s$ which is true by definition. Hence, $v_W(\emptyset, 0) \ge v_W(\emptyset, \tau)$ for any $\tau \le \bar{\tau}$.

[21] $\alpha_l < 3\alpha_s - 5\tau$ ensures $v_l(g^c) > v_l\left(g_m^H\right)$ and $\alpha_l < 3\alpha_m - 6\tau$ ensures $v_l\left(g_m^H\right) > v_l(g_{sm})$.

[22] When $\frac{\tau}{\alpha_s}$ and $\frac{\alpha_m}{\alpha_s}$ are very low, the last preference ordering in part i) is violated when $\frac{\alpha_l}{\alpha_s}$ is very high. In this cases, the FDNE becomes more tedious but the main results remain.

insider exclusion incentive even if global free trade maximizes the aggregate payoff of the governments.

7   Discussion

I now discuss how two important assumptions that helped maintain analytical tractability of the model can be relaxed without affecting the key insights of the model.

First, I assumed that at most one link could form in any given period. This ruled out a direct move to the complete network from the empty and insider–outsider networks and also a direct move to the hub–spoke network from the empty network. Would the importance of the insider exclusion incentive and the related result that the FDNE is merely a permanent insider–outsider network diminish if such moves were possible?

It is straightforward to show the answer is no when allowing a direct move to the complete network in any period. The basic intuition is as follows. Suppose the action space is expanded to include an announcement c indicating a player announces a move to the complete network and that such a move takes place if and only if all players announce c.[23] Further, suppose the No Exclusion condition holds for a pair of insiders in an insider–outsider network and becoming insiders is strictly most preferred for each insider in the empty network so that the unique FDNE is that these players become and permanently remain insiders. Despite the announcement c, remaining insiders at the insider–outsider network and becoming insiders at the empty network is still strictly most preferred for each such insider. Thus, an appropriate extension of Lemma 1 establishes that the FDNE remains unchanged.

The answer is also no when allowing a direct move to the hub–spoke network from the empty network. The permanence of an insider–outsider network as an FDNE arises when $\delta > \max\left\{\bar{\delta}_{m,l}^{NE}(\theta), \bar{\delta}_{l,m}^{NE}(\theta)\right\}$. However, given a pair of insiders, say m and l, each insider prefers remaining an insider forever rather than moving directly to the hub–spoke if and only if $\frac{1}{1-\delta}v_i(g_{ml}) > v_i\left(g_i^H\right) + \frac{\delta}{1-\delta}v_i(g^c)$ for $i = m, l$ which holds if and only if $\delta > \max\left\{\bar{\delta}_{m,l}^{NE}(\theta), \bar{\delta}_{l,m}^{NE}(\theta)\right\}$. Thus, the importance of the insider exclusion incentive and the possibility of an FDNE not leading to the complete network does not depend on the assumption that at most one link can form in any given period.

A second important assumption of the model is there are only three players. Indeed, using an evolutionary game theoretic model, Zhang et al. (2014) have recently showed that whether the complete network obtains can hinge on whether there are exactly three players.

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[23]Adding the announcement c to the action space means the action space for player i at the empty network is $A_i(\emptyset) = \{\phi, j, k, c\}$. This is effectively equivalent to allowing each player to announce multiple links in a given period, i.e $A_i(\emptyset) = \{\phi, j, k, \{j, k\}\}$, *if* it is also assumed that refusal by any player to participate in a proposed link vetoes *all* proposed links.

To see this is not the case in my model, consider a model with four symmetric players i, j, k, l. Moreover, assume each player can only form one link per period but multiple links can form in a given period. Consider an "insider–outsider" network $g_1 = (ij, kl)$ and assume that link monotonicity holds so that the "hub–spoke" network $g_2 = (ij, kl, ik, jl)$ will expand to the complete network in the following period.[24] Then, given the network g_1, i prefers to maintain the status quo g_1 permanently rather than form a link with k if $\frac{1}{1-\delta} v_i(g_1) > v_i(g_2) + \frac{\delta}{1-\delta} v_i(g^c)$ which is analogous to violation of the earlier No Exclusion condition. Indeed, imposing $v_i(g_1) > v_i(g^c)$ would be analogous to the insider exclusion incentive earlier. Thus, even in a four player model, a type of No Exclusion condition and insider exclusion incentive will still drive whether the complete network obtains.

