An Amplification Mechanism in a Model of Energy

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Abstract

This paper investigates a propagation mechanism of the energy price shock in a model where capital utilization is associated with costly energy consumption. Endogenous depreciation is an important element of the model, as it has been shown to produce a significant negative effect of energy prices on output. I show that the amplifying effect of endogenous depreciation is determined by the choice of the functional form and calibration strategy for the energy cost function. My estimates of the energy cost function allow to conclude that the energy price shock has only a moderate effect on output in this model, while endogenous depreciation mitigates rather than amplifies the effect of the energy price shock.

JEL classifications: E32, Q43
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1 Introduction

Since the work of Kim and Loungani (1992), macroeconomic models often introduce energy as an added factor of production. Because the estimated share of energy as a constituent of total U.S. gross domestic product (GDP) is very small, such models have experienced criticism based on their inability to explain the sizeable effects of energy price increases on economic activity observed in the 1970s (Rotemberg and Woodford (1996)). This criticism has, in turn, encouraged the development of new approaches to modeling the energy sector, including Finn's (2000) suggestion to exploit the idea of the technological relationship between energy and capital utilization, recognizing that energy is necessary to produce the service flow from capital (Throughout the article, I refer to Finn's framework as the energy-as-cost, or EC model.) Another crucial element of the EC model is its assumption of a variable capital depreciation rate, which is tied to the degree of capital utilization. Finn (2000) demonstrates that the EC model with endogenous depreciation can produce a sizeable output drop after a rise in energy prices.[1]

Economists generally agree that endogenous depreciation tied to variable capital utilization creates a strong amplifying effect in the propagation of different shocks in the economy. For example, Burnside and Eichenbaum (1996) document this result for the neutral

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[1]The contribution of variable depreciation in the propagation of energy price shocks is easy to recognize in Finn's model, because without variable depreciation, that model is observationally equivalent to a more standard energy-in-production model.

technology and government spending shocks; Finn (2000), Leduc and Sill (2004) and Aguiar-Conraria and Wen (2007) emphasize the amplifying effect of endogenous depreciation in their models with exogenous energy prices. In this paper, I argue that endogenous depreciation in the EC model may not necessarily serve as an amplifier for the effect of the energy price shock. In Section 2, I demonstrate that the choice of the functional form for the capital's energy cost function is important to generate the amplified effect on output. This sensitivity of the amplifying effect to the functional form reveals the need for further empirical evidence regarding the energy costs function. In Section 3, I estimate the elasticity of the energy cost function with respect to capital utilization using the data on energy use, capital stock and capacity utilization. I find that this elasticity is generally smaller than 1, which is impossible to justify assuming a standard, constant elasticity function. I also demonstrate that the elasticity of the marginal energy costs with respect to capital utilization is an important parameter to determine the quantitative response of the economy to the energy price shock. The estimates of this elasticity are in the range of 7 and 10. I calibrate the model using the empirically estimated energy cost function and conclude that the effect of the energy price shock on output is only moderate in this model.

2 Simple Model and Analysis

Consider a standard real business cycle model modified to include the energy sector in a way similar to Finn (2000). The representative household's utility is a standard concave function of consumption c_t and leisure $1 - h_t$

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - h_t), \tag{1}$$

where $\beta \in [0, 1)$ is the intertemporal discount factor and $\mathcal{E}_t$ denotes conditional expectations at time t. Households invest to build future capital k_{t+1} according to the dynamic law

$$k_{t+1} = (1 - \delta(u_t))k_t + i_t, \tag{2}$$

where i_t is investment and $\delta(u_t)$ is the rate of capital depreciation, which varies with the intensity of capital utilization ($\delta'(u_t), \delta''(u_t) > 0$). As in Finn (2000), energy e_t is necessary to provide capital services for final goods- producing firms. The quantity of energy needed (e_t) is determined by the following technological constraint:

$$e_t \geq a(u_t)k_t. \tag{3}$$

That is, the required minimum quantity of energy is proportional to the existing capital stock and depends on the intensity of capital use u_t. The necessary ratio of energy to capital $a(u_t)$ varies with the intensity of capital utilization u_t. The energy-to-capital ratio $a(u_t)$ satisfies the properties of a cost function: $a'(\cdot)$, $a''(\cdot) > 0$. In what follows, I refer to $a(u_t)$ as the energy cost function.

The representative household's budget constraint can now be written as

$$c_t + p_{e,t}e_t + i_t = w_t h_t + r_t^k u_t k_t,$$

where $p_{e,t}$ is the real price of energy, w_t is the wage rate and r_t^k is the rental rate of capital services. Production of final goods y_t involves capital services, $u_t k_t$ and labor h_t according to the constant returns to scale production technology:

$$y_t \leq f(u_t k_t, h_t), \tag{4}$$

where $f(\cdot,\cdot)$ is increasing and concave. I assume that all energy is produced within the economy and that the cost of extraction is zero,[2] which gives rise to the following resource constraint:

$$c_t + i_t \leq y_t. \tag{5}$$

Finally, the real price of energy follows an exogenous stochastic AR(1) process,

$$\log\left(\frac{p_{e,t}}{p_e}\right) = \rho_{p_e}\log\left(\frac{p_{e,t-1}}{p_e}\right) + \epsilon_t, \tag{6}$$

where p_e is the steady-state price of energy, $\rho_{p_e} \in [0,1)$ is the autoregressive parameter and ϵ_t is $i.i.d(0, \sigma_{p_e})$, with $\sigma_{p_e} > 0$.

