Credit Attribution and Collaborative Work*

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Abstract

We examine a dynamic model of teamwork in which the public attributes credit for success based on its perception of individual efforts. The collaborative behavior varies starkly depending on the shape of marginal effort cost, or project's "difficulty." In the unique (interior) equilibrium, higher ability collaborators work less and thus receive lower credit and payoff for "easy" projects, while the reverse holds for "difficult" projects. Despite free-riding, the team equilibrium may involve over-investment. Social efficiency requires over-rewarding collaborative work and under-rewarding solo work. The incentives to team up and the impact of effort monitoring on credit attribution are also investigated.

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1 Introduction

Credit for scientific discovery is vital for the reward system, and hence, the progress of science. According to Flier (2019), "absent credit, it is impossible [for scientists] to secure appointments, promotions, research funding, access to students, and other necessities of research." Essential to credit attribution is the inference of individual contributions to knowledge production. This inference, however, has been complicated by the steady rise in collaborative work across disciplines.[1] While various conventions such as authorship order have developed to overcome this complication, public perception of each collaborator's contribution remains crucial in assigning credit.[2]

Individual recognition for collaborative success is not specific to scientific teams. In business, managers pay discretionary bonuses to their subordinates for reaching company goals (Rajan and Reichelstein, 2006). In politics, voters reward or punish parties in a coalition differently for policy outcomes (Marsh and Tilley, 2010). And in education, teachers assign individualized grades on group projects (Zhang and Ohland, 2009).

The issue of credit attribution in teamwork raises some obvious positive and normative questions. Do higher ability (or more productive) team members always deserve more credit for collaborative success? Do team members always underinvest in the joint project due to free-riding? How should society reward collaborative work? How about solo work? Does credit assignment by the public encourage or discourage collaboration? And, do team members favor public monitoring of their activities for proper credit?

To address these and related questions, we present a dynamic model of teamwork building on Lee and Wilde's (1980) seminal paper on R&D races; see Reinganum (1989) for a review of this literature. We envision that instead of working independently, agents work as a team toward a breakthrough, e.g., a scientific discovery.

[1]Based on all the articles registered from 1980 to 2013 in the Social Science Citation Index (SSCI), Henriksen (2016) documents a significant increase in co-authorship with the average number of authors per article in economics rising from 1.3 to 2.3 over the period. Wuchty et al. (2007) report that in the natural sciences, team size has grown each year and nearly doubled from 1.9 to 3.5 authors per article between 1955 and 2000.

[2]As Flier (2019) convincingly argues, it is often difficult to assign credit given the current authorship practices since "broadly accepted conventions that specify the meaning of authorship as regards to type and extent of contributions by each author are lacking." Authorship conventions range from alphabetical (mathematics, economics), to descending importance (biology, high energy physics), to listing the lead author first and the principal investigator last (chemistry, psychology); see Shen and Barabási (2014). Recently the American Economic Association (AEA) has started permitting random order of coauthors (Ray ⓡ Robson, 2018).

Agents have heterogenous abilities, where higher ability implies lower effort cost. To isolate from career concerns, we assume abilities to be publicly known, perhaps, because of agents' past achievements. Our main departure from the teamwork literature (discussed below) is that the sharing rule for the team's output is not exogenous; rather, it is determined by the public's perception in equilibrium. Specifically, unable to observe individual efforts but conjecturing them, the Bayesian public, e.g., the scientific community, attributes the breakthrough to an agent with a probability equal to his effort relative to the team's total.[3] We call this probability the agent's "credit" for collaborative success. In equilibrium, the team's effort profile and the public's credit allocation must be consistent.

We focus on the interior equilibrium in which all team members are active, and show that one generically exists.[4] Our analysis reveals that an agent's equilibrium behavior is driven by his "perception" of the project's difficulty. This perception depends on the discount rate, the anticipation of teammates' efforts, and the project's intrinsic cost of effort. For a wide range of projects, the project's intrinsic technology dominates the agents' perceptions. We refer to a project as "intrinsically easy" if the intrinsic *marginal* cost of effort is concave, and "intrinsically difficult" if that marginal cost is sufficiently convex. For moderately convex marginal cost, we refer to a project as "non-intrinsic" since the agents' perceptions can vary, depending on what each expects of teammates and on the discount rate.

Central to our investigation are the intrinsic projects, since they admit a unique interior equilibrium, and more importantly, agents' behaviors and credits strikingly differ across them. For an intrinsically easy (resp. difficult) project, the higher ability agent works less (resp. more) and, as a result, receives less (resp. more) credit in equilibrium. If he were to receive more credit for an intrinsically easy project, the higher ability agent would work disproportionately harder than the credit allocation, which would be inconsistent with the public's perception. Thus, the higher ability agent must be under-rewarded in equilibrium. Under-rewarding does not arise in an intrinsically difficult project because, by definition, the marginal cost of effort is too steep to expend disproportionate effort. Agents also fare very differently across intrinsic projects. In equilibrium, the higher ability agent is worse off (resp. better off) than his lower ability teammates under an intrinsically easy (resp. difficult) project. By working harder, an agent improves his equilibrium credit but also incurs a greater effort cost. Our result indicates that the more diligent, not necessarily more able, agent fares better in teamwork.

[3]We use the male pronoun for team members and the female pronoun for the social planner later.

[4]An inactive agent can effectively be removed from the team in our model.

These insights from intrinsic projects extend to non-intrinsic ones in that the agents may have mixed perceptions in the latter. As such, there may be multiple interior equilibria, and the equilibrium credit allocation and payoffs can be non-monotonic in ability. In particular, we numerically illustrate that in a three-member team, it may be the medium-ability member who works the hardest and receives both the highest credit and payoff in equilibrium.

Our next set of results focus on inefficiencies that arise from credit attribution, regardless of agents' perceptions of the project. Free riding, a well-known feature of teamwork, often leads to under-investment. Our model also delivers under-investment when team members are identical and thus expect equal credit for success. For a significant ability gap, however, over-investment by some member is possible. This inefficiency is obvious for an intrinsically easy project: contrary to desired socially, the low ability agent expends more effort in this case. Consider, thus, two highly heterogeneous agents in an intrinsically difficult project. Anticipating most of the credit, the high ability agent would perform virtually solo in equilibrium. Such a work allocation, however, would be too unequal from the social viewpoint: by shifting some work to the low ability agent, the team's payoff could improve.

We also determine the optimal credit profile that would correct for these inefficiencies. As can be expected in light of Holmstrom (1982), efficiency requires "budget-breaking": the total optimal credit for team members must exceed one. What is surprising in our setting is that teamwork must be substantially over-rewarded. Each team member must be offered more than the full credit for the team's success. This is because, by marginally increasing his effort, an agent not only speeds up the breakthrough, but also saves the future costs of his teammates. In fact, since he is asked to work harder than teammates at the social optimum and thus, his cost savings for others are smaller, the optimal credit for a higher ability agent must be lower.

The optimal over-rewarding of teamwork suggests that when able to choose between team- and solo work, agents should have incentives to team up. To establish these incentives, we examine a complementary setting in which agents work solo and compete for the breakthrough, as in a typical R&D race. We show that, while rational, assigning the winner full credit for solo success is too generous from the social viewpoint. Put differently, it is socially optimal to under-reward solo work. This stark contrast with teamwork is explained by the competitive pressure that over-motivates agents in a race to success. Faced with the optimal credit schemes, the agents would have a strict incentive to team up.[5]

[5] Also important in this observation is the fact that the socially optimal effort allocation is the same across team and solo work settings, which we formally establish in the analysis. Hence, the credit –

Incentives to form a team can also arise under the equilibrium credit attribution by the public: sufficiently patient agents who are also similar in ability prefer to work together to avoid costly competition. Interestingly, this finding implies that a high ability agent can team up with a low ability agent on an intrinsically easy project, despite anticipating a lower credit and payoff from its completion than his teammate.

Last, but not least, we investigate how public monitoring of individual efforts may impact credit attribution.[6] We show that with observable efforts, teamwork is strategically equivalent to solo work for the agents, as they now compete for credit. Our previous findings thus indicate that with public monitoring, the higher ability agent always works harder and deserves more credit, but he may dislike observability due to the implied competition.

Related Literature. Aside from those mentioned above, our paper is related to three strands of the literature on team incentives. The first strand examines contracts that elicit efficient actions in teams. These contracts often feature nonlinear sharing rules and large penalties for (potential) deviators; e.g., Holmstrom (1982), Rasmusen (1987), Legros and Matthews (1993) and Winter (2004). Our optimal credit profile also elicits efficient actions but, given our main interest in scientific credit, it is restricted to be a simple, linear contract without penalties. The second strand fixes the sharing rule and views teamwork as voluntary contributions to a public good; e.g., Olson (1965) and Andreoni (1988). These papers employ static models and identify free riding among team members. Building on this insight, the third – and more recent – strand of the literature views teamwork as dynamic public good provision. It characterizes non-stationary team dynamics when agents aim to reach a pre-specified project scale (e.g., Admati and Perry, 1991; Yildirim, 2006; Georgiadis, 2015; Bowen et al. 2019) or they learn about the project's potential (e.g., Bonatti and Horner, 2011; Cetemen et al. 2019). As in Lee and Wilde (1980), our model exhibits stationary strategies. Most importantly, we endogenize the sharing rule as an equilibrium credit allocation by the public.[7] As such, unlike in these studies, equilibrium effort can be non-monotonic in ability, and over-investment by both high and low ability agents can occur in our model.

The scant theoretical work on credit attribution in economics has mostly focused on authorship order as a mechanism to signal relative contributions. Engers et al.

not the cost of effort – is what matters for an agent's payoff comparison.

[6]In practice, such monitoring is not uncommon. In academia, researchers often discuss their ongoing projects with their colleagues, and in business, managers can track employees' work hours.

[7]With few exceptions such as Bowen et al. (2019) and Cetemen et al. (2019), dynamic teamwork models assume symmetric agents and focus on symmetric equilibrium, so credit allocation would also be trivial.

(1999) show that the alphabetical order may emerge as the unique equilibrium, as any deviation from it hurts the author with the early name in the alphabet more than it benefits the other. Also in a model with two co-authors, Ray ⓡ Robson (2018) show that a certified random order is fairer, distributes credit evenly on the alphabet and will invade an environment in which alphabetical order is dominant. While these insights are valuable and empirically relevant (Einav and Yariv, 2006), authorship order can only provide a noisy signal of individual contributions. With more than two authors, this signal can be even noisier, as evident from the lack of consensus on a name-order convention across disciplines and the reliance on peer reviews in tenure/promotion decisions.[8] Therefore, in our model the public infers relative contributions of an arbitrary number of collaborators using no other information than the commonly known and heterogeneous abilities.

Another set of papers focus on the incentives of scientists to invite collaboration when they have full ownership rights on projects. Motivated by the structure of scientific labs, Gans and Murray (2013) show that a senior scientist's decision to co-author with a junior one depends on whether their efforts are complements and the exact timing of the co-authoring decision. In Bikard et al. (2015), collaborative work increases the quality of research but lowers individual rewards due to credit attribution, which is modeled as an exogenous function that only depends on the number of collaborators. In particular, credit does not depend on the public perception of the relative efforts and identities of the collaborators.[9]

The paper is organized as follows. Section 2 describes the model. Sections 3.1 and 3.2 describe the social optimum and the team equilibrium, respectively. Section 4.1 illustrates the exogenous (non-Bayesian) credit profile that would implement the socially optimal efforts in equilibrium. Section 4.2 shows that some team members may overinvest if the team is sufficiently heterogenous. Section 5 analyzes solo work and incentives to form a team. Section 6 considers credit attribution with observable efforts. Section 7 concludes. The Appendix contains the proofs and the technical details omitted from the main text.

[8]The proposals for credit allocation have come a long way from assigning full or equal fractional credit to each co-author to unequal schemes such as arithmetic, geometric and harmonic counting; see Kim and Kim (2015) for a review. Yet, there is always a gap between peer perception and the credit allocated by the specific scheme (Wren et al. 2007). Kim and Kim (2015) quantify this gap by using credit allocations from surveys in chemistry, biomedicine, economics, marketing, and psychology.

[9]While our model and focus are very different, credit attribution by the public is conceptually similar to blame-sharing in groups for unpopular decisions; see, e.g., Bartling et al. (2015) and Falk and Szech (2017) for experimental evidence and Name-Correa and Yildirim (Forthcoming) for a theoretical analysis.