8 Conclusion

It is well known that the equilibrium of sequential move games can be very sensitive to the protocol governing which players can act when (e.g. Ray and Vohra (1997), Jackson (2008)). In the context of dynamic network formation games, the protocol governs which pair of players have an opportunity to form a link in any given period. I develop a three player dynamic network formation model where players are farsighted and the identity of the players who form a link in any given period depends endogenously on player characteristics. I do this by embedding a simultaneous move announcement game in each period of the dynamic game where each player announces who it wants to form a link with. In doing so, I define a new equilibrium concept called Farsighted Dynamic Network Equilibrium.

Of course, endogenizing which players form a link in a given period is not necessarily important in and of itself. However, I show that it can lead to substantively different predictions regarding the equilibrium path of network formation and, in particular, attainment of the complete network relative to models with an exogenous protocol. This difference arises even if link formation is always myopically attractive and the complete network is efficient. Specifically, the presence of an insider exclusion incentive can mean the complete network fails to obtain when the identity of which players form a link in a given period is endogenous. In this case, the "most attractive" players become insiders and remain so forever. However, when the pair of players who have the opportunity to form a link in a given period is randomly chosen, the chosen players may not want to wait for their ideal partner but instead take the link formation opportunity currently available. This has significant implications

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[24]g_1 is a type of insider–outsider network in the sense that (for example) i and j are insiders with respect to each other but outsiders with respect to j and k. g_2 is a hub–spoke network in the sense that (for example) i is a hub between j and k which makes j and k spokes with respect to i.

for the equilibrium path of network formation because, when players are asymmetric, which players become insiders affects the strength of the insider exclusion incentive and hence whether an insider–outsider network eventually leads to the complete network. Thus, endogenizing which players form a link in a given period substantively affects the equilibrium path of network formation.

Extending a framework that endogenizes which players form a link in a given period beyond a model of three players remains an avenue for future research. On the surface, the analytical tractability of the three player model relied on the complete network being obtained in at most three periods. However, the complete network can also be obtained in three periods with four players if the one link per period assumption is relaxed so that *each player* can form one link per period. As discussed in Section 7, the idea of an insider exclusion incentive can still drive the equilibrium path of network formation in a model with four players. This observation provides motivation and hope for extension to a setting with many players.

Appendix

A   Proofs

PROOF OF LEMMA 2

Note, for any hub–spoke network $g = g_i^H$, $G(N, g) = g^c$ by link monotonicity and Lemma 1.

Let $\delta > \bar{\delta}^{NE}(\theta)$. Then, $\langle g_{ij} \rangle = g_{ij}$ is strictly most preferred for i and j. Thus, by Lemma 1, $g_{ij} = G(N, g_{ij})$.

Now let $\delta < \bar{\delta}^{NE}(\theta)$. Lemma 1 implies $g_i^H \in G(N, g_{ij})$ and $g_j^H \in G(N, g_{ij})$ because g_i^H is most preferred for i and k while g_j^H is most preferred for j and k. Moreover, $g_{ij} \notin G(N, g_{ij})$ given $S = ik$ have a self enforcing deviation from g_{ij} to $g_i^H \in G(P_{ik}, g_{ij})$. $g_i^H \in G(P_{ik}, g_{ij})$ follows given g_i^H is most preferred for i and k. Hence, $G(N, g_{ij}) = \{g_i^H, g_j^H\}$.■

PROOF OF PROPOSITION 1

The proof proceeds by backward induction. Consider the subgame at a hub–spoke network $g = g_i^H$. Given link monotonicity, Lemma 1 implies $G(N, g_i^H) = g^c$. Now consider the subgame at an insider–outsider network $g = g_{ij}$. By Lemma 2, $G(N, g_{ij}) = \{g_i^H, g_j^H\}$ if $\delta < \bar{\delta}^{NE}(\theta)$ and $G(N, g_{ij}) = g_{ij}$ if $\delta > \bar{\delta}^{NE}(\theta)$.