The set of equilibrium conditions includes the resource constraint (5), the energy requirement (3), the ratio of intratemporal optimality conditions for consumption c_t and labor h_t,

$$-\frac{U_2(c_t, 1-h_t)}{U_1(c_t, 1-h_t)} = f_2(u_t k_t, h_t), \tag{7}$$

the intertemporal optimality condition for the choice of future capital k_{t+1}:

$$\begin{aligned} U_1(c_t, 1-h_t) = \beta\mathcal{E}_t U_1(c_{t+1}, 1-h_{t+1})[f_1(u_{t+1}k_{t+1}, h_{t+1})u_{t+1}+ \\ (1-\delta(u_{t+1})) - p_{e,t+1}a(u_{t+1})] \end{aligned} \tag{8}$$

and the optimal choice of capital utilization u_t,

$$f_1(u_t k_t, h_t) = \delta'(u_t) + p_{e,t}a'(u_t). \tag{9}$$

To understand the propagation mechanism of energy prices on output in this simple model and the role of the endogenous rate of depreciation, it is convenient to start with Figure 1. This figure depicts the response of output and capital utilization to a 10% increase in the energy price in the two versions of the model. The solid blue line represents the responses in the model with variable utilization rate and the dashed red line shows the responses in the model when the depreciation rate is constant ($\delta'(u_t) = 0$). The models' parameters are calibrated in a standard way, as reported in Table 1, and the energy cost function assumes

[2]This assumption helps to eliminate the negative wealth effect from the rise in the price of energy and allows to focus on the transmission of the energy price shock through the capital utilization margin.

a constant elasticity form, as in Finn (2000):

$$a(u_t) = \frac{\upsilon_I}{\upsilon_{II}} u_t^{\upsilon_{II}}, \tag{10}$$

where $\upsilon_I > 0$ and $\upsilon_{II} > 1$. In the figure, quarters are shown along the horizontal axis and percentage deviations in output and utilization from their steady state values are presented along the vertical axis. The impulse responses in the figure reveal that, when energy prices increase, output and capital utilization decrease. The maximum drop in output is observed on impact in both models. Output falls as much as 1.5% in a model with endogenous depreciation, while it decreases by only 0.7% if the depreciation rate is constant. Therefore, the figure demonstrates that the presence of endogenous depreciation is associated with an amplified effect of the energy shock.

For further analysis, it is useful to decompose the effect of the shock into a direct and indirect effects. The direct effect is associated with the impact of the energy price only, assuming all other endogenous variables stay unchanged. The direct effect propagates in the economy through the optimality condition (9), which equalizes the marginal product of capital services with their marginal costs.[3] Higher energy prices increase the marginal costs of capital due to energy use, resulting in the inefficiently large level of capital utilization. In addition, if the energy price shock is long-lasting, then higher future energy prices will decrease the return on capital according to Equation (8), resulting in a drop in investment. The indirect effect of the energy price shock is due to the adjustment of endogenous variables to an increase in $p_{e,t}$. For example, an increase in the energy price causes a drop in the capital utilization rate, which at least to some extent, lowers the marginal cost of rented capital.

The final effect of the shock on the economy is determined by the strength of the direct and indirect impact of the energy price increase. To evaluate the quantitative contribution of each effect, I use the first-order log-linear approximation to Equation (9):

$$\epsilon_{f_1}^{uk/h} (\frac{\hat{uk}}{h})_t = \gamma \hat{u}_t + \omega \hat{p}_{e,t}, \tag{11}$$

where $\hat{x}_t = log(x_t/x)$ is the log deviation of a variable x_t from its steady state value x, $\epsilon_x^y = \partial log(x)/\partial log(y)$ is the elasticity of x with respect to y evaluated at a steady state, $\omega = p_e a'(u)/(p_e a'(u) + \delta'(u))$, with $\omega \in [0, 1]$, can be interpreted as the strength of the direct effect of the energy shock and $\gamma = (1-\omega)\epsilon_{\delta'}^u + \omega \epsilon_{a'}^u > 0$ measures the indirect effect of the shock. Notice that the larger ω implies a stronger negative effect from higher energy prices, while larger values of γ, on the other hand, ensure that the mitigating effect of lowering utilization rates is more pronounced.[4] It is important to note that both ω and γ differ across the versions of the models with and without variable depreciation. For example, if the rate of capital depreciation is constant, then $\omega = 1$; otherwise, $\omega < 1$. This implies that the direct negative effect of energy prices on capital cost is quantitatively smaller in the presence of endogenous depreciation. While this finding may seem surprising in light of the evidence presented in Figure 1, it simply suggests that the indirect effect, measured by γ

[3]The marginal costs consist of marginal depreciation and energy costs of capital use.

[4]This is so because the larger γ is, the less the utilization rate needs to fall to counter the direct effect of higher energy prices.

must also be different in the two versions of the model. Notice that γ is a weighted average of elasticities of the marginal costs components with respect to capital utilization, with the weights determined by the relative importance of depreciation or energy in marginal capital costs. Because these quantities are determined by the slope and curvature of $a(u_t)$ and $\delta(u_t)$, how these functions are calibrated is important for the resulting quantitative response of output to the energy price shock.

To some extent, parametrization of the cost functions is dictated by steady state equilibrium conditions. For example, one may notice that the following steady state relationship must be satisfied for the energy cost functions in the two versions of the model:

$$p_e a'_{cons}(u) = p_e a'_{var}(u) + \delta'(u), \tag{12}$$

where subscripts *cons* and *var* help to distinguish between the constant and the variable depreciation rate models, respectively, and the no-subscript variables denote their steady-state values.[5] Because $\delta'(u) > 0$, Equation (12) implies

$$a'_{var}(u) < a'_{cons}(u). \tag{13}$$

In addition to the steady state restrictions, the form of the energy cost function may also be important in determining the direct and indirect effects of the energy shock, especially if it restricts parameters determining the slope and curvature of the cost function. For example, when the energy cost function takes the constant elasticity form, as in Equation (10), then $\epsilon^u_a = \upsilon_{II}$ and $\epsilon^u_{a'} = \upsilon_{II} - 1$. Therefore, only one parameter, υ_{II}, determines the two elasticities. Notice that because $a'(u) = \epsilon^u_a(u/a(u))$, and because u and $a(u) = e/k$ are usually calibrated using the data statistics, elasticity ϵ^u_a defines the slope of the energy costs function. Therefore, the choice of υ_{II} determines both the slope $a'(u)$ and the elasticity of marginal costs $\epsilon^u_{a'}$. One implication of this restriction is that, in addition to marginal energy costs, the elasticity $\epsilon^u_{a'}$ has to differ across the constant and variable depreciation versions of the model as well. More specifically, due to relationship (13), elasticity $\epsilon^u_{a'}$ is smaller in the version of the model with variable depreciation,[6] and the same is true regarding parameter γ in Formula (11), which is smaller in the model with endogenous depreciation. Consequently, it turns out that when the energy cost is the constant elasticity function, energy shocks are more costly, and output falls more in the presence of variable depreciation than it does in the constant depreciation version.