2 Model

Our model is a simple adaptation of Lee and Wilde's (1980) on R&D races. Rather than competing toward a single breakthrough, e.g., a scientific discovery, a team of $n > 1$ risk-neutral agents indexed by $i = 1, ..., n$ undertakes a joint project toward it. They continuously and independently choose their efforts over an infinite time horizon, $t \in [0, \infty)$. Let $x_i(t) \in [0, \infty)$ be agent i's instantaneous effort at time t, which is unobservable to others. The flow cost of effort is given by

$$c_i(x_i(t)) = \frac{c(x_i(t))}{a_i}, \tag{1}$$

where $a_i > 0$, $c' > 0$, $c'' > 0$, and $c(0) = c'(0) = 0$.

We refer to the parameter a_i as agent i's "ability" and assume that it is publicly known, perhaps due to his track record, ruling out any reputational concern for the agent. And we refer to the function $c(.)$ as the project's "intrinsic" cost technology, which signifies its intrinsic level of difficulty. For expositional convenience, we adopt the following iso-elastic form for $c(.)$ in the text:[10]

$$c(x) = \frac{x^k}{k}, \quad k > 1. \tag{2}$$

As in Lee and Wilde (1980), we assume no knowledge accumulation and that agent i's instantaneous probability of a breakthrough at time t is also his effort, $x_i(t)$.[11] Without loss of generality, we can, therefore, drop the time index and focus on stationary strategies, x_i, throughout. Such stationarity implies that agent i's random time for the breakthrough, denoted by $T_i \in [0, \infty)$, is exponentially distributed with rate x_i.[12] Consequently, the team's random time for completing the project, which

[10] In the proofs of the formal results, we offer general cost conditions all of which are satisfied by the iso-elastic specification. Besides its expositional ease, we have adopted this specification in the text for its wide use in the continuous-time teamwork studies; e.g., Bonatti and Horner (2011) and Georgiadis (2015).

[11] The assumption of no knowledge accumulation is obviously unrealistic, but it greatly simplifies our analysis and appears reasonable for highly innovative projects. The linearity of the discovery rate is, however, without loss of generality. We could assume it to be some increasing and concave function $R(x_i)$. Then, by a change of variables: $x_i := R^{-1}(x_i)$, it is evident that the nonlinearity in the rate would be absorbed by the project's intrinsic technology, $c(.)$. By a change of variables: $x_i := x_i/a_i$, it is also evident that our model is robust to an alternative specification of the discovery rate: $a_i x_i$.

[12] The exponential arrival time is assumed by Lee and Wilde (1980), but it need not be. Without knowledge accumulation, agent i's discovery follows a Poisson process, with the rate of $x_i(t)$. Moreover, given the stationarity, the process is homogenous, with exponential interarrival times. The stationarity also implies that it is immaterial whether or not agents commit to their effort strategies in our model; see Reinganum (1982) for a similar observation.

is $\min_i T_i$, is exponentially distributed with rate $X = \sum_i x_i$. Let $\omega_t \in \{0, 1\}$ be the state of the project at time t, with $\omega_t = 1$ representing its completion. The value of a completed project is normalized to *one* while an incomplete project is worth nothing. Agents discount the future benefit and costs at a common rate $r > 0$.

We depart from the existing literature on team incentives (discussed above) and consider an endogenous allocation of the reward based on the public's belief as to who is responsible for success.[13] Specifically, unable to observe the individual efforts of the team members but conjecturing their profile $\mathbf{x} = (x_1, ..., x_n)$, the Bayesian public would credit the breakthrough at time t to agent i with the probability:

$$q_i = \Pr(T_i = \min_j T_j | \omega_t = 1) = \frac{x_i}{X}. \tag{3}$$

Hence, if the breakthrough occurs, agent i receives the following expected reward:[14]

$$q_i(1) + (1 - q_i)(0) = q_i.$$

To derive his expected discounted payoff, note that given the exponential arrival time, the probability of no breakthrough until time t is e^{-Xt}. In the next instant dt, agent i incurs his flow cost $c_i(x_i)dt$ and receives his reward q_i if the team succeeds with probability Xdt. If the team fails, the game is reset to $t = 0$. As a result, agent i's expected discounted payoff at any time without a breakthrough is

$$\begin{aligned} u_i &= \int_0^\infty e^{-rt} e^{-Xt} \left(Xq_i - c_i(x_i) \right) dt \\ &= \frac{X}{r + X} q_i - \frac{c_i(x_i)}{r + X}. \end{aligned} \tag{4}$$

(4) is, perhaps, best interpreted if the discount rate r is viewed as the nature's fixed flow effort to "steal" the discovery. Then, given the exponential rates, the term $X/(r + X)$ becomes the team's probability of winning against the nature, in which case agent i receives the reward q_i. Furthermore, with this interpretation, the term $1/(r + X)$ corresponds to the expected length of time that the agent expends effort and explains his expected cost in (4).

3 Analysis

We begin our analysis by establishing the social optimum and then turn our attention to the team equilibrium.

[13]We assume that communication between a team member and the public about the breakthrough is either infeasible or noncredible, as each member would claim responsibility for it.

[14]Given risk-neutral agents, the credit q_i can also be interpreted as i's share from the unit surplus.

3.1 Social optimum

Suppose that a social planner chooses agents' efforts to maximize their expected joint payoff or welfare: $W = \sum_i u_i$. Using (4) and the fact that $\sum_i q_i = 1$ from (3), it readily follows that the welfare is

$$W(\mathbf{x}; r, \mathbf{a}) \equiv \frac{X}{r+X} - \frac{\sum_j c_j(x_j)}{r+X}, \tag{5}$$

where the first term on the right-hand side represents the team's expected total benefit and the second term represents its expected total flow cost.

Accordingly, the planner solves

$$\max_{\mathbf{x}} W(\mathbf{x}; r, \mathbf{a}). \tag{SO}$$

Let the effort profile $\mathbf{x}^S$ be a solution to (SO), resulting in the individual credit $q_i^S = x_i^S / X^S$ and the expected payoff u_i^S for agent i. Lemma 1 establishes the intuitive properties of the social optimum.

Lemma 1 *There is a unique solution, $\mathbf{x}^S$, to (SO). At the social optimum,*

(a) *every agent exerts positive effort: $x_i^S > 0$ for all i,*

(b) *the higher ability agent works harder and thus receives more credit for the success: $x_i^S > x_j^S$ and $q_i^S > q_j^S$ for $a_i > a_j$,*

(c) *the higher ability agent obtains a higher payoff: $u_i^S > u_j^S$ for $a_i > a_j$.*

To understand Lemma 1, we write the marginal impact of agent i's effort from (5):

$$\frac{\partial}{\partial x_i} W(\mathbf{x}; r, \mathbf{a}) = \frac{r + \sum_j c_j(x_j)}{(r+X)^2} - \frac{c_i'(x_i)}{r+X}. \tag{6}$$

It is evident from the first term on the right-hand side of (6) that, by bringing forward the breakthrough date, a marginal increase in agent i's flow effort contributes to welfare both directly through success and indirectly through future cost savings to the team. The planner trades off these benefits against the expected increase in agent i's flow cost, as reflected in the second term. Since the marginal cost of a small effort is assumed to be negligible, i.e., $c_i'(0) = 0$, (6) implies that the optimal effort must be positive for every team member. Moreover, at the optimum, the first-order condition, namely $\frac{\partial}{\partial x_i} W(\mathbf{x}; r, \mathbf{a}) = 0$, requires that

$$c_i'(x_i) = \frac{r + \sum_j c_j(x_j)}{r+X}. \tag{7}$$

Clearly, the right-hand side of (7) is the same across agents. Hence, as one would predict, the planner chooses the optimal efforts to equalize marginal costs across team members. This also explains why a higher ability member is asked to work harder. Such hard work implies more credit for the higher ability agent upon success, although it is obvious from (5) that being able to dictate their efforts at the social optimum, the planner is neutral to the credit distribution among the agents. Finally, despite a higher cost of effort, the social optimum allocates a greater expected payoff to a more able agent. These intuitive observations from Lemma 1 are useful to understand team incentives, which we turn to next.

3.2 Team Equilibrium

In teamwork, efforts are independently chosen and unobservable to others, including the public. Thus, taking as given the public's belief q_i and the total effort $X_{-i} = \sum_{j \neq i} x_j$ by teammates, agent i chooses his flow effort x_i to maximize his expected utility in (4). That is, agent i solves

$$\max_{x_i} u_i = \frac{X}{r+X} q_i - \frac{c_i(x_i)}{r+X}. \tag{8}$$

The first-order condition of (8) equates the expected marginal benefit to the expected marginal cost:[15]

$$\underbrace{\frac{r}{(r+X)^2} q_i}_{=MB^i} = \underbrace{\frac{c_i'(x_i)(r+X) - c_i(x_i)}{(r+X)^2}}_{=MC^i}. \tag{9}$$

In (Nash) equilibrium, both the agents' and the public's beliefs must be correct. In particular, the public's credit allocation to team members must be consistent with (3). Therefore, substituting for $q_i = x_i/X$, the equilibrium effort profile $\mathbf{x}^* = (x_1^*, ..., x_n^*)$ is the solution to (9) for $i = 1, ..., n$, resulting in the equilibrium credit allocation: $\mathbf{q}^* = (q_1^*, ..., q_n^*)$.

Before characterizing the equilibrium, we note a trivial multiplicity: if an agent expects no credit from the public for a successful project, i.e., $q_i^* = 0$, then (9) implies that he will exert no effort, $x_i^* = 0$, which will, in turn, confirm the public's belief.[16] Since an inactive agent receives an expected payoff of zero by (4), he can essentially be removed from the team, perhaps, by requiring a small participation cost. Hence, in

[15] The second-order condition is easily verified.

[16] Note, however, that $x_i^* > 0$ for some i because $c_i'(0) = c_i(0) = 0$ and $r > 0$. Hence, $X^* > 0$.

what follows we focus on the *interior equilibria* at which all team members are active; i.e., $x_i^* > 0$ for all i.

To better understand agent i's equilibrium behavior, we divide both sides of (9) by $q_i = x_i/X$ and re-write it in terms of the credit-adjusted marginal benefit and marginal cost:

$$\underbrace{\frac{r}{(r+X)^2}}_{=mb(X,r)} = \underbrace{\left(\frac{c_i'(x_i)(r+X) - c_i(x_i)}{x_i}\right)\frac{X}{(r+X)^2}}_{=mc^i(x_i,X,r)}. \tag{10}$$

Evidently, the credit-adjusted marginal benefit, $mb(X,r)$, is equal across team members since, besides discounting, the breakthrough rate depends only on the aggregate effort, X. The credit-adjusted marginal cost, $mc^i(x_i, X, r)$, is, however, more involved, and its properties play a key role in our equilibrium characterization. In particular, central to agent i's equilibrium behavior turns out to be his "perception" of the project, which we define based on whether mc^i is decreasing or increasing in x_i.

Definition 1 ***(easy vs. difficult projects)*** *Agent i is said to perceive the project as "easy" if $\partial mc^i(x_i^*, X^*, r)/\partial x_i < 0$, and "difficult" if $\partial mc^i(x_i^*, X^*, r)/\partial x_i > 0$. Moreover, agent i is said to perceive the project as "intrinsically easy" if $\partial mc^i(x_i, X, r)/\partial x_i < 0$, and "intrinsically difficult" if $\partial mc^i(x_i, X, r)/\partial x_i > 0$ for all x_i, X_{-i}, r.*

In words, an agent views the project as "easy" or "difficult" if his credit-adjusted marginal cost is strictly decreasing or strictly increasing in his own effort, respectively. Two remarks are in order here. First, an agent's perception of the project is defined by fixing the aggregate effort, X, which includes his own. This approach is more convenient in the analysis because agents in our setting play an "aggregative game" in that agent i cares about his teammates' efforts to the extent of their sum, X_{-i}. And because of this feature, aggregative games have received much attention in various literatures in economics; see Acemoglu and Jensen (2013) and the references therein.