Finally, consider the subgame at the empty network $g = \emptyset$. Suppose $\delta > \bar{\delta}^{NE}(\theta)$. Given g_{ij} is most preferred for any $i, j \in N$, two observations establish the proof. First, Lemma 1 implies $G(N, \emptyset) \supseteq \Omega^{I-O}$ where $\Omega^{I-O} \equiv \{g_{ij}, g_{ik}, g_{jk}\}$. Second, $g_{ij} \in G(P_{ij}, \emptyset)$ and thus

the self enforcing deviation by $S = ij$ from $\emptyset$ to $g_{ij} \in G(P_{ij}, \emptyset)$ implies $\emptyset \notin G(N, \emptyset)$. Therefore, $G(N, \emptyset) = \Omega^{I-O}$ and, given $G(N, g_{ij}) = g_{ij}$, the set of FDNE is Ω^{I-O}.

Now suppose $\delta < \bar{\delta}^{NE}(\theta)$. Thus, for any $g \in \Omega^{I-O}$, Lemma 2 implies g^c eventually obtains via a hub–spoke network. Without loss of generality, suppose each country is the hub on one such path. Given Conditions 1 and 2, $\left(g_{ij}, g_i^H, g^c\right) \succ_i \left(g_{ij}, g_j^H, g^c\right) \succ_i \left(g_{jk}, g_j^H, g^c\right)$ and $\left(g_{ij}, g_j^H, g^c\right) \succ_i \langle \emptyset \rangle$. Thus, Lemma 3 applies and says the EBA networks are $\underset{S \subset N}{\cup} G(P_S, \emptyset) = \Omega^{I-O}$. In turn, the set of FDNE is any path of bilateral links leading to g^c.∎

PROOF OF LEMMA 4

Note, for any hub–spoke network $g = g_i^H$, $G(N, g) = g^c$ by link monotonicity and Lemma 1. Throughout the proof let $\alpha_i > \alpha_j$.

If $\delta > \max\left\{\bar{\delta}_{i,j}^{NE}(\theta), \bar{\delta}_{j,i}^{NE}(\theta)\right\}$, (2) and Condition 3 imply g_{ij} is strictly most preferred for i and j. Thus, by Lemma 1, $g_{ij} = G(N, g_{ij})$.

If $\delta < \bar{\delta}_{i,j}^{NE}(\theta)$, (2) and Condition 3 imply g_i^H is strictly most preferred for i and k. Thus, by Lemma 1, $g_i^H = G(N, g_{ij})$.

Now consider the final case where $\delta \in \left[\bar{\delta}_{i,j}^{NE}(\theta), \bar{\delta}_{j,i}^{NE}(\theta)\right)$ noting that $\gamma(P^*, g_{ij}) \supseteq \left\{g_{ij}, g_j^H\right\}$ and $\gamma(P^*, g_{ij}) = g_j^H$ if $a_j(g_{ij}) = k$ or $a_k(g_{ij}) = j$. Four observations establish the proof. First, $g_j^H \in G(P_{jk}, g_{ij})$ because i) $g_j^H \in \gamma(P_{jk}, g_{ij})$, ii) g_j^H is most preferred for j, and iii) $g_{ij} \in \gamma(P^*, g_{ij})$ deters k's to $a_k(g_{ij}) = i$. Second, $g_j^H \notin G(P_{ik}, g_{ij})$ because $\langle g_i^H \rangle \succ_{ik} \langle g_j^H \rangle$. Third, $g_i^H \in G(P_{ik}, g_{ij})$ because i) $g_i^H \in \gamma(P_{ik}, g_{ij})$, ii) g_i^H is strictly most preferred by k, and iii) $g_j^H \in \gamma(P^*, g_{ij})$ deters any deviation by i. Fourth, $g_{ij} \notin G(P_S, g_{ij})$ for any $S \subset N$: j (or k) has a self enforcing deviation to $a_j(g_{ij}) = k$ (or $a_k(g_{ij}) = j$) and $g_j^H = \gamma(P^*, g_{ij})$.