To break the relationship between elasticities ϵ^u_a and $\epsilon^u_{a'}$ imposed by the constant elasticity function, one may consider the following modification of the energy cost function:

$$a(u_t) = \upsilon_0 + \frac{\upsilon_I}{\upsilon_{II}} u_t^{\upsilon_{II}}, \tag{14}$$

where υ_0 represents the energy expenditures required to maintain a certain capital level, even when capital is not in use. While as previously $\epsilon^u_{a'} = \upsilon_{II} - 1$, the elasticity ϵ^u_a is now

[5] Appendix provides more details about the calibration strategy.

[6] For example, in Finn (2000), the ratio $a''_{cons}/a'_{cons} \approx 7$ and $a''_{var}/a'_{var} \approx 0.6$. Thus, $(\epsilon^u_{a'})^{cons}$ is more than 10 times greater than $(\epsilon^u_{a'})^{var}$.

determined by both parameters υ_0 and υ_{II}, as follows

$$\epsilon_a^u = \upsilon_{II}\left[\frac{E/K - \upsilon_0}{(E/K)}\right].$$

Therefore, ϵ_a^u and $\epsilon_{a'}^u$ are determined by two parameters, and can be calibrated independently. I use the specification in Equation (14) to evaluate the effect of the elasticity of marginal energy costs $\epsilon_{a'}^u$ on the implied dynamics of the model following an energy price shock. Figure 2 produces the contemporaneous impulse response of output to the energy price shock of 10% for a variety of values of $\epsilon_{a'}^u$, from 0.3 to 10, with a step of 0.2. The output drop is presented in percentages from the steady state along the vertical axis of the graph, and $\epsilon_{a'}^u$ is shown on the horizontal axis. The solid blue line in the figure depicts the response in the model with endogenous depreciation, while the dashed green line represents the response in the model, in which the depreciation rate is constant. In both cases, a negative relationship is observed between the elasticity $\epsilon_{a'}^u$ and the size of the negative response of output to the shock. This agrees well with the intuition described above: larger $\epsilon_{a'}^u$ implies larger γ in Formula (11). Therefore, a smaller adjustment of the capital utilization rate is sufficient to counter the negative direct effect of the energy price shock, which makes the shock less expensive in terms of output.

Point A in Figure 2 corresponds to the response of output in the baseline calibration with constant elasticity specification and endogenous depreciation. As follows from Table 1, this model imposes $\epsilon_{a'}^u = 0.66$. Output drops by 1.48% in this model after the energy shock. Point B shows the response of output in the corresponding model with constant depreciation. The constant elasticity specification of the energy cost implies the elasticity $\epsilon_{a'}^u$ is 5.97. The resulting drop of output on impact is approximately 0.74%. If $\epsilon_{a'}^u$ were 0.66 instead of 5.97 in the model where the depreciation rate is constant, then output would drop by approximately 2.81%, which is almost four times larger than the output drop at point B. This situation is represented by point C in the figure, which corresponds to the EC model with constant depreciation and the implied energy cost function determined by Equation 14. Comparing the response of output at points A and C, one may conclude that if the elasticity of the marginal energy cost did not change when producing output responses in Figure 1, then endogenous depreciation would actually mitigate the effect of the energy shock, and would result in an output drop twice as small as that in the model with constant depreciation rate.

The response of output according to the blue line in Figure 2 assumes the elasticity ϵ_a^u is the same as in the baseline calibration: $\epsilon_a^u = 1.66$. I now evaluate the role the elasticity of the energy cost function ϵ_a^u plays in determining the quantitative response of output to the energy price shock. This can be done by adopting a more general, quadratic functional form for the energy cost function:

$$a(u_t) = a_0 + a_I(u_t - u) + a_{II}(u_t - u)^2. \tag{15}$$

The benefit of this function is that the marginal costs in the steady state $a'(u) = a_I$, and are independent of either a_0 or a_{II}. As a result, the steady state does not restrict the parameters of the slope and curvature of this function, allowing to choose different values for ϵ_a^u and $\epsilon_{a'}^u$.[7] To see how the choice of ϵ_a^u affects the predicted response of output to the shock, I replicate the blue curve in Figure 2 for different values of ϵ_a^u: 0.5, 1, 1.5, 2 and 2.5. The thin grey curves in Figure 2 show the resulting responses. The grey curve on top of the figure corresponds to the calibration $\epsilon_a^u = 0.5$, and the lowest curve assumes $\epsilon_a^u = 2.5$. Such positioning of output response curves is very intuitive: the larger the elasticity ϵ_a^u, the closer ω is to 1, and the more substantial is the direct effect of the energy price shock on the economy, resulting in a greater output drop. Therefore, the bigger ϵ_a^u, the lower the output response curve in this graph.

To summarize, Figure 2 reveals that output response to energy shocks in the model is extremely sensitive to the choice of the functional form and the calibration of the energy cost function. Depending on what the elasticities of the energy cost function $a(u)$ are, the output drop immediately after a 10% energy price increase may vary anywhere between 0.5 and 4%. Additional evidence on the relationship between energy-capital ratio and capital utilization is therefore needed to help identify the effect from energy price shocks in the EC model.