Second, an agent's perception of the project is defined as an equilibrium object because it is likely to be influenced by his anticipation of the team's total effort. As (10) implies, however, the agent's perception is also influenced by the project's technology, $c(.)$, signifying its intrinsic difficulty, and by the discount rate, r, signifying its urgency for completion.[17] It is possible that the project's technology dominates

[17]The agent's perception of the project does not directly depend on the team's ability profile as his ability only affects its scale, and the information about the others' is subsumed in the equilibrium efforts, which the agent cares about.

the agent's perception, in which case we call the project "intrinsically easy" or "intrinsically difficult." In particular, an intrinsically easy project requires that the agent perceive it to be easy even when he is very impatient ($r \to \infty$), whereas an intrinsically difficult project requires that he perceive it to be difficult even when he is very patient ($r \to 0$).[18]

Given that $c_i(x_i) = c(x_i)/a_i$ by (1), a useful observation from (10) is that the credit-adjusted marginal cost, mc^i, is separable in the agent's ability, a_i. Thus, if a team member perceives the project as intrinsically easy or intrinsically difficult, so do his teammates; that is, the team's perception is uniform over such projects. The following result characterizes team members' perceptions of the project.

Lemma 2 *For* $1 < k \leq 2$*, the project is intrinsically easy whereas, for* $k \geq \frac{3+\sqrt{5}}{2} \approx 2.62$*, it is intrinsically difficult. For* $2 < k < \frac{3+\sqrt{5}}{2}$*, team members' perceptions of the project may differ, depending on the equilibrium efforts and discount rate.*

Lemma 2 reveals that the project is intrinsically easy if its intrinsic *marginal* cost, c', is concave or $k \leq 2$ for the iso-elastic specification, and intrinsically difficult if it is sufficiently convex or $k \geq 2.62$.[19] Otherwise, if the project's intrinsic marginal cost is moderately convex, the equilibrium efforts and the discount rate also affect how an agent views the project. We numerically demonstrate this point below, but first, we establish the existence of an interior equilibrium.

Proposition 1 *There is an (interior) equilibrium if and only if* $k \neq 2$*, or* $k = 2$ *and agents are not too heterogenous in terms of ability. Furthermore, if the project is intrinsically easy or intrinsically difficult, then the equilibrium is unique.*

As alluded to above, the proof of Proposition 1 heavily exploits the fact that agents play an aggregative game, allowing us to write each agent's strategy as a function of the total effort. For $k \neq 2$, an interior equilibrium obtains independent of the team's ability profile since, in this case, the marginal cost of effort satisfies Inada-like conditions: $c_i''(0) = 0$ or ∞. For $k = 2$ (the quadratic cost), $c_i''(0) = 1/a_i$, so the

[18]The reader will probably agree that given the state of knowledge, proving the Pythagorean theorem is intrinsically easy, whereas proving the Riemann Hypothesis, one of the Clay Mathematics Institute's millennium problems, is intrinsically difficult; see https://www.claymath.org/millennium-problems.

[19]Specifically, we show that $\partial mc^i(x_i, X, r)/\partial x_i < 0$ for all x_i, X_{-i}, r if and only if $c'''(x_i) \leq 0$ for all x_i, and $\partial mc^i(x_i, X, r)/\partial x_i > 0$ for all x_i, X, r if and only if $(c'(x_i)/x_i)' x_i - (c(x_i)/x_i)' \geq 0$ for all x_i, where it is verified that $(c'(x_i)/x_i)' \overset{sign}{=} c'''(x_i)$ and $(c(x_i)/x_i)' \overset{sign}{=} c''(x_i)$.

existence of an interior equilibrium requires that agents not be too heterogenous in ability; otherwise, the free-riding incentive would be too strong to elicit positive effort from all team members. Perhaps more importantly, the interior equilibrium is also unique when the project is intrinsically easy or intrinsically difficult. When the project is non-intrinsic, there can be multiple equilibria because each agent's perception of the project depends on what he expects of his teammates and on the discount rate.

Tables 1-3 demonstrate Proposition 1 for a three-member team with the ability profile $\mathbf{a} = (3.1, 3.05, 3)$ and project technologies $k = 2$, 2.1, and 3. In light of Lemma 2, Tables 1 and 3 refer to intrinsically easy and intrinsically difficult projects, respectively, and thus, the equilibrium is unique in each.[20]

r	x_1^*	x_2^*	x_3^*	u_1^*	u_2^*	u_3^*
.01	.056	.061	.067	.286	.313	.340
.05	.120	.132	.145	.263	.289	.316
.50	.315	.361	.407	.189	.214	.239
.75	.363	.422	.481	.169	.194	.219
1	.397	.468	.539	.154	.179	.204
5	.516	.741	.966	.065	.090	.112
10	.443	.840	1.236	.032	.057	.078

Table 1. Intrinsically Easy Project: $k = 2$; $(a_1, a_2, a_3) = (3.1, 3.05, 3)$

r	x_1^*	x_2^*	x_3^*	u_1^*	u_2^*	u_3^*	Agent 1	Agent 2	Agent 3
.01	.054	.072	.085	.243	.323	.379	Easy	Easy	Easy
.05	.103	.154	.183	.208	.307	.364	Easy	Easy	Easy
.50	.291 .366	.344 .168	.507 .622	.172 .210	.199 .099	.285 .340	Diff. Easy	Easy Diff.	Easy Easy
.75	.509 .344	.682 .212	.152 .784	.225 .157	.292 .098	.071 .329	Easy Diff.	Easy Diff.	Diff. Easy
1	.240 .514	.182 .777	1.076 .190	.093 .192	.071 .276	.356 .074	Diff. Easy	Diff. Easy	Easy Diff.
5	1.040	.700	.527	.120	.086	.066	Diff.	Diff.	Diff.
10	1.072	.828	.656	.071	.057	.047	Diff.	Diff.	Diff.

Table 2. Non-Intrinsic Project: $k = 2.1$; $(a_1, a_2, a_3) = (3.1, 3.05, 3)$

[20]Simulations were done with Mathematica, using six different initial points to check convergence.

r	x_1^*	x_2^*	x_3^*	u_1^*	u_2^*	u_3^*
.01	.157	.154	.152	.331	.325	.319
.05	.265	.261	.256	.316	.310	.305
.50	.528	.519	.509	.249	.244	.240
.75	.587	.577	.566	.227	.224	.220
1	.630	.619	.608	.211	.207	.204
5	.852	.838	.823	.104	.102	.101
10	.920	.904	.889	.065	.064	.063

Table 3. Intrinsically Difficult Project: $k = 3$; $(a_1, a_2, a_3) = (3.1, 3.05, 3)$

Table 2 shows that for $k = 2.1$, there are two equilibria for $r = .5$, .75, and 1. Moreover, the agents' perceptions of the project are mixed in each. Consider, for instance, $r = .5$ in Table 2. Note that in the first equilibrium, only the most able agent ($a_1 = 3.1$) while in the second equilibrium, only the middle agent ($a_2 = 3.05$) perceives the project to be difficult. Note also that the agents' perceptions of the project are affected by the discount rate, signifying their patience to complete the project. Specifically, when the agents are sufficiently patient, i.e., $r = .01$ or .05, they all view the project to be easy, and when the agents are sufficiently impatient, i.e., $r = 5$ or 10, they all view it to be difficult.

The regions of the unique equilibrium in Proposition 1 are of special interest to us because agents' equilibrium behaviors sharply differ across the intrinsic projects. Inspecting Tables 1 and 3, it is evident that the higher ability agent exerts lower (resp. higher) effort for an intrinsically easy (resp. difficult) project. Proposition 2 generalizes these observations.

Proposition 2 *Suppose $a_1 \geq a_2 \geq ... \geq a_n$. Then, in the unique (interior) equilibrium*

(a) *for an intrinsically easy project, the higher ability agent works less and thus receives less credit: $x_1^* \leq x_2^* \leq ... \leq x_n^*$ and $q_1^* \leq q_2^* \leq ... \leq q_n^*$,*

(b) *for an intrinsically difficult project, the higher ability agent works more and thus receives more credit: $x_1^* \geq x_2^* \geq ... \geq x_n^*$ and $q_1^* \geq q_2^* \geq ... \geq q_n^*$,*

with strict inequalities whenever $a_i \neq a_j$.

Proof. Without loss of generality, take $a_i \geq a_j$. In equilibrium, $mb(X, r) = mc^i(x_i, X, r)$ by (10). Clearly, the left-hand side is equal for all agents. As for the right-hand side, $mc^i(x_i, X, r)$ is strictly decreasing in a_i since $c_i(x_i) = c(x_i)/a_i$. Moreover, by definition, $mc^i(x_i, X, r)$ is strictly decreasing in x_i for an intrinsically easy project. Hence, it must be that $x_i^* \leq x_j^*$ and in turn, $q_i^* \leq q_j^*$ by (3), with strict inequalities for $a_i \neq a_j$, proving part (a). Part (b) similarly follows because, by definition, $mc^i(x_i, X, r)$ is strictly increasing in x_i for an intrinsically difficult project. ■

To provide intuition for part (a), consider a two-member team with abilities $a_1 > a_2$ and a quadratic cost of effort, $c_i(x_i) = x_i^2/(2a_i)$, so that the project is intrinsically easy. Recall that given the public's credit allocation (q_1, q_2) and the teammate's effort choice, agent i's optimal effort satisfies the first-order condition in (9). Dividing them side by side for both agents and factoring out, we observe

$$\frac{q_1}{q_2} = \frac{(r + X - x_1/2)\,/a_1}{(r + X - x_2/2)\,/a_2}\left(\frac{x_1}{x_2}\right). \tag{11}$$

Suppose $q_1 \geq q_2$. Then, the public must believe that the higher ability agent has worked (weakly) harder: $x_1 \geq x_2$. Given $a_1 > a_2$, this would imply $\frac{q_1}{q_2} < \frac{x_1}{x_2}$ from (11). That is, expecting more credit for collective success, the higher ability agent would exploit the project's intrinsic ease and work disproportionately harder. Hence, his diligence would be inconsistent with the public's credit allocation in equilibrium because by (3), the allocation must be proportional to team members' efforts: $\frac{q_1}{q_2} = \frac{x_1}{x_2}$. Such inconsistency means that for an intrinsically easy project, the public cannot sufficiently compensate the higher ability agent in equilibrium if he is expected to work harder. As a result, the only consistent behavior in an interior equilibrium is that the higher ability agent exerts *lower* effort and receives less credit.[21] For an intrinsically difficult project, the problem of under-compensation does not arise since the marginal cost is too steep for the higher ability agent to exert disproportionate effort.

[21] Here, one may argue that the public can overcome the problem of insufficient credit in equilibrium by merely giving the higher ability agent the full credit for success: $q_1 = 1$ and $q_2 = 0$, which would mean a non-interior equilibrium and no collaboration between the two agents over an intrinsically easy project. Note, however, that agents cannot preclude each other from working solo on the breakthrough. Thus, as we show in Section 5, the higher ability agent may strictly prefer to team up to avoid costly competition. Besides, the agents may have committed to teaming up for various other reasons, including the use of shared resources and an assignment by the company's management. Hence, our focus on the unique interior equilibrium for the team is appropriate.

The equilibrium credit allocation in Proposition 2 raises an important question: can a higher ability agent be worse off than his lower ability teammate in equilibrium? Tables 1 and 3 suggest that the answer can be affirmative for an intrinsically easy project but not for an intrinsically difficult one. Furthermore, Table 2 shows that the expected payoffs can be non-monotonic in ability. For instance, for $r = .5$ in Table 2, the lowest expected payoff accrues to the highest ability team member in the first equilibrium and to the medium ability member in the second. The intuition for these observations lies in the tradeoff between an agent's desire to receive more credit for success and his dislike for working harder. To see the tradeoff formally, let u_i^* denote agent i's expected equilibrium payoff. From (4) and (9),

$$u_i^* = \frac{X^*}{r + X^*} q_i^* - \frac{c_i(x_i^*)}{r + X^*} = q_i^* - c_i'(x_i^*). \tag{12}$$

That is, in equilibrium, agent i's expected payoff is simply the difference between his expected credit and the marginal cost of effort. Despite the tradeoff, the next result confirms our numerical findings in Tables 1 and 3.

Proposition 3 *Suppose $a_1 \geq a_2 \geq ... \geq a_n$. Then, in the unique (interior) equilibrium*

- **(a)** *for an intrinsically easy project, the higher ability agent fares worse: $u_1^* \leq u_2^* \leq ... \leq u_n^*$,*
- **(b)** *for an intrinsically difficult project, the higher ability agent fares better: $u_1^* \geq u_2^* \geq ... \geq u_n^*$,*

with strict inequality whenever $a_i \neq a_j$.