The first observation implies $g_{ij} \notin G(N, g_{ij})$: $S = jk$ have a self enforcing deviation to $g_j^H \in G(P_{jk}, g_{ij})$. The third observation implies $g_j^H \notin G(N, g_{ij})$: $S = ik$ have a self enforcing deviation to $g_i^H \in G(P_{ik}, g_{ij})$. To establish $G(N, g_{ij}) = g_i^H$, note that the potentially profitable deviations from g_i^H are i) j to g_j^H and ii) i and/or j to g_{ij}. However, these respective deviations are not self enforcing because of, respectively, observations i) two and ii) one and four. ∎

PROOF OF PROPOSITION 2

The proof proceeds by backward induction. Consider the subgame at a hub–spoke network $g = g_i^H$. By link monotonicity and Lemma 1, $G(N, g_i^H) = g^c$. Now consider the subgame at an insider–outsider network $g = g_{ij}$. Let $\alpha_i > \alpha_j$. For $\delta > \max\left\{\bar{\delta}_{i,j}^{NE}(\theta), \bar{\delta}_{j,i}^{NE}(\theta)\right\}$, Lemma 4 implies $g_{ij} = G(N, g_{ij})$. For $\delta < \max\left\{\bar{\delta}_{i,j}^{NE}(\theta), \bar{\delta}_{j,i}^{NE}(\theta)\right\}$, Lemma 4 implies $G(N, g_{ij}) = g_i^H$.

For later purposes note that $\delta < \max\left\{\bar{\delta}_{m,l}^{NE}(\theta), \bar{\delta}_{l,m}^{NE}(\theta)\right\}$ implies not only $g_l^H \in G(N, g_{ml})$ but also $g_l^H \in G(N, g_{sl})$ and, unless $\bar{\delta}_{m,l}^{NE}(\theta) < \bar{\delta}_{l,m}^{NE}(\theta)$, $g_m^H \in G(N, g_{sm})$. The latter follows

because Condition 3 implies $\bar{\delta}_{m,l}^{NE}(\theta) < \bar{\delta}_{m,s}^{NE}(\theta)$. The former follows because, using Condition 3, either i) $\delta < \bar{\delta}_{l,s}^{NE}(\theta)$ or ii) $\delta \in \left(\bar{\delta}_{l,s}^{NE}(\theta), \bar{\delta}_{m,s}^{NE}(\theta)\right)$ which, via part ii) of Assumption 1, implies $\delta < \bar{\delta}_{m,l}^{NE}(\theta) < \bar{\delta}_{s,l}^{NE}(\theta)$.

Now consider the subgame at the empty network $g = \varnothing$. First, let $\delta > \max\left\{\bar{\delta}_{m,l}^{NE}(\theta), \bar{\delta}_{l,m}^{NE}(\theta)\right\}$. Condition 3 implies g_{ml} is strictly most preferred for m and l regardless of $G(N, g_{sm})$ and $G(N, g_{sl})$. Hence, Lemma 1 implies $g_{ml} = G(N, \varnothing)$ and thus g_{ml} is the unique FDNE.

Second, let $\delta < \max\left\{\bar{\delta}_{m,l}^{NE}(\theta), \bar{\delta}_{l,m}^{NE}(\theta)\right\}$. Suppose Condition 4 holds. For $\delta < \bar{\delta}^m(\theta)$, Condition 3 implies g_{ml} is strictly most preferred for m and l. So Lemma 1 implies $g_{ml} = G(N, \varnothing)$ and $\left(g_{ml}, g_l^H, g^c\right)$ is the unique FDNE. For $\delta > \bar{\delta}^m(\theta)$, the conditions of Lemma 3 are satisfied given Condition 3 and part i) of Assumption 1. Hence, Ω^{I-O} are the EBA networks and thus $\hat{\Omega}^c$ are the FDNE. Now suppose Condition 3 fails. Given Condition 3 implies $\bar{\delta}_{m,l}^{NE}(\theta) < \bar{\delta}_{m,s}^{NE}(\theta)$ then $\bar{\delta}_{m,l}^{NE}(\theta) < \bar{\delta}_{l,m}^{NE}(\theta)$ must hold. Thus, part iii) of Assumption 1 implies $\langle g_{sm} \rangle = g_{sm}$ is strictly most preferred for s and m when $\delta > \bar{\delta}^m(\theta)$ and Condition 4 fails. Hence, Lemma 1 implies $g_{sm} = G(N, \varnothing)$ and, in turn, g_{sm} is the unique FDNE.∎

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