3 Quantitative Evaluation of the Energy Cost Function

The empirical literature does not provide much evidence on the elasticities of the energy cost function. Finn (2000) was the first to model the relationship of energy-to -capital ratio and capital utilization. However, Finn relies on the steady state equilibrium conditions to calibrate this ratio. In her paper, the elasticity of the marginal energy cost is 0.64. Leduc and Sill (2004) also calibrate these parameters to match the energy-capital ratio so that the elasticity of marginal energy cost is 0.94. Both studies rely on the constant elasticity specification for the energy cost function.

There is a growing amount of literature in macroeconomics that estimates parameters of the capital cost function within medium-scale DSGE models, although the cost of capital use is not necessarily tied to energy use. Such studies usually conclude that the parameters of the capital cost function are small and difficult to identify. For example, Christiano, Eichenbaum, and Evans (2005) fix the elasticity of marginal cost at a small number due to identification difficulties. Kormilitsina (2011) estimates the parameters of a quadratic energy cost function modeled in a DSGE framework by matching impulse responses, and finds that the energy cost function is virtually flat, with the reported elasticity of marginal energy cost being very small at 0.03 and not significantly different from zero. In Christiano, Eichenbaum, and Trabandt (2015), the estimate of the cost function for the adjustment of capital utilization is $\epsilon_{a'}^u = 0.053$.[8] Therefore, the existing evidence suggests that for the cost

[7]Notice that because only the first two derivatives of $a(u_t)$ are important for the linear approximate solution of the model, the power specification in Equation 14 is equivalent in terms of impulse responses to the quadratic form, as in Equation (15) when $a_0 = \alpha_0 + \frac{\upsilon_I}{\upsilon_{II}} u^{\upsilon_{II}}$, $a_I = \upsilon_I u^{\upsilon_{II}-1}$ and $a_{II} = 0.5\upsilon_I(\upsilon_I - 1)u^{\upsilon_{II}-2}$. For a power specification of $a(u_t)$, the steady state requires that at least one of these parameters to be fixed and therefore cannot be varied.

[8]The slope of the function is similar in these studies, as it is pinned down by the steady state equilibrium conditions.

function is approximately linear and almost flat. However, the results in these studies may be biased, as they do not use the data on energy in estimation.

To provide a plausible calibration of the energy cost function, I collect quarterly data on energy use, capital and capacity utilization rate and estimate the technological relationship of the type described in Equation (3). The data on energy use consist of the total primary use of energy in commercial, transportation and industrial sectors.[9] Capital is the chain-type quantity index for the net stock of private nonresidential fixed assets, provided by BEA. Because these data are only available at an annual frequency, I obtain the quarterly data by interpolation. Capital utilization rates are not directly observable. However, in many studies, the capacity utilization rate is used as a proxy for capital utilization.[10] I aggregate monthly data on capacity utilization by simple averaging. The data set covers the first quarter of 1973 until the end of 2013, determined by availability of the energy use data.

Figure 3 plots the energy-capital ratio over time. The data exhibit a clear, negative trend. This observation is in agreement with the modeling strategy in Kormilitsina (2011), which assumes that the growing energy prices induce technological improvements that require reduced energy use per unit of capital over time. I detrend the data by estimating the log-linear regression:

$$log(\frac{E_t}{K_t}) = -\underset{(0.005)}{3.27} - \underset{(1.1\times10^{-4})}{0.005}(t-\bar{t}) + err_t,$$

where err_t is the error term and $\bar{t}$ is the mean of the time trend variable t, to ensure the detrended ratio retains the mean value of the original data. In the analysis below, I use the detrended series: $(\widehat{E/K})_t = E_t/K_t e^{0.005(t-\bar{t})}$, which is shown with a green dashed line in Figure 3. Figure 4 presents the scatter plot of the detrended energy-capital ratio, shown along the vertical axis, and capacity utilization along the horizontal axis. The plot reveals the positive relationship between utilization and energy-capital ratio.[11] This relationship also appears convex.

To evaluate the parameters of the energy cost function, I first estimate the elasticity with respect to capital utilization by a simple OLS regression of energy-capital ratio on capacity utilization and a constant, both in logs. The estimated regression model is[12]

$$log(\frac{\hat{E}_t}{K_t}) = -\underset{(0.02)}{3.12} + \underset{(0.09)}{0.67}log(CU_t) + \xi_t,$$

The resulting estimate of the elasticity is $\epsilon_a^u = 0.67$ and it is statistically significant, with the standard deviation of 0.09. Clearly, because ϵ_a^u smaller than 1, this elasticity value cannot justify a constant elasticity function for the energy cost $a(u_t)$, because when $\epsilon_a^u < 1$, the cost function is concave rather than convex. However, this smaller value does not necessarily violate the convexity assumption in the case of specifications (14) or (15). Therefore, it is important to consider alternative, less restrictive functional forms for $a(u_t)$.

[9] The data are in quadrillion of BTU, and are available on the EIA web site in monthly frequencies, which are aggregated into quarterly frequencies and correct for seasonality.

[10] See, for example, Christiano, Eichenbaum, and Trabandt (2015) and Finn (2000), among others.

[11] The correlation coefficient of the two series is moderately positive at 0.51.

[12] I obtain very similar results when I include the deterministic trend as a regressor and do not detrend the energy to capital ratio.

I proceed by estimating a quadratic specification for the energy cost function. In particular, I use the ratio of energy to capital as the dependent variable, and the demeaned capacity utilization rate, together with its squared value and a constant as three independent variables.[13] The estimated regression model is:

$$(\frac{\hat{E}}{K})_t = \underset{(0.0002)}{0.038} + \underset{(0.005)}{0.036}(CU_t - CU) + \underset{(0.075)}{0.17}(CU_t - CU)^2 + \xi_t,$$

where CU is the mean value of CU_t. The implied elasticities of the fitted energy cost function are $\epsilon_a^u = 0.77$ and $\epsilon_{a'}^u = 7.62$, both statistically significant, indicating the importance of the non-linearity in the relationship of energy to capital ratio and utilization.