To understand part (a), note that when taking on an intrinsically easy project, the higher ability agent commands a clear cost advantage both because he puts less effort in equilibrium and because the same level of effort is less onerous for him; that is, $a_1 > a_2$ implies $x_1^* < x_2^*$ and $c_1'(x_1^*) < c_2'(x_2^*)$. However, the higher ability agent is also attributed proportionally less credit by the public: $x_1^*/X^* < x_2^*/X^*$. Part (a) shows that the credit effect dominates. Put another way, the lower ability agent earns disproportionately more credit when the team undertakes an intrinsically easy project. Part (b) follows because when taking on an intrinsically difficult project, the diligence of the higher ability agent also discourages his teammates, allowing him to obtain disproportionately more credit for success than the increase in his cost. We complete this section by reporting some intuitive comparative statics with respect to agents' abilities and the discount rate.

Proposition 4 *Suppose $a_1 \geq a_2 \geq ... \geq a_n$. Then, for intrinsic projects, the equilibrium total effort is increasing in each agent's ability and the discount rate: $\partial X^*/\partial a_i > 0$ for all i, and $\partial X^*/\partial r > 0$. Moreover, as his teammates grow more able, each agent works harder in equilibrium for an intrinsically easy project, i.e., $\partial x_i^*/\partial a_j > 0$ for $i \neq j$, but less hard for an intrinsically difficult project, i.e., $\partial x_i^*/\partial a_j < 0$ for $i \neq j$.*

The fact that the inclusion of a more able agent improves the team's success rate, X^*, is expected. It is also expected that the team's success rate increases with discounting since impatient agents frontload their efforts. The second part of Proposition 4 follows because each agent views his effort as a "strategic complement" to the team effort (including his own), i.e., $\partial x_i/\partial X > 0$, for the intrinsically easy project while he views it to be a "strategic substitute" for the intrinsically difficult project, i.e., $\partial x_i/\partial X < 0$.[22] In particular, the agent is more concerned about cost savings when working for the latter type of project.

Armed with the characterizations of the social optimum and team equilibrium, we now compare them to identify inefficiencies.

4 Social optimum vs. team equilibrium

Given that each team member maximizes his own utility, it is unlikely that he will choose the socially optimal effort in equilibrium. For an intrinsically easy project, the inefficiency is evident since a higher ability team member works less in equilibrium, even though the social optimum characterized in Lemma 1 would dictate otherwise. For other types of projects, the inefficiency is not immediate, but we know that team members do not internalize the impact of their efforts on others. To determine the

[22]Due to the aggregative nature of the game, we find it more convenient to define the concepts of strategic complement and strategic substitute based on an agent's response to the total effort in our team setting. The reason is that, as mentioned above, an agent's optimal strategy can be expressed in terms of the team effort: $x_i = f(X, r, a_i)$.

equilibrium inefficiency, here we ask a more direct question: what exogenous, possibly non-Bayesian, credit profile would implement the socially optimal efforts in equilibrium? We answer this question next and then show that, contrary to conventional wisdom, some team members may overinvest if the team is sufficiently heterogenous.

4.1 Efficiency via exogenous credits

Suppose that before team members choose their efforts, the social planner publicly announces the following credit profile:

$$\mathbf{q}^{ST} = (q_1^{ST}, ..., q_n^{ST}) \in \mathbb{R}^n.$$

To focus purely on team incentives, we assume that $\mathbf{q}^{ST}$ has no explicit cost to the planner. For this credit profile to implement the socially optimal efforts in equilibrium, i.e., to engender $\mathbf{x}^S = \mathbf{x}^*$, it must satisfy agents' first-order conditions in (9). After simplification, this requires that

$$rq_i^{ST} = c_i'(x_i^S)(r + X^S) - c_i(x_i^S) \text{ for all } i. \tag{13}$$

In addition, $\mathbf{x}^S$ must satisfy the planner's first-order conditions in (7), which, by rearranging, implies that

$$r = c_i'(x_i^S)(r + X^S) - \sum_j c_j(x_j^S) \text{ for all } i. \tag{14}$$

From (13) and (14), Proposition 5 is immediate.

Proposition 5 *The socially optimal efforts are implemented as a team equilibrium by the following credit profile:*

$$q_i^{ST} = 1 + \frac{\sum_{\ell \neq i} c_\ell(x_\ell^S)}{r} \text{ for all } i. \tag{15}$$

Under (15), a higher ability agent receives less credit for team's success and a lower expected payoff: $q_i^{ST} < q_j^{ST}$ and $u_i^{ST} < u_j^{ST}$ for $a_i > a_j$.

Proof. (15) directly obtains from (13) and (14). To prove the rest, take $a_i > a_j$. Then, $c_i'(x_i^S) = c_j'(x_j^S)$ and $x_i^S > x_j^S$ by Lemma 1. Moreover, since $c_i(x_i) = \frac{x_i^k}{ka_i}$, we have that $c_i(x_i) = \frac{x_i}{k}c_i'(x_i)$ and, in turn, $c_i(x_i^S) > c_j(x_j^S)$. Next, note that (15) can be re-written: $q_i^{ST} = 1 + \frac{\sum_\ell c_\ell(x_\ell^S)}{r} - \frac{c_i(x_i^S)}{r}$. Hence, $q_i^{ST} < q_j^{ST}$ and $u_i^{ST} < u_j^{ST}$ by (12). ■

As expected, the planner assigns credit so that each team member receives the full return to his effort. In particular, agent i is offered a unit credit for marginally increasing the rate of discovery, i.e., $\partial X/\partial x_i = 1$, and an additional credit for saving his teammates from future effort costs owing to a faster discovery, i.e., $\left(\sum_{\ell \neq i} c_\ell(x_\ell^S)\right)/r$. Agent i is, however, not compensated for his own cost savings, as he internalizes them.

Proposition 5 reveals that to induce the efficient efforts in equilibrium, the team must be over-rewarded for success in that $\sum_i q_i^{ST} > 1$. That is, the team has to be distributed more surplus than it produces.[23] While interesting in itself, such "budget-breaking" requirement for efficiency in teams is known at least from Holmstrom (1982). An important implication of this observation in our context is that the (Bayesian) public necessarily balances the budget in allocating credits, namely $\sum_i q_i^* = 1$, and thus must provide too weak incentives for some team members. Proposition 5, however, goes further: for efficiency, *each* team member must be given more than the full credit for success. It is clear from (15) that such excessive rewarding of success is due to team externalities as well as the dynamics of the breakthrough. In particular, observe that agent i would receive no more than the full credit, $q_i^{ST} = 1$, if he worked solo or the agents were all myopic, i.e., $r \to \infty$.

Perhaps surprisingly, Proposition 5 also reveals that for efficiency, the higher ability agent must receive *less* credit for success and a *lower* utility, regardless of whether the project is perceived to be easy or difficult. The reason is that by Lemma 1, the higher ability agent works harder and thus incurs a higher cost of effort at the social optimum. Such diligence, however, means that his cost savings for others must be less than those of a lower ability agent's and in turn, deserves a less generous compensation via a credit. The utility comparison is then straightforward from (12).

[23] There is some evidence for this observation. Using the annual research activity of 661 MIT faculty over 31 years, Bikard et al. (2015) find that each scientist expects to receive 70% of the total credit when collaborating. In the same vein, an earlier study by Lindsey (1980) suggests giving the full credit to each co-author to account for peer perception (see also footnote 8). Incidentally, universities often reward their productive faculty internally in the form of reduced teaching, salary increases, or promotions above and beyond the recognition they receive from the scientific community for the same research.

It is important to note that the inverse relationship between one's ability and expected payoff in Proposition 5 obtains because the planner elicits the efficient efforts in equilibrium through individual credits. If the planner could contract directly on the effort levels, the higher ability agent would receive more credit and a higher payoff, as found in Lemma 1. Moreover, with such contractibility, the planner would be less generous in her credit assignment: $q_i^S < 1 < q_i^{ST}$ for all i. This also makes sense since when efforts are unobservable, the planner must over-reward the agents or pay them "rents" to induce the efficient efforts as a team equilibrium. In Section 5.2, we will show that the planner's over-rewarding of teams will encourage collaboration among agents.

4.2 Overinvestment in teams

The exogenous credit profile, $\mathbf{q}^{ST}$, stated in (15) implements the efficient efforts as a team equilibrium by compensating each agent for positive externalities on his peers. Since $q_i^{ST} \neq q_i^*$ for all i, it suggests that the equilibrium efforts are inefficient. Furthermore, since $q_i^* < 1 < q_i^{ST}$ for all i, i.e., the social planner over-rewards each team member with respect to the equilibrium, it also suggests that the equilibrium efforts must be all inefficiently low. This intuition is, however, incomplete in our model with endogenous credits. In the following two results, we show that while agents underinvest in a homogenous team, some may overinvest in a sufficiently heterogenous one.

Lemma 3 *Suppose the team is homogenous, i.e., $a_i = a$ for all i. Then, every agent underinvests, i.e., $x_i^* < x_i^S$ for all i.*

Lemma 3 obtains because, in a homogenous team, each agent expects to receive equal credit for the team's success in any equilibrium, which amounts to teamwork with an exogenous sharing rule of $1/n$. Hence, the agent's behavior is plagued with the standard free-riding incentive, resulting in the underinvestment.

This straightforward logic, however, does not extend to a heterogenous team, even though the agents still take the sharing rule or the credit allocation, $\mathbf{q}^*$, as given when choosing their efforts. To gain some intuition, consider a two-member team with abilities $(a_1, a_2) = (50, 2)$ and discount rate $r = .1$. Table 4 reports the unique equilibrium and socially optimal effort levels for project technologies $k = 1.5$ and $k = 3$.

k	x_1^*	x_1^S	x_2^*	x_2^S
1.5	0.004	5.877	0.516	0.009
3	1.866	1.801	0.052	0.360

Table 4. Over- vs. Under-investment in Teams

Note that for the intrinsically difficult project ($k = 3$), agent 1, being substantially more able, works virtually solo in equilibrium and expects to collect most of the credit for success ($q_1^* = .973$). Such equilibrium effort allocation is too unequal from the social perspective: the high ability agent overinvests ($x_1^* > x_1^S$) while the low ability agent underinvests ($x_2^* < x_2^S$) in the project. Unconcerned about the credit distribution within the team (see (5)), the planner would optimally allocate more balanced workload to the agents so that their marginal costs are equal. A similar argument also explains the direction of the inefficiency for the intrinsically easy project ($k = 1.5$): the high ability agent underinvests ($x_1^* < x_1^S$) while the low ability agent overinvests ($x_2^* > x_2^S$). In this case, we know from Proposition 2 that it is the low ability agent who is expected to undertake most of the work, which is clearly in contrast to the optimal effort allocation. We generalize these observations in Proposition 6.

Proposition 6 *Consider a two-member team with $a_1 > a_2$. For a sufficiently large a_1,*

(a) *the high ability agent underinvests whereas the low ability agent overinvests in an intrinsically easy project: $x_1^* < x_1^S$ and $x_2^* > x_2^S$,*

(b) *the high ability agent overinvests whereas the low ability agent underinvests in an intrinsically difficult project: $x_1^* > x_1^S$ and $x_2^* < x_2^S$.*

5 Solo work

Up to now, we have assumed that agents are committed to collaborating on the project. Such commitment may be unavoidable in many contexts. The project may be part of a broader research agenda; it may be assigned to a group of employees by their employer, or it is a policy proposal that a congressional committee brings forward. For many projects, though, agents are flexible in choosing whether to collaborate or work solo. Proposition 3 suggests that the highest ability agent would prefer to work solo at least for the intrinsically easy projects since he would fare the worst in a team. We will, however, argue that despite sharing the credit for success and enjoying no direct synergies from collaboration, agents may team up to avoid costly competition when they are of similar ability and sufficiently patient. To this end, we next consider the case of solo work and compare its equilibrium payoffs with those of teamwork. We then characterize the non-Bayesian credit profile that implements the efficient effort levels under solo work and show that the social planner would also encourage teamwork by *under-rewarding* solo work.

5.1 Equilibrium in solo work and incentives to team up

Suppose that each agent i takes on the project alone and exerts an unobservable flow effort x_i toward its completion. This means that the first successful agent receives the full credit of one from the public while the rest receives no credit. Hence, the competition among the agents becomes a standard R&D race, as in Lee and Wilde (1980), with a unit prize. Similar to (4), it is readily verified that conditional on having no winner yet, agent i's expected payoff is found to be[24]

$$\widehat{u}_i = \frac{x_i}{r+X} - \frac{c_i(x_i)}{r+X}. \tag{16}$$

Interpreting, again, the interest rate r as the nature's fixed flow effort to "steal" the discovery, the first term $x_i/(r+X)$ is simply the probability that agent i wins the race, and the second term $c_i(x_i)/(r+X)$ is his expected total cost until there is a winner.