Alternatively, I estimate parameters a_0, a_1 and a_2 of specification (14) by fitting the data with the non-linear least squares estimator. The resulting non-linear regression model is

$$(\frac{\hat{E}}{K})_t = \underset{(0.0012)}{0.035} + \underset{(0.011)}{0.024}CU_t^{\overset{9.99}{(4.11)}} + \xi_t. \tag{16}$$

The implied elasticities are $\epsilon_a^u = 0.78$ and $\epsilon_{a'}^u = 8.99$, which agree with the estimates in other model specifications. Figure 5 plots the estimated non-linear relationship in Equation (16), together with the scatter plot of the data. I find that, at least over the range of the available data for capacity utilization, the resulting curve is very similar to the one I obtain using the fitted values of the quadratic regression.

Table 2 summarizes the estimates of the energy cost elasticities obtained from the three regression models. The estimates for the log-linear model provide smaller elasticity ϵ_a^u than do those in the non-linear regression models. This specification, however, is not consistent with the convexity assumption of the energy cost function. The estimates of elasticities in the quadratic and nonlinear models are very similar. The elasticity of the energy cost function is in the range of 0.7 to 0.8, and the estimate of the elasticity of the marginal energy costs varies around 7 and 9. If these estimates are mapped into Figure 2, then the response of output to the energy price increase would only be moderate. Point D in Figure 2 illustrates this result for the calibration of $\epsilon_a^u = 0.75$ and $\epsilon_{a'}^u = 8$. Notice that the difference in output responses with and without endogenous depreciation will be very small, provided the elasticity of the marginal energy costs remains constant. Figure 6 verifies this observation. The figure shows the response of output and capital utilization for 10 quarters after an unexpected 10% increase in the energy price in the EC models with and without variable depreciation. Strikingly, at no time after the shock, does the response of output exceed 0.7%. Moreover, the output drop is slightly larger in the model with constant depreciation rate than it is in the case of variable depreciation, pointing to a mitigating effect from variable depreciation. The responses of capital utilization are related in a similar way. If compared with the responses in Figure 1, one can see that both the magnitude, and the difference in the responses of output and capital utilization is smaller in Figure 6.

[13] I use demeaned regressors for convenience, following the majority of the empirical literature. Although the demeaned utilization rate and its squared value are not technically independent regressors, I find that correlation between them is very small, while the raw utilization rate and its squared value are almost perfectly correlated.

It is important to point out that, everywhere in this analysis, I assume the depreciation function has a constant elasticity form, similar to the one in Equation (10). This depreciation function is calibrated to ensure the steady state equilibrium conditions are satisfied. At Point D of Figure 2, the elasticity of the marginal depreciation cost is 0.48. This value is close to the one obtained by Burnside and Eichenbaum (1996), who use the same constant elasticity functional form for $\delta(u_t)$. However, because this function does not seem to be a good choice for the energy cost function, the same may be true in the case of the depreciation function. Indeed, Basu and Kimball (1997) argue for the choice of the power with a constant specification similar to Equation (14), to describe the depreciation function. These authors find that the elasticity of marginal depreciation is approximately 1. To evaluate how an alternative specification for the depreciation function might influence the results in this paper, I obtain the responses of output and utilization, as in Figure 6, assuming the specification proposed by Basu and Kimball (1997) and calibrating $\epsilon_{\delta'}^{u} = 1$. The resulting responses are very similar, with the response of output even more moderate under variable depreciation (output falls 0.52% on impact versus 0.6% in Figure 6). Therefore, the results reported in Figures 1, 2 and 6 are robust to an alternative specification of the depreciation function.

4 Conclusion

The following conclusions can be made to summarize the results of this paper. Firstly, because the estimate of ϵ_a^u is below 1, the constant elasticity formulation is not an appropriate functional form for the energy cost function. A quadratic function, or a power functional form with a constant coefficient is therefore a preferred approximation of the relationship between the energy-to-capital ratio and capital utilization. Secondly, as the theoretical model suggests, knowing the slope and the curvature of the energy cost function is crucial to determine the quantitative effects of energy shocks. This finding emphasizes the importance of additional empirical research that would shed more light on the appropriate calibration of the energy cost function as well as on other sources of costs for capital use. The estimates obtained in this paper suggest that energy price shocks generate only moderate effects on output, contrary to the substantial effect observed using the conventional calibration. Moreover, in an empirically calibrated EC model, endogenous depreciation is unlikely to amplify the negative effect of energy price shocks.

5 Tables and Figures

Table 1: Parametrization of the EC model

		variable δ	constant δ
ϵ_a^u	elasticity of energy cost	1.66	6.97
$\epsilon_{a'}^u$	elasticity of marginal energy cost	0.66	5.97
β	Intertemporal discount factor	0.99	
θ	Share of capital	0.3	
δ	Depreciation	0.025	
h	Labor	0.3	
u	Utilization	0.81	
p_e	Energy price	1	
s^e	Share of energy	0.043	

Table 2: Elasticities of the energy cost function: Empirical evidence

	Log-log	Quadratic	Power
ϵ_a^u	0.67	0.77	0.78
	(0.09)	(0.1)	(0.1)
$\epsilon_{a'}^u$	-	7.62	8.99
	-	(3.17)	(4.11)
R^2	0.26	0.26	-