In (Nash) equilibrium, agent i maximizes (16) with respect to x_i given others' total effort X_{-i}. The first-order condition of his maximization requires that[25]

$$r + X_{-i} = c_i'(x_i)(r+X) - c_i(x_i). \tag{17}$$

The equilibrium profile of efforts denoted by $\mathbf{x}^L = (x_1^L, ..., x_n^L)$ solves (17) for $i = 1, ..., n$. Let $\widehat{u}_i^L$ be the resulting expected payoff for agent i.

[24] Formally, having no winner by time t, agent i wins the race in the next instant with probability $x_i dt$ and obtains the full credit of 1. Hence, (16) obtains from

$$\widehat{u}_i = \int_0^\infty e^{-rt}e^{-Xt}\left(x_i - c_i(x_i)\right)dt.$$

[25] The second-order condition is satisfied.

Lemma 4 *Under solo work, there is an equilibrium. Moreover, in every equilibrium, the higher ability agent competes harder and receives a higher expected payoff:* $x_i^L > x_j^L$ *and* $\widehat{u}_i^L > \widehat{u}_j^L$ *for* $a_i > a_j$.

Under solo work, every equilibrium must be interior, i.e., $x_i^L > 0$ for all i because, unlike in teamwork, each agent expects an exogenous reward from the success.[26] In addition, exploiting his cost advantage toward the same reward, the higher ability agent exerts greater effort and is better off than his lower ability rivals. To understand the nature of the competition among the agents, it can be seen from (17) that their efforts are strategic complements: agents are motivated by others' effort.[27] As such, the competition is expected to grow most intense among homogenous agents. We, therefore, predict that such agents will have an incentive to team up even if that means sharing the credit for success. Proposition 7 confirms our prediction when the agents are sufficiently patient.

Proposition 7 *Assume homogenous agents, i.e.,* $a_i = a$ *for all i. Then, each agent is strictly worse off by working solo than by working with others as a team:* $\widehat{u}^L < u^*$ *as* $r \to 0$.

Note that when they work as a team and thus, face no competition, patient agents can afford to postpone the discovery by exerting little effort each time. The same strategy is not optimal for patient agents racing to success. Moreover, in a homogenous population, each agent expects to receive a reward of $1/n$ in equilibrium due to equal credit sharing under teamwork and an equal probability of winning under solo work. Hence, patient agents strictly prefer to team up and avoid costly competition when they are identical.

Three remarks about Proposition 7 are in order. First, the result holds regardless of whether agents perceive the project to be easy or difficult. Second, the result offers an equilibrium theory of team formation when there are only two agents, e.g., two authors, which is common in scientific research.[28] And third, by the continuity

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[26] Formally, $\left.\frac{\partial \widehat{u}_i}{\partial x_i}\right|_{x_i=0} = \frac{1}{r+X} > 0$.

[27] Differentiating (17) reveals $\frac{\partial x_i}{\partial X_{-i}} = \frac{1-c_i'(x_i)}{c_i''(x_i)(r+X)+1-c_i'(x_i)} > 0$ since, combining (16) and (17), we observe that $1 - c_i'(x_i) = \widehat{u}_i > 0$ in equilibrium.

[28] For an arbitrary number of agents, one needs to consider all possible subteams. We leave this interesting issue for future research.

of payoffs, the result extends to agents who are not too heterogeneous and not too impatient. We illustrate the last two remarks by a numerical example.

Consider two agents with abilities $(a_1, a_2) = (3, 2.8)$, and a quadratic cost technology, $k = 2$, referring to an intrinsically easy project. Table 5 reports that agents have a strict incentive to team up for interest rates $r \leq .2$.

r	x_1^*	x_1^L	x_2^*	x_2^L	u_1^*	u_1^L	u_2^*	u_2^L
.01	.085	1.976	.106	1.895	.416	.341	.518	.323
.05	.179	1.989	.228	1.908	.380	.337	.479	.319
.1	.244	2.006	.315	1.922	.355	.331	.452	.313
.15	.289	2.021	.379	1.936	.336	.326	.432	.308
.2	.325	2.036	.431	1.950	.322	.321	.416	.304

Table 5. Team- vs. Solo Work

Take, for instance, $r = .1$. Consistent with Proposition 2(a), the high ability agent fares worse in teamwork, $u_1^* = .355 < .452 = u_2^*$. This does not, however, mean that he would operate solo since he would then have to compete with the low ability agent for the breakthrough. Although the high ability agent would be better off than his rival in this competition, $u_1^L = .331 > .313 = u_2^L$, he would be worse off than working as a team, i.e., $u_1^L < u_1^*$. Hence, agent 1 has a strict incentive to team up. Since the same preference is shared by the low ability agent, the two would collaborate and do so despite having no direct synergies between them in our model.[29]

5.2 Efficient credits in solo work

Proposition 7 identifies agents' incentives to collaborate when the rewards are endogenously given by the public. It is also edifying and policy relevant to determine team formation incentives when the rewards are exogenously given by a social planner who wants to induce the optimal efforts. Note that the optimal effort profile under solo work is the same as under teamwork, $\mathbf{x}^S$, found in Lemma 1 since, summing up

[29]For $r > .2$, our simulations indicate that agent 1 is worse off whereas agent 2 is better off teaming up.

(16), welfare coincides with (5):[30]

$$\sum_i \widehat{u}_i = \frac{X}{r+X} - \frac{\sum_i c_i(x_i)}{r+X} = W.$$

Proceeding as in Section 4.1, suppose that before the agents choose their efforts under solo work, the social planner publicly announces the following credit profile for success:

$$\mathbf{q}^{SL} = (q_1^{SL}, ..., q_n^{SL}) \in \mathbb{R}^n. \tag{18}$$

Given $\mathbf{q}^{SL}$, agent i's expected payoff in (16) is modified to be

$$\widehat{u}_i = \frac{x_i}{r+X} q_i^{SL} - \frac{c_i(x_i)}{r+X}, \tag{19}$$

which implies the first-order condition:

$$(r + X_{-i})\, q_i^{SL} = c_i'(x_i)(r+X) - c_i(x_i). \tag{20}$$

For $\mathbf{q}^{SL}$ to implement the optimal effort profile under solo work, it must be that $\mathbf{x}^S = \mathbf{x}^L$. Hence, evaluating (20) at $\mathbf{x}^S$ and comparing it with the social planner's first-order condition in (14), we reach the next conclusion.

Proposition 8 *The socially optimal efforts are implemented as an equilibrium under solo work by the following credit profile:*

$$q_i^{SL} = \frac{r + \sum_{\ell \neq i} c_\ell(x_\ell^S)}{r + X_{-i}} \text{ for all } i. \tag{21}$$

In addition, under (21),

(a) *each agent is under-rewarded for success:* $q_i^{SL} < 1$ *for all* i*,*

(b) *a higher ability agent receives more credit for success and a higher expected payoff:* $q_i^{SL} > q_j^{SL}$ *and* $\widehat{u}_i^{SL} > \widehat{u}_j^{SL}$ *for* $a_i > a_j$*.*

As with teamwork, the social planner sets the credit profile $\mathbf{q}^{SL}$ so that agent i is compensated for the externalities on his peers. Comparing (21) with (15), it is evident that the only difference between them is that under solo work, agent i's credit is

[30] This coincidence is not surprising because, in both settings, the planner is concerned about the rate of discovery and the total cost but not about who makes the discovery.

further discounted by the discovery rate of his rivals', X_{-i}. Since agent i internalizes such competitive pressure when choosing his effort, he is not rewarded for it. In fact, Proposition 8 says that the presence of competition causes him to be under-rewarded: $q_i^{SL} < 1$. That is, to induce the optimal efforts, the planner would commit to recognizing an agent's achievement only partially, thereby softening the competition. This clearly contrasts with the Bayesian public who would assign the full credit of one ex post to the successful solo achiever. Proposition 8 also says that a higher ability agent receives more credit for success and is better off than his lower ability rivals as a result. While intuitive, this finding clashes with that for teamwork in Proposition 5. The reason is that the competitive pressure mentioned above is lower for a higher ability agent, and to motivate him, the planner discounts his reward less heavily.

From Propositions 5 and 8, it is immediate that $q_i^{SL} < 1 < q_i^{ST}$ for all i; that is, relying on the competition as an incentive mechanism, the planner rewards each agent less generously under solo work than under teamwork to elicit the same optimal effort profile, $\mathbf{x}^S$. Hence, given the exogenous credit schemes $\mathbf{q}^{ST}$ and $\mathbf{q}^{SL}$, the agents strictly prefer to team up, as we formally state in the following result.

Proposition 9 *Given the credit profiles $\mathbf{q}^{ST}$ and $\mathbf{q}^{SL}$ in (15) and (21), respectively, each agent strictly prefers teamwork to solo work; i.e., $\widehat{u}_i^{SL} < u_i^{ST}$ for all i.*

Note that Proposition 9 holds for all agents regardless of their abilities and patience because each is over-rewarded under teamwork for exerting the same effort, x_i^S. As such, compared with Proposition 7, it implies that the society, e.g. the scientific community, provides the agents with stronger incentives to team up than the Bayesian public.

6 Observable effort

A key obstacle for credit attribution in teamwork is that individual efforts are unobservable to the public. While this monitoring problem is likely to be severe in cases where the public is in an arms-length relationship with the team, it may be less so in others. For instance, in academia, it is not uncommon that researchers discuss their ongoing projects with their colleagues and regularly present them at conferences. Similarly, in business, managers can often track employees' work hours.

Since credit attribution affects team incentives, it is natural to ask how the observability of effort by the public may change these incentives. To this end, suppose that, unlike in Section 3.2, the public can perfectly observe, though cannot dictate, team members' flow of efforts. The same level of monitoring may or may not be available within the team, but this is irrelevant in our setting because efforts are chosen simultaneously. Hence, given his teammates' flow effort X_{-i}, agent i expects to receive the credit $q_i = \frac{x_i}{x_i + X_{-i}}$, which is increasing in his own effort, x_i. This means that the observability of effort by the public creates competition for credit within the team.

To see the amount of competition, we substitute for q_i in (4) and find the following expected payoff for agent i under the observability:

$$\begin{aligned} u_i^O &= \frac{X}{r+X}\left(\frac{x_i}{X}\right) - \frac{c_i(x_i)}{r+X} \\ &= \frac{x_i}{r+X} - \frac{c_i(x_i)}{r+X}. \end{aligned} \tag{22}$$

Clearly, (22) coincides with (16). In words, teamwork with observable efforts is strategically equivalent to solo work for the agents. From this equivalence and our results in Section 5.1, two main insights emerge.

First, Lemma 4 implies that when the public can monitor efforts, a higher ability agent always works harder and receives more credit from success. Second, Proposition 7 reveals that team members dislike such public monitoring if they are of similar ability and sufficiently patient. The reason is that the lack of monitoring allows them to commit not to compete for credit. Interestingly, as was demonstrated in Table 5, despite his competitive advantage, a higher ability agent would also endorse no monitoring by the public even though he would fare worse than his lower ability teammate when collaborating in an intrinsically easy project.

7 Conclusion

Proper credit for scientific discovery plays a key role in the progress of science, as it affects appointments, promotions, and funding for researchers. Assigning credit, however, has grown complicated by the increasing dominance of collaborative work across many disciplines. A similar problem also exists in non-academic settings such as business and politics, where teamwork is prevalent. In this paper, we have examined in some detail the endogenous relationship between credit attribution and team incentives in a tractable model. Our main results are as follows.

First, it is the most diligent – not necessarily the most able – team member who deserves the most credit for collective success. And the identity of that member depends on the team's composition and the project's difficulty. In particular, it is the least able member who works the hardest for an "intrinsically easy" project and thus receives the most credit for success, while the opposite holds for an "intrinsically difficult" project. Second, team equilibrium may exhibit over-investment. We show that in a two-member team with sufficient ability differential, expecting to receive most of the credit, the low (resp. high) ability agent almost works solo in an intrinsically easy (resp. difficult) project. This workload allocation, however, is too unbalanced from the social viewpoint. Third, to induce efficient efforts, the team must be substantially over-rewarded to mitigate the moral hazard. In contrast, efficiency requires under-rewarding solo work to soften competition. Fourth, both the equilibrium and social credit attributions give agents incentives to team up, which may explain the prevalence of teamwork even without synergies. Last, but not least, we show that team members may dislike public monitoring of their efforts for proper credit because such monitoring also creates competition within the team.

While contributing to the ongoing debate on credit attribution in teamwork, our investigation has only scratched the surface. For one, unlike in our model, many team projects require multiple breakthroughs or milestones to be completed. It would be interesting to explore credit allocation in such complex projects. Furthermore, team projects may involve a deadline, especially when they are sponsored. The deadline is likely to render effort choices non-stationary and introduce credit attribution dynamics. For instance, if the project is completed much earlier than the deadline, will the low- or high-ability team member get more of the credit? Finally, though assumed away in our model for tractability, agents often accumulate knowledge, i.e., they learn from their past failures. Such learning is likely to affect their perceptions of the project's difficulty. Thus, as with deadlines, knowledge accumulation is expected to generate nontrivial effort and credit attribution dynamics. We leave these extensions for future research.

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1 Appendix for online publication

Proof of Lemma 1. As established in the text, the solution to (SO) must equalize marginal costs across agents; that is,

$$c_i'(x_i) = z \geq 0 \text{ for all } i. \tag{A-1}$$

Since $c_i(x_i) = \frac{c(x_i)}{a_i}$ and $c' > 0$, (A-1) can be inverted as

$$x_i = \phi(a_i z), \tag{A-2}$$

where $\phi \equiv c'^{-1}$, $\phi(0) = 0$, and $\phi' > 0$.

Using (A-2), the first-order condition in (7) can be re-written:

$$\Gamma(z) \equiv (r + \sum_i \phi(a_i z))z - \sum_i c_i(\phi(a_i z)) - r = 0. \tag{A-3}$$

Clearly, $\Gamma(0) = -r < 0$. Moreover,

$$\begin{aligned} \Gamma(1) &= \sum_i \phi(a_i) - \sum_i c_i(\phi(a_i)) \\ &= \sum_i (x_i - c_i(x_i)) \\ &> \sum_i x_i \left(1 - c_i'(x_i)\right) \\ &= 0, \end{aligned}$$

where we employ the facts that $\frac{c_i}{x_i} < c_i'$ (since $c_i'' > 0$) and $c_i'(x_i) = z = 1$.

Hence, $\Gamma(1) > 0$ and, by continuity, there is an interior solution, $z^S \in (0,1)$, to (A-3). Next, we observe that

$$\begin{aligned} \Gamma'(z) &= r + \sum_i \phi(a_i z) + z \sum_i \phi'(a_i z) a_i - \sum_i \underbrace{c_i'(\phi(a_i z))}_{=z} \phi'(a_i z) a_i \\ &= r + \sum_i \phi(a_i z) \\ &> 0, \end{aligned} \tag{A-4}$$

which establishes that z^S is unique.

As a result, there is a unique optimal solution to (SO) such that $x_i^S = \phi(a_i z^S) > 0$, proving part (a).

Part (b) readily follows because, for $a_i > a_j$,

$$x_i^S = \phi(a_i z^S) > \phi(a_j z^S) = x_j^S,$$

and in turn, $q_i^S = x_i^S / X^S > x_j^S / X^S = q_j^S$.

Finally, by definition,

$$u_i^S \equiv \frac{X^S}{r + X^S} q_i^S - \frac{c_i(x_i^S)}{r + X^S} = \frac{x_i^S - c_i(x_i^S)}{r + X^S}.$$

Moreover, $\partial(x_i - c_i(x_i))/\partial x_i = 1 - c_i'(x_i) > 0$ for $c_i'(x_i) < 1$, which is true at the optimum. Hence, $u_i^S > u_j^S$ since $x_i^S > x_j^S$. ■

Proof of Lemma 2. First, define

$$\Phi(x_i, X, r) \equiv \frac{c'(x_i)(r + X) - c(x_i)}{x_i}. \tag{A-5}$$

Next, observe from (10) that $mc^i(x_i, X, r) = \Phi(x_i, X, r) \frac{X}{(r+X)^2 a_i}$, which implies

$$mc^i_{x_i} \overset{sign}{=} \Phi_{x_i}, \tag{A-6}$$

where the subscripts refer to partial derivatives throughout.

Clearly,

$$\Phi_{x_i} = \left(\frac{c'(x_i)}{x_i} \right)' (r + X) - \left(\frac{c(x_i)}{x_i} \right)'. \tag{A-7}$$

Since

$$\left(\tfrac{c(x_i)}{x_i} \right)' \overset{sign}{=} c''(x_i) > 0 \quad \text{and} \quad \left(\tfrac{c'(x_i)}{x_i} \right)' \overset{sign}{=} c'''(x_i),$$

it follows that if $c'''(x_i) \leq 0$ for all x_i, then

$$\Phi_{x_i} < 0 \text{ for all } x_i, X_{-i}, r. \tag{A-8}$$

Conversely, suppose (A-8) holds, but $c'''(x_i) > 0$ for some x_i. Then, (A-7) would imply that $\Phi_{x_i} > 0$ for $r + X_{-i} \to \infty$, a contradiction. Hence, by (A-6), the project is *intrinsically easy* if and only if $c'''(x_i) \leq 0$ for all x_i. For the iso-elastic specification, $c(x_i) = x_i^k / k$, it is readily verified that

$$c'''(x_i) \leq 0 \Longleftrightarrow 1 < k \leq 2.$$

Next, by (A-6), the project is *intrinsically difficult* if and only if $\Phi_{x_i} > 0$ for all x_i, X_{-i}, r, which, by (A-7), requires that $c'''(x_i) > 0$ for all x_i. Hence, the project is intrinsically difficult if and only if $\Phi_{x_i} \geq 0$ for $r = X_{-i} = 0$ and all x_i, or equivalently,

$$\left(\frac{c'(x_i)}{x_i}\right)' x_i - \left(\frac{c(x_i)}{x_i}\right)' \geq 0 \text{ for all } x_i. \tag{A-9}$$

For $c(x_i) = x_i^k/k$, (A-9) is satisfied if and only if $(k-2) - \frac{k-1}{k} \geq 0$, or $k \geq \frac{3+\sqrt{5}}{2} \approx 2.62$, as claimed. ■

Proof of Proposition 1. We offer a more general proof here by imposing the following three cost conditions. It is straightforward to verify that the iso-elastic specification, $c(x) = x^k/k$ with $k > 1$, satisfies them.

$$\lim_{x\to\infty} \frac{c(x)}{xc'(x)} < 1 \tag{C1}$$

$$\lim_{x\to 0} \frac{xc''(x)}{c'(x)} < 1 \text{ if } c''(0) = \infty. \tag{C2}$$

$$c''(0) = 0 \text{ if } c''' > 0. \tag{C3}$$

As a preliminary, we use $\Phi(x_i, X, r)$ from (A-5) and define

$$m(x_i, X, r) \equiv X\Phi(x_i, X, r). \tag{A-10}$$

Then, given that $c_i(x_i) = \frac{c(x_i)}{a_i}$, the first-order condition in (10) can be written as

$$m(x_i, X, r) = ra_i. \tag{A-11}$$

Clearly,

$$m_{x_i} \overset{sign}{=} \Phi_{x_i}. \tag{A-12}$$

Depending on the $sign[\Phi_{x_i}]$, we consider two cases in turn.

Case 1. $c'''(x_i) \leq 0$ *for all* x_i *(*$k \in (1, 2]$ *for* $c(x) = x^k/k$*).*

From Lemma 2, this case refers to the intrinsically easy project. In particular, $\Phi_{x_i} < 0$ and thus, $m_{x_i} < 0$. Given $X > 0$, there is a unique solution $x_i \in [0, X]$ to (A-11) if and only if

$$A(X) \leq ra_i \leq B(X)$$

where

$$A(X) \equiv m(X,X,r) = c'(X)(r+X) - c(X), \tag{A-13}$$

$$B(X) \equiv m(0,X,r) = c''(0)(r+X)X.$$

Clearly, $A(0) = 0$ and, by (C1), $A(\infty) = \infty$. Furthermore, $A'(X) = c''(X)(r+X) > 0$. Hence, there is a unique cutoff $0 < X_i < \infty$ such that $A(X) \leq ra_i$ for $X \leq X_i$. For $B(X) \geq ra_i$, we consider two subcases.

Case 1.1. *$c''(0) = \infty$ ($k < 2$ for $c(x) = x^k/k$).*

Then, $B(X) > ra_i$ for all $X > 0$. This implies that there is a unique solution

$$x_i = f(X, r, a_i), \tag{A-14}$$

to (A-11) if and only if $X \in [0, X_i]$, with $f(0, r, a_i) = 0$ and $f(X_i, r, a_i) = X_i$.

Substituting (A-14) into (A-11), we re-write (A-11) as

$$\Phi(f(X,r,a_i), X, r) = \frac{ra_i}{X}, \tag{A-15}$$

which implies

$$f_X(., a_i) = -\frac{\frac{ra_i}{X^2} + \Phi_X}{\Phi_{x_i}} \tag{A-16}$$

and

$$f_{a_i}(., a_i) = \frac{r}{X\Phi_{x_i}}. \tag{A-17}$$

From (A-5), we obtain

$$\Phi_X = \frac{c'(x_i)}{x_i} > 0 \text{ and} \tag{A-18}$$

$$\Phi_{x_i} = \frac{1}{x_i}\left[c''(x_i)(r+X) - c'(x_i) - \Phi\right]. \tag{A-19}$$

Substituting (A-18) and (A-19) into (A-16), and using $\Phi_{x_i} < 0$, it follows that

$$\begin{aligned} f_X(., a_i) &= \frac{\frac{x_i}{X}\Phi + c'(x_i)}{\Phi + c'(x_i) - c''(x_i)(r+X)} \\ &> \frac{\frac{x_i}{X}\Phi + c'(x_i)}{\Phi + c'(x_i)} \\ &\geq \frac{x_i}{X}. \end{aligned} \tag{A-20}$$

Hence, using (A-14),

$$f_X(.,a_i) > \frac{f(X,r,a_i)}{X}. \tag{A-21}$$

Next, note that summing (A-14) across agents, the equilibrium total effort X must solve

$$h(X,r,\mathbf{a}) \equiv \sum_i f(X,r,a_i) - X = 0. \tag{A-22}$$

Moreover, note from (A-21) that

$$h_X(.) = \sum_i f_X(.,a_i) - 1 > 0 \text{ whenever } h(.) = 0. \tag{A-23}$$

Hence, if a solution to (A-22) exists, it must be unique.

We now establish the existence. First, observe that for any j,

$$\begin{aligned} h(X_j,r,\mathbf{a}) &= \sum_{i\neq j} f(X_j,r,a_i) + \underbrace{f(X_j,r,a_j) - X_j}_{=0} \\ &> 0. \end{aligned} \tag{A-24}$$

Second, since $h(0,.) = 0$, to complete the proof, we need to establish that $h(\widehat{X},.) < 0$ for some $\widehat{X} > 0$. To do so, it suffices to show that $\lim_{X\to 0} f_X(.,a_i) = 0$, which, by (A-23), will reveal that $\lim_{X\to 0} h_X(.) = -1 < 0$.

Dividing both the numerator and denominator on the right-hand side of (A-20) by Φ, we have that

$$f_X(.,a_i) = \frac{\frac{x_i}{X} + \frac{c'(x_i)}{\Phi}}{1 + \frac{c'(x_i)}{\Phi} - \frac{c''(x_i)(r+X)}{\Phi}}. \tag{A-25}$$

Using the definition of $\Phi(.)$ in (A-5), we can re-write the last term in the denominator in (A-25) as

$$\frac{c''(x_i)(r+X)}{\Phi} = \frac{x_i c''(x_i)}{c'(x_i)(r+X) - c(x_i)}(r+X). \tag{A-26}$$

Since $x_i = f(X,r,a_i)$ and $f(0,r,a_i) = 0$, it follows, from (A-26) and (C2), that

$$\lim_{X\to 0} \frac{c''(x_i)(r+X)}{\Phi} = \lim_{x_i\to 0} \frac{x_i c''(x_i)}{c'(x_i)} < 1. \tag{A-27}$$

For the other terms in (A-25), we observe

$$\lim_{X\to 0} \Phi(x_i,X,r) = \infty, \quad \lim_{X\to 0} f(X,r,a_i) = 0 \text{ and } \quad \lim_{X\to 0} \frac{f(X,r,a_i)}{X} = 0. \tag{A-28}$$

Applying (A-27) and (A-28) in (A-25), we, therefore, obtain

$$\begin{aligned}\lim_{X\to 0} f_X(.,a_i) &= \lim_{X\to 0} \frac{\frac{x_i}{X}+\frac{c'(x_i)}{\Phi}}{1+\frac{c'(x_i)}{\Phi}-\frac{c''(x_i)(r+X)}{\Phi}} \\ &= 0.\end{aligned}$$

Hence, there exists some $\widehat{X} > 0$ such that $h(\widehat{X}, .) < 0$, and in turn, a unique solution $X^* > 0$ to (A-22). Given $f_X(., a_i) > 0$ by (A-21), we find $x_i^* = f(X^*, r, a_i) > 0$, proving the existence and uniqueness of an interior equilibrium when $c''(0) = \infty$.