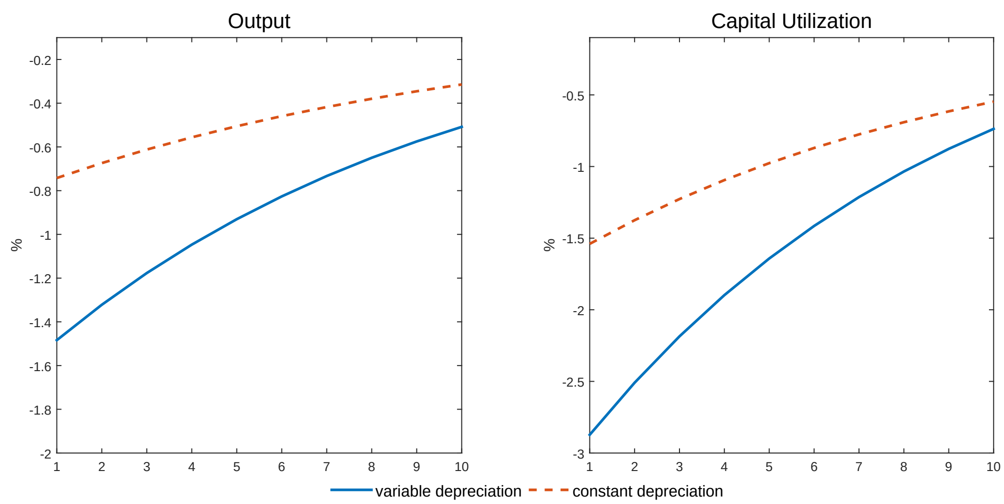

Figure 1: Impulse response of output with and without endogenous depreciation

Notes: The figures show the impulse response functions as percentage deviations from steady states in response to a 10% increase in the price of energy p_e. Percentages are shown on the vertical axis and quarters are shown on the horizontal axis. The solid blue lines correspond to Model 1, which features the constant depreciation rate and power energy cost function. The dotted red line corresponds to Model 2, which features variable depreciation and power energy cost function.

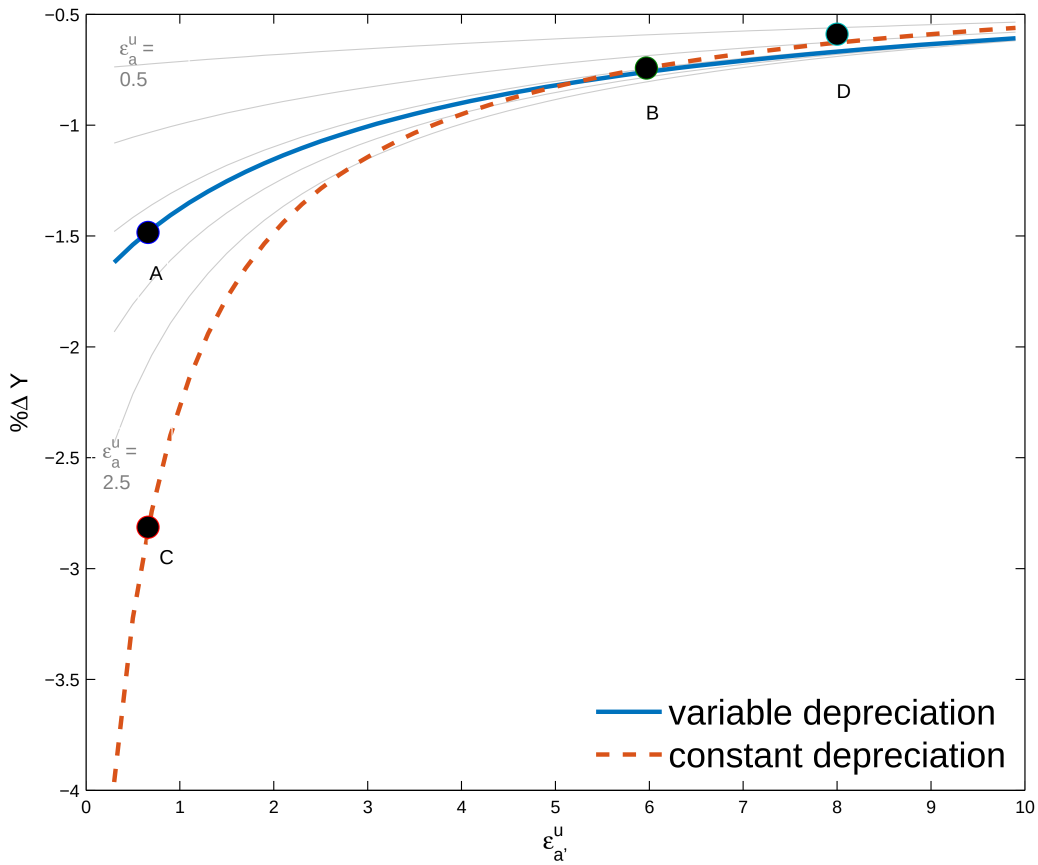

Figure 2: Immediate output response as a function of the marginal energy cost elasticity

Notes: The graph demonstrates that the amplifying effect of the energy price shock in the presence of variable depreciation results from increased elasticity of the marginal energy cost function. The figure shows the immediate response of output to a 10% increase in the energy price depending as a function of the steady state elasticity of the marginal energy cost. The solid blue line shows the response in the model featuring endogenous depreciation rate, while the red dashed line depicts the output response assuming the depreciation rate is constant. Point A shows the response when calibration is induced by power (constant elasticity) specification for $a(u_t)$. Point B represents the response in the same model with a constant depreciation, and reveals a smaller output drop in the model with constant depreciation rate. Point C represents the response of output in the model with constant depreciation, when the elasticity of the marginal energy costs coincides with that in the model with endogenous depreciation. Point D shows the response in the model with endogenous depreciation, where the depreciation function is parameterized using the estimated elasticities of the marginal energy costs. The thin grey lines provide the response of output versus $\epsilon^u_{a'}$ for five alternative calibrations of ϵ^u_a ranging from 0.5 to 2.5.