Case 1.2. $c''(0) < \infty$ *($k = 2$ for $c(x) = x^k/k$).*

Then, from (A-13), we have that $B(X) > ra_i$ if and only if $X > Z_i$, where

$$Z_i = \frac{1}{2}\left(-r+\sqrt{r^2+\frac{4ra_i}{c''(0)}}\right) > 0.$$

Recall that $A(X) \le ra_i$ for $X \le X_i$. We next show $Z_i < X_i$. To do so, it suffices to show that $A(X) < B(X)$ for $X > 0$. Let $\Delta(X) \equiv B(X) - A(X)$. Then, from (A-13),

$$\Delta(X) = c''(0)(r+X)X - c'(X)(r+X) + c(X).$$

Clearly, $\Delta(0) = 0$, and

$$\begin{aligned}\Delta'(X) &= c''(0)(r+2X) - c''(X)(r+X) \qquad \text{(A-29)} \\ &\ge c''(X)(r+2X) - c''(X)(r+X) \text{ (since } c'''(X) \le 0) \\ &= Xc''(X). \\ &> 0 \text{ for } X > 0.\end{aligned}$$

Hence, $\Delta'(X) > 0$ and, in turn, $\Delta(X) > 0$ for $X > 0$, establishing that $A(X) < B(X)$ for $X > 0$. As a result, $f(X, a_i, r) \in [0, X]$ if and only if $X \in [Z_i, X_i]$, with $f(Z_i, r, a_i) = 0$.

In sum, together with (A-24), when $c''(0) < \infty$, an interior equilibrium exists if and only if

$$h(Z_i, r, \mathbf{a}) < 0 \text{ for all } i.$$

Case 2. $c'''(x_i) > 0$ *for all* x_i *($k > 2$ for $c(x) = x^k/k$).*

First, by (C3), $c''(0) = 0$. Hence, $B(X) = 0$ in (A-13). Moreover, $ra_i \leq A(X)$ whenever $X \geq X_i$. Hence, there is a solution

$$x_i = f(X, r, a_i) \in [0, X]$$

to (A-11) if and only if $X \geq X_i$, with $f(X_i, r, a_i) = X_i$. It follows from (A-22) that $h(X_i, r, \mathbf{a}) > 0$ for any i. Furthermore, $\lim_{X \to \infty} h(X, r, \mathbf{a}) = -\infty$. Therefore, there exists an $X^* > 0$ that solves (A-22). In particular, $X^* > \max_i X_i$, implying that $x_i^* > 0$ for all i.

Suppose now the project is intrinsically difficult, i.e., $mc_{x_i}^i > 0$ (and hence $\Phi_{x_i} > 0$) for all i. It follows from (A-16) that $f_X(., a_i) < 0$. This further implies, from (A-23), that $h_X < 0$ whenever $h(.) = 0$, proving the uniqueness of the interior equilibrium in this case. ■

Proof of Proposition 3. To prove part (a), suppose the project is intrinsically easy: $c'''(x_i) \leq 0$ and thus $\Phi_{x_i} < 0$, where $\Phi = \Phi(x_i, X, r)$ is as defined in (A-5). In this part, we also impose the following cost condition:

$$\left(\frac{c(x_i)}{c'(x_i)}\right)'' \leq 0 \text{ for all } x_i, \tag{C4}$$

which is, again, satisfied by the iso-elastic cost: $c(x_i) = x_i^k/k$ with $k > 1$.

Consider agent i's equilibrium utility described in (12). Given $q_i^* = x_i^*/X^*$, it can be re-written as

$$u_i^* = \frac{x_i^*}{X^*} - \frac{c'(x_i^*)}{a_i}. \tag{A-30}$$

Next recall from the previous proof that $x_i^* = f(X^*, r, a_i)$. Substituting for x_i^* and dropping the star sign for simplicity here, (A-30) becomes

$$u_i = \frac{f(X, r, a_i)}{X} - \frac{c'(f(X, r, a_i))}{a_i} \equiv U(X, r, a_i). \tag{A-31}$$

It is evident from (A-31) that to obtain the reverse utility ordering in part (a), it suffices to show that

$$U_{a_i}(X, r, a_i) < 0.$$

To this end, we partially differentiate (A-31) with respect to a_i and substitute back for $x_i = f(X, r, a_i)$ to find

$$U_{a_i} = \left[\frac{1}{X} - \frac{c''(x_i)}{a_i}\right] f_{a_i} + \frac{c'(x_i)}{a_i^2}. \tag{A-32}$$

Since $f_{a_i} = \frac{r}{X\Phi_{x_i}}$ by (A-17), we observe

$$\begin{aligned} U_{a_i} \text{ ¡ } 0 \iff & \frac{c'(x_i)}{a_i^2} < -\left[\frac{1}{X} - \frac{c''(x_i)}{a_i}\right]\frac{r}{X\Phi_{x_i}} \\ \iff & c'(x_i) < -\left[\frac{a_i}{X} - c''(x_i)\right]\frac{ra_i}{X\Phi_{x_i}} \\ \iff & -c'(x_i)\Phi_{x_i} < \left[\frac{a_i}{X} - c''(x_i)\right]\frac{ra_i}{X}, \end{aligned} \tag{A-33}$$

where the last line follows because $\Phi_{x_i} < 0$.

Collecting terms, we further observe

$$\begin{aligned} U_{a_i} \text{ ¡ } 0 \iff & -c'(x_i)\Phi_{x_i} + c''(x_i)\frac{ra_i}{X} < \frac{ra_i^2}{X^2} \\ \iff & r\left[-c'(x_i)\Phi_{x_i} + c''(x_i)\frac{ra_i}{X}\right] < \left(\frac{ra_i}{X}\right)^2 \\ \iff & r\left[-c'(x_i)\Phi_{x_i} + c''(x_i)\Phi\right] < \Phi^2 \\ \iff & r\frac{-c'(x_i)\Phi_{x_i} + c''(x_i)\Phi}{\Phi^2} < 1 \\ \iff & r\frac{\partial}{\partial x_i}\left[\frac{c'(x_i)}{\Phi}\right] < 1, \end{aligned} \tag{A-34}$$

where the third line follows because $\Phi = \frac{ra_i}{X}$ by (A-10) and (A-11).

Next, by the definition of Φ from (A-5), note that

$$\begin{aligned} \frac{c'(x_i)}{\Phi} &= \frac{c'(x_i)}{\frac{c'(x_i)(r+X)-c(x_i)}{x_i}} \\ &= \frac{x_i}{r + X - \underbrace{\frac{c(x_i)}{c'(x_i)}}_{\equiv g(x_i)}}. \end{aligned}$$

Hence,

$$\frac{\partial}{\partial x_i}\left[\frac{c'(x_i)}{\Phi}\right] = \frac{r + X - g(x_i) + x_i g'(x_i)}{(r + X - g(x_i))^2}.$$

Since $g''(x_i) \leq 0$ by the cost condition C4 above, we have that $-g(x_i) + x_i g'(x_i) \leq 0$. Moreover, $g(x_i) \leq x_i$ since $c''(x_i) > 0$. Therefore, from (A-34),

$$U_{a_i} < 0 \text{ if } \frac{r(r+X)}{(r+X-x_i)^2} < 1.$$

Now recall from Proposition 2 that for an intrinsically easy project, the ability profile $a_1 \geq ... \geq a_n$ implies $x_1 \leq ... \leq x_n$ in equilibrium. Hence, $x_i \leq \frac{X}{2}$ for all $i \neq n$ since for a positive set of real numbers, there can be *at most* one that is strictly larger than the half of their sum. For $i \neq n$, we thus conclude

$$\frac{r(r+X)}{(r+X-x_i)^2} \leq \frac{r(r+X)}{(r+X-\frac{X}{2})^2} = \frac{r(r+X)}{r(r+X)+\frac{X^2}{4}} < 1,$$

which proves that $U_{a_i}(X, r, a_i) < 0$ for $i \neq n$ and in turn,

$$U(X, r, a_1) \leq U(X, r, a_2) \leq ... \leq U(X, r, a_n),$$

with strict inequality whenever $a_i \neq a_j$, as desired.

To prove part (b), suppose the project is intrinsically difficult: $mc^i_{x_i} > 0$ and thus, $\Phi_{x_i} > 0$. From (4), it follows that in equilibrium,

$$\begin{aligned} u_i^* &= \frac{X^*}{r+X^*} q_i^* - \frac{c_i(x_i^*)}{r+X^*} \\ &= \frac{x_i^*}{r+X^*} - \frac{c_i(x_i^*)}{r+X^*} \\ &\propto x_i^* - \frac{c(x_i^*)}{a_i} \\ &= f(X^*, r, a_i) - \frac{c(f(X^*, r, a_i))}{a_i}. \end{aligned} \quad \text{(A-35)}$$

Recall from (A-17) that $f_{a_i} > 0$ when $\Phi_{x_i} > 0$. Hence, given X^*, we find

$$\frac{\partial u_i^*}{\partial a_i} \propto \underbrace{[1 - c_i'(f(.))]}_{(+)} \underbrace{f_{a_i}}_{(+)} + \frac{c(.)}{a_i^2} > 0, \quad \text{(A-36)}$$

where $1 - c_i'(f(.)) > 0$ by (12). From (A-36), the utility ordering in part (b) is immediate. ■

Proof of Proposition 4. We prove the results for each intrinsic project.

(a) Consider an intrinsically easy project: $c''' \leq 0$ and thus $\Phi_{x_i} < 0$, where $\Phi = \Phi(x_i, X, r)$ is as defined in (A-5). Below we show that **(i)** $\frac{\partial X^*}{\partial a_i} > 0$ for all i, **(ii)** $\frac{\partial x_i^*}{\partial a_j} > 0$ for $i \neq j$, and **(iii)** $\frac{\partial X^*}{\partial r} > 0$.

(a-i) Differentiating $h(X^*, r, \mathbf{a}) = 0$ in (A-22) with respect to a_i reveals

$$\frac{\partial X^*}{\partial a_i} = -\frac{h_{a_i}}{h_X} = -\frac{f_{a_i}(., a_i)}{h_X} > 0,$$

since $h_X > 0$ by (A-23) and $f_{a_i}(., a_i) < 0$ by (A-17) when $\Phi_{x_i} < 0$.