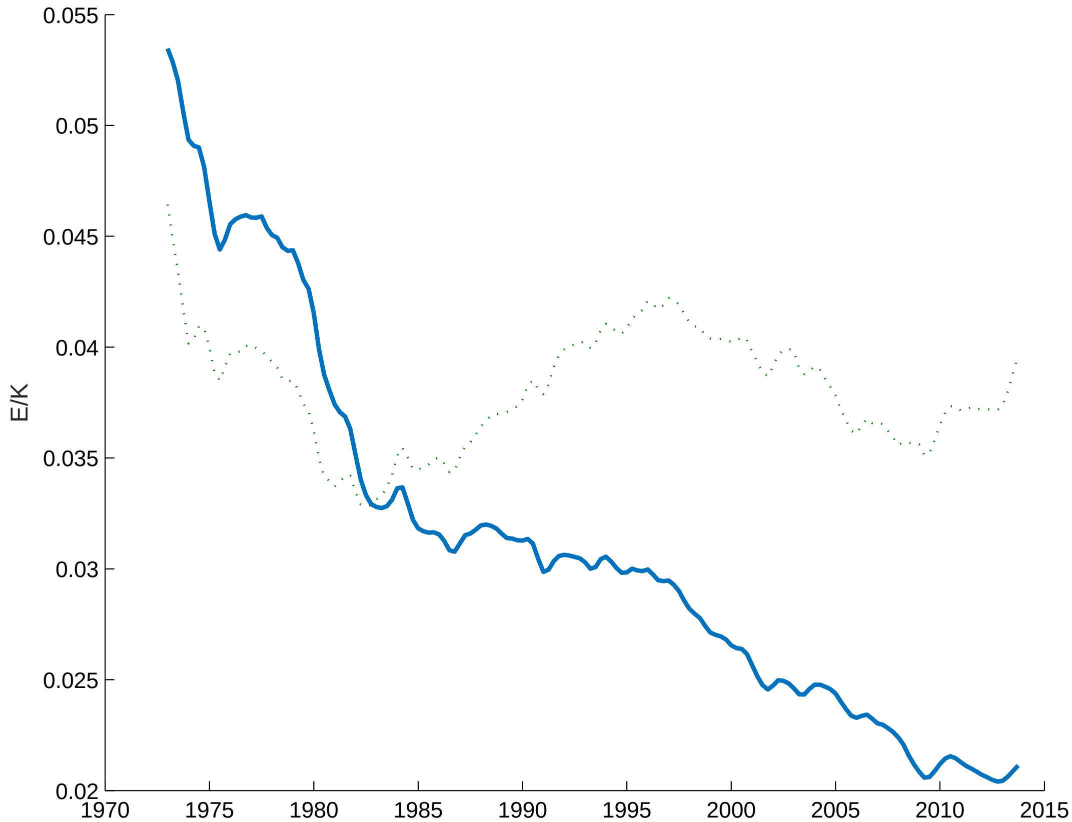

Figure 3: Energy-to-capital ratio

Notes: The solid blue line is the energy to real capital ratio in trillion BTU (seasonally adjusted) per unit of capital in dollars of 1999. The dotted green line is the detrended series.

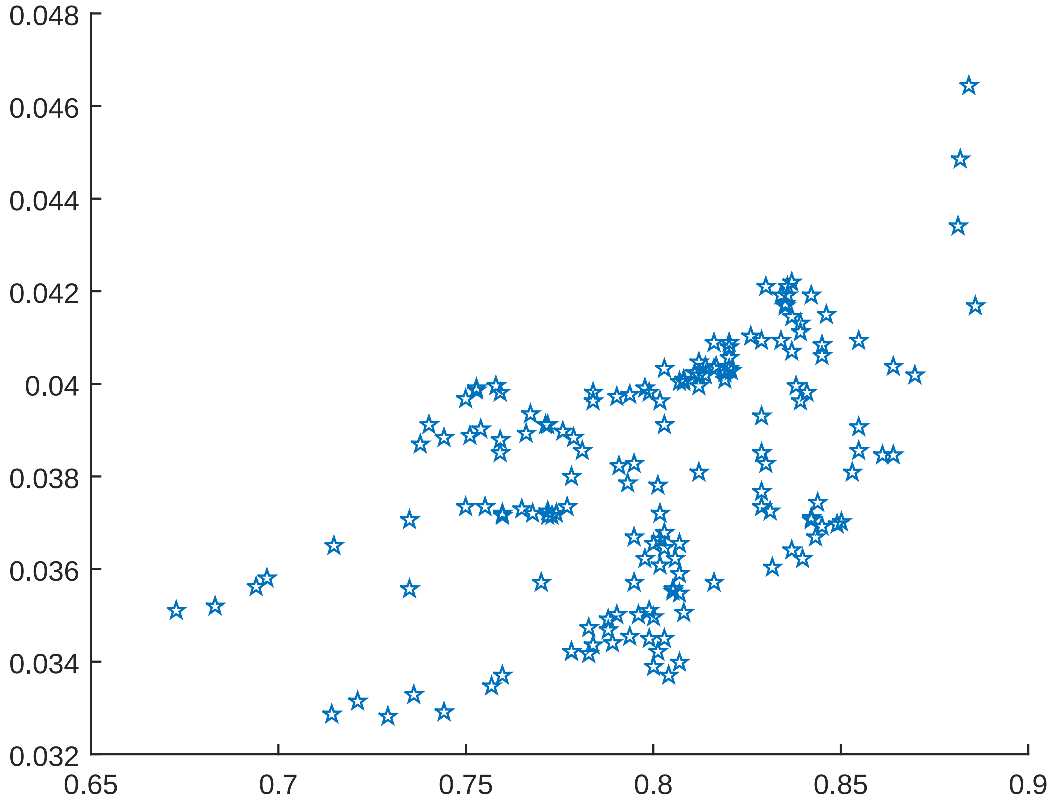

Figure 4: Energy-capital ratio and capacity utilization

Notes: The top picture displays the time series of energy to capital ratio in trillion BTU (seasonally adjusted) per unit of real capital.

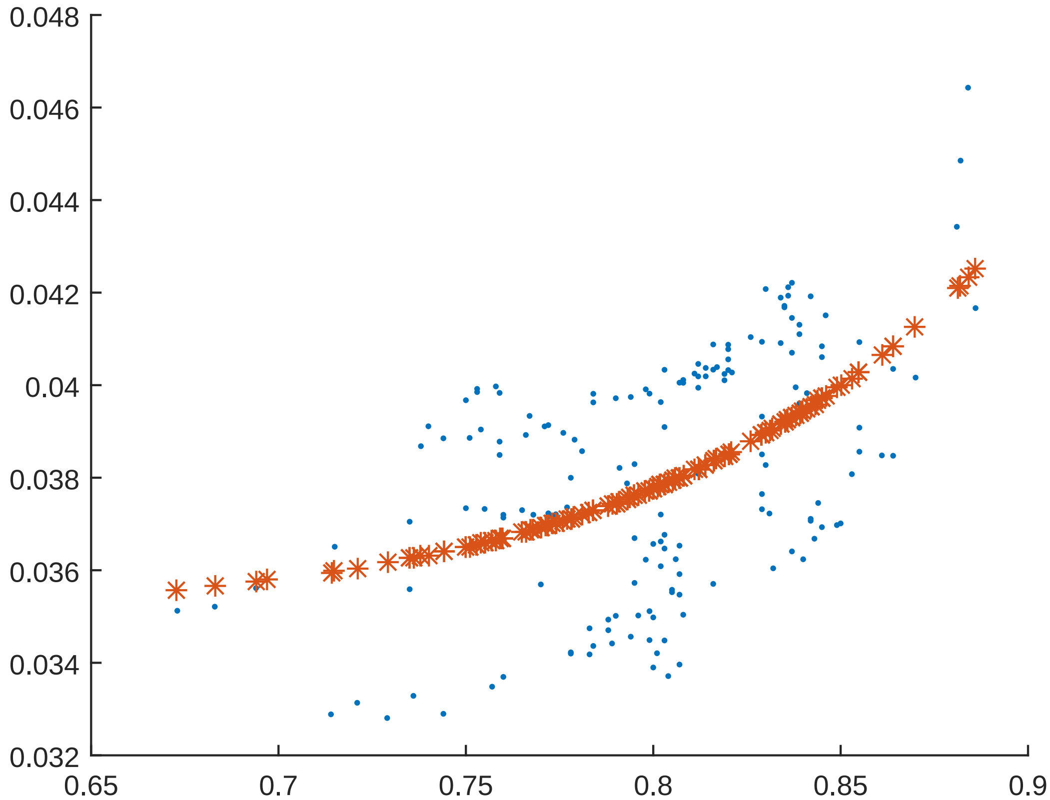

Figure 5: Energy-to-capital ratio and capacity utilization, data and the fitted model

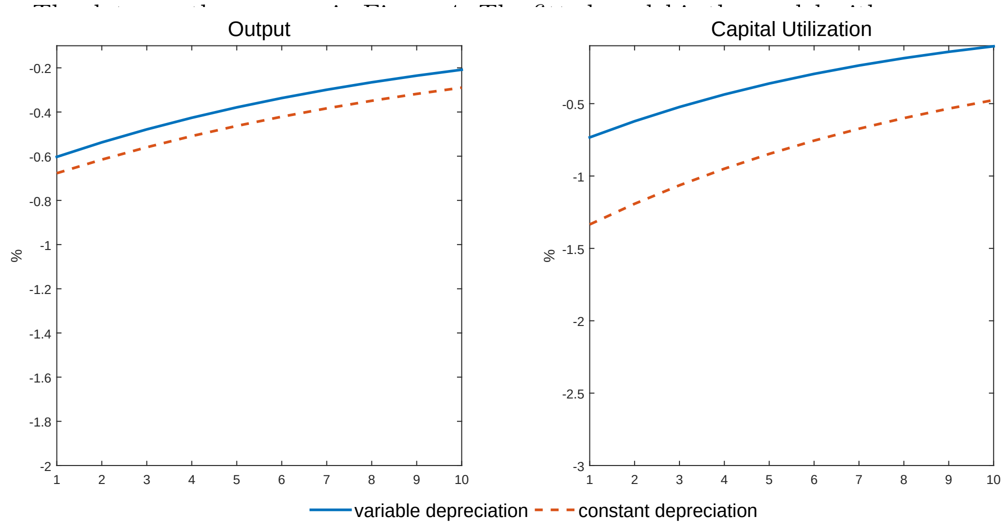

Figure 6: Response of output with estimated parameters of energy cost function

Notes: See notes to Figure 1.

6 Appendix. Proof of the steady state relationship (12)

First note that the parameters of the production technology and steady state quantities of factors are identical across the two models. Therefore, the values of the marginal product of capital on the left-hand side of equation (9) are the same in the models with and without endogenous depreciation. As a result, the total marginal costs on the right-hand side of formula (9) are also identical, which results in the relationship 12.

In the steady state, the marginal productivity of capital services on the left-hand side of Equation (9) depends exclusively on the calibration of the production block, which stays unchanged across models. Specifically, in the case of the Cobb-Douglass production technology, where $f(uk, h) = Auk^{\theta}h^{1-\theta}$, the marginal productivity of capital in the steady state is

$$f_1(uk, h) = \theta \frac{y}{uk}. \tag{17}$$

Substituting this expression into the steady state of condition (8), the latter can be simplified as

$$1 = \beta(\theta \frac{y}{k} + 1 - \delta - \frac{p_e e}{k}). \tag{18}$$

Parameters δ, θ and β are calibrated according to Table 1. The steady-state ratio of energy expenditures to capital is

$$\frac{p_e e}{k} = \frac{p_e e}{y}\frac{y}{k} = s_e \frac{y}{k}, \tag{19}$$

where s_e is the calibrated share of energy. Now, one can substitute this formula into Equation (18) to derive

$$\frac{y}{k} = \frac{\frac{1}{\beta} - 1 + \delta}{\theta - s_e}.$$

Because β, θ, δ and s_e are the same in the models with and without variable depreciation, and since steady state capital utilization rate is determined by calibration, the steady-state ratio y/k, the marginal productivity of capital services, and thus the total marginal costs of capital are invariant to the assumption of variable depreciation.

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