(a-ii) Given $x_i^* = f(X^*, r, a_i)$ and $f_X(., a_i) > 0$ by (A-21), we find

$$\frac{\partial x_i^*}{\partial a_j} = f_X(., a_i)\frac{\partial X^*}{\partial a_j} > 0 \text{ for } i \neq j. \tag{A-37}$$

(a-iii) Using (A-10), the first-order condition in (A-11) for agent i can be rewritten as

$$X\Phi(x_i, X, r) = ra_i. \tag{A-38}$$

Differentiating both sides of (A-38) with respect to r and re-arranging terms yield

$$x_i' + \underbrace{\left(\frac{x_i\Phi + c'X}{x_iX}\right)}_{\equiv T_i}\frac{X'}{\Phi_{x_i}} = \frac{a_i}{x_i\Phi_{x_i}}\underbrace{\left(\frac{x_i}{X} - \frac{c'}{a_i}\right)}_{=q_i - c_i'}. \tag{A-39}$$

where we let $x_i' \equiv \partial x_i / \partial r$ and $X' \equiv \sum x_i'$ here for convenience. Hence, summing both sides of (A-39) across agents, we obtain

$$X'\left(1 + \sum_i \frac{T_i}{\Phi_{x_i}}\right) = \sum_i \frac{a_i\left(q_i - c_i'\right)}{x_i\Phi_{x_i}}. \tag{A-40}$$

Since $\Phi_{x_i} < 0$, and $q_i - c_i' = u_i > 0$ by (12),

$$X' > 0 \Longleftrightarrow \left(1 + \sum_i \frac{T_i}{\Phi_{x_i}}\right) < 0. \tag{A-41}$$

Substituting for Φ_{x_i} from (A-19) and T_i from (A-39) into (A-41) reveals

$$X' > 0 \Longleftrightarrow \sum_i \frac{x_i\Phi + c'X}{X\left(\Phi + c' - (r+X)c''\right)} > 1. \tag{A-42}$$

First, observe that since $c'' > 0$,

$$Z_i \equiv x_i\Phi = (r+X)c' - c > 0. \tag{A-43}$$

Next, observe that multiplying both the numerator and denominator by x_i, we can rewrite (A-42) as

$$X' > 0 \Longleftrightarrow \Omega \equiv \sum_i \underbrace{\left(\frac{Z_i + c'X}{Z_i + x_i c'(x_i) - x_i(r+X)c''} \right)}_{\equiv Y_i} \frac{x_i}{X} > 1. \tag{A-44}$$

Note that $Y_i > 1$ because $Z_i + c'X > 0$ and

$$\begin{aligned} Z_i + x_i c' - x_i(r+X)c'' &= \left[(r+X)c' - c\right] + x_i c' - x_i(r+X)c'' \\ &> (r+X)c' - x_i(r+X)c'' \\ &= (r+X)\left(c' - x_i c''\right) \\ &\geq 0. \end{aligned}$$

where the second line follows from $\frac{c}{x_i} < c'$ since $c'' > 0$, and the last line follows from $\frac{c'}{x_i} \geq c''$ since $c''' \leq 0$. Therefore, as desired,

$$\Omega = \sum_i Y_i \frac{x_i}{X} > \sum_i \frac{x_i}{X} = 1.$$

(b) Consider an intrinsically difficult project: (A-9) or equivalently, $\Phi_{x_i} > 0$ holds. Below we show that **(i)** $\frac{\partial X^*}{\partial a_i} > 0$ for all i, **(ii)** $\frac{\partial x_i^*}{\partial a_j} < 0$ for $i \neq j$ and $\frac{\partial x_i^*}{\partial a_i} > 0$, and **(iii)** $\frac{\partial X^*}{\partial r} > 0$.

(b-i) Recall from (A-16) that $f_X(., a_i) < 0$ for all i when $\Phi_{x_i} > 0$. Thus, from (A-23), $h_X = \sum f_X(., a_i) - 1 < 0$. Recall also that, when $\Phi_{x_i} > 0$, we have $f_{a_i}(., a_i) > 0$ from (A-17). Differentiating $h(X^*, r, \mathbf{a}) = 0$ in (A-22) with respect to a_i, therefore, yields

$$\frac{\partial X^*}{\partial a_i} = -\frac{h_{a_i}}{h_X} = -\frac{f_{a_i}(., a_i)}{h_X} > 0.$$

(b-ii) From $x_i^* = f(X^*, r, a_i)$, we obtain

$$\frac{\partial x_i^*}{\partial a_j} = f_X(., a_i)\frac{\partial X^*}{\partial a_j} < 0. \tag{A-45}$$

Furthermore,

$$\begin{aligned}\frac{\partial x_i^*}{\partial a_i} &= f_X(.,a_i)\frac{\partial X^*}{\partial a_i} + f_{a_i}(.,a_i) \\ &= f_X(.,a_i)\left(-\frac{f_{a_i}(.,a_i)}{h_X}\right) + f_{a_i}(.,a_i) \\ &= f_{a_i}(.,a_i)\left(1-\frac{f_X(.,a_i)}{h_X}\right).\end{aligned} \tag{A-46}$$

Since $h_X < 0$ and $f_{a_i}(.,a_i) > 0$, we observe from the last line in (A-46) that

$$\frac{\partial x_i^*}{\partial a_i} \overset{sign}{=} 1 - \frac{f_X(.,a_i)}{h_X} \overset{sign}{=} f_X(.,a_i) - h_X.$$

Given $h_X = \sum_j f_X(.,a_j) - 1 < 0$ and $f_X(.,a_j) < 0$, it follows that

$$\frac{\partial x_i^*}{\partial a_i} \overset{sign}{=} 1 - \sum_{j\neq i} f_X(.,a_j) > 0. \tag{A-47}$$

(b-iii) Since $T_i > 0$ by (A-39) and $q_i - c_i' = u_i > 0$ by (12), we observe from (A-40) that $\frac{\partial X^*}{\partial r} > 0$ since $\Phi_{x_i} > 0$.

■

Proof of Lemma 3. Suppose $a_i = a$ for all i. Then, by Proposition 1, there is an (interior) equilibrium, which must be symmetric. That is, $x_i^* = x^* = f(X^*, r, a) > 0$ and $q_i^* = \frac{1}{n}$ for all i.[31] In addition, (9) implies that x^* uniquely solves

$$c'(x^*)(r + nx^*) - c(x^*) = \frac{ra}{n}. \tag{A-48}$$

Next, the uniqueness in Lemma 1 implies that $x_i^S = x^S > 0$. Hence, simplifying (5), x^S uniquely solves

$$\max_x W(x;a,r) \equiv \frac{nx - nc(x)/a}{r + nx}.$$

Differentiating the welfare with respect to x, we obtain

$$\frac{\partial}{\partial x} W(x;.) = \frac{arn - nc'(x)(r + nx) + n^2 c(x)}{a(r + nx)^2}. \tag{A-49}$$

[31]By Proposition 1, the equilibrium is also unique when the project is intrinsically easy or intrinsically difficult, but the equilibrium uniqueness is not needed in this proof.

Note that

$$\begin{aligned}\left.\frac{\partial}{\partial x}W(x;.)\right|_{x=x^*} &\propto ar - c'(x^*)(r+nx^*) + nc(x^*)\\ &= ar - \frac{ra}{n} - c(x^*) + nc(x^*)\\ &= \left(1-\frac{1}{n}\right)ar + (n-1)c(x^*)\\ &> 0,\end{aligned}$$

where the second line employs (A-48). Hence, $x^* < x^S$. ■

Proof of Proposition 6. Consider a two-member team with abilities $a_1 > a_2$, and fix r and a_2. To prove part (a), suppose that the project is intrinsically easy. Then, by Proposition 2, $x_1^* < x_2^*$. Moreover, $x_2^* < \infty$ by (12). Next, we show that $x_1^* \to 0$ as $a_1 \to \infty$. From the first-order condition in (9), note that

$$c'(x_1^*)(r+X^*) - c(x_1^*) = rq_1^* a_1. \tag{A-50}$$

Clearly, the left-hand side of (A-50) is finite, but its right-hand side would grow unbounded as $a_1 \to \infty$ if $x_1^* \nrightarrow 0$ and thus $q_1^* \nrightarrow 0$. Hence, $x_1^* \to 0$, which implies $q_2^* \to 1$ and in turn, $x_2^* \nrightarrow 0$. To compare with the social optimum, recall from Lemma 1 that $0 < x_2^S < x_1^S$, and by (A-1),

$$\frac{c'(x_1^S)}{a_1} = \frac{c'(x_2^S)}{a_2} = z^S \in (0,1). \tag{A-51}$$

Suppose that $z^S \nrightarrow 0$ as $a_1 \to \infty$, which would imply $x_2^S \nrightarrow 0$. But inspecting (5), it is clear that the planner could do strictly better by shifting the effort x_2^S to agent 1. Hence, $z^S \to 0$ and thus $x_2^S \to 0$. Moreover, $x_1^S \nrightarrow 0$. Together, we conclude that $x_1^* < x_1^S$ and $x_2^* > x_2^S$ for a sufficiently large a_1.

To prove part (b), suppose that the project is intrinsically difficult. Then, $x_2^* < x_1^*$. Moreover, since $c_i(x_i) = \frac{x_i}{k}c_i'(x_i)$ for the iso-elastic cost, (9) reveals

$$\begin{aligned}c_i'(x_i^*) &= \frac{rq_i^*}{r+X^*-\frac{x_i^*}{k}} \qquad \text{(A-52)}\\ &= \frac{rx_i^*}{\left(r+X^*-\frac{x_i^*}{k}\right)X^*}\end{aligned}$$

and in turn, $c_2'(x_2^*) < c_1'(x_1^*)$. For part (b), it, therefore, suffices to show that $c_2'(x_2^*) < z^S < c_1'(x_1^*)$.

To the contrary, suppose that $z^S \le c_2'(x_2^*)$. Then, $x_2^S \le x_2^*$ and $x_1^S < x_1^*$ by (A-51) since $c_i'' > 0$. From the first-order conditions (7) and (9), this implies

$$rq_1^* = c_1'(x_1^*)(r + x_1^* + x_2^*) - c_1(x_1^*) > c_1'(x_1^S)(r + x_1^S + x_2^S) - c_1(x_1^S) = r + c_2(x_2^S), \tag{A-53}$$

because $c_i'(x_i)(r + x_i + x_j) - c_i(x_i)$ is strictly increasing in x_i. Hence, $rq_1^* > r + c_2(x_2^S)$ and in turn, $q_1^* > 1$, a contradiction.

Next suppose, to the contrary, that $c_1'(x_1^*) \le z^S$ for a sufficiently large a_1. Then, $x_2^* < x_2^S$ and $x_1^* \le x_1^S$, implying $X^* < X^S$. By the iso-elastic cost, $c_i(x_i^S) = \frac{x_i^S}{k} c_i'(x_i^S) = \frac{x_i^S}{k} z^S$. Inserting this fact into (7), we obtain

$$z^S = \frac{r}{r + X^S - \frac{X^S}{k}}. \tag{A-54}$$

Using (A-52) and (A-54), we observe

$$\begin{aligned} c_1'(x_1^*) \le z^S &\iff \frac{rx_1^*}{\left(r + X^* - \frac{x_1^*}{k}\right) X^*} \le \frac{r}{r + X^S - \frac{X^S}{k}} \\ &\iff x_1^* \le \frac{X^*(r + X^*)}{r + X^S - \frac{X^S}{k} + \frac{X^*}{k}}. \end{aligned} \tag{A-55}$$

We now claim that $x_1^* \approx X^*$ for a sufficiently large a_1. To prove, suppose that $x_1^* \nrightarrow \infty$ as $a_1 \to \infty$. Then, the left-hand side of (A-50) is finite, but its right-hand side would grow unbounded since $q_1^* \nrightarrow 0$. Hence, $x_1^* \to \infty$, which implies that $q_1^* \to 1$ (since $x_2^* < \infty$) and $x_2^* \to 0$, proving the claim. Plugging $x_1^* \approx X^*$ into (A-55) and simplifying terms, we find that $X^S \le X^*$, a contradiction.

As a result, $c_2'(x_2^*) < z^S < c_1'(x_1^*)$ for a sufficiently large a_1, and in turn,

$$x_2^* < x_2^S \text{ and } x_1^* > x_1^S \text{ by (A-51), as desired.}$$

■

Proof of Lemma 4. As discussed in the text, every equilibrium under solo work must be interior since $\left.\frac{\partial \hat{u}_i}{\partial x_i}\right|_{x_i=0} = \frac{1}{r+X} > 0$, so we restrict attention to $x_i > 0$ and

$X > 0$. Moreover, substituting the first-order condition (17) into (16) and simplifying terms, we observe that in equilibrium,

$$\widehat{u}_i = 1 - c_i'(x_i) > 0. \tag{A-56}$$

(In equilibrium, $\widehat{u}_i > 0$ because agent i would otherwise choose $x_i = 0$). Hence, in equilibrium,

$$x_i < c'^{-1}(a_i).$$

Next, given $X_{-i} = X - x_i$, we define

$$\Lambda(x_i, X, r, a_i) = c_i'(x_i)(r + X) - c_i(x_i) + x_i - X,$$

so that (17) becomes

$$\Lambda(x_i, X, r, a_i) = r. \tag{A-57}$$

Proceeding as in the proof of Proposition 1, fix X and note that

$$\Lambda(0, X, r, a_i) = -X < 0 \text{ and } \Lambda(X, X, r, a_i) = c_i'(X)(r + X) - c_i(X) > 0.$$

Moreover, $\Lambda_X = -\left[1 - c_i'(x_i)\right] < 0$. Hence, (A-57) admits a unique solution:

$$x_i = \widehat{f}(X, r, a_i) \tag{A-58}$$

if and only if

$$\Lambda(X, X, r, a_i) \geq r. \tag{A-59}$$

Since $\Lambda(0, 0, r, a_i) = 0$ and $d\Lambda(X, X, r, a_i)/dX = c_i''(X)(r + X) > 0$, (A-59) is satisfied if and only if

$$X > \underline{X}_i$$

where $\underline{X}_i > 0$ is the unique cutoff such that $\widehat{f}(\underline{X}_i, r, a_i) = \underline{X}_i$.

Now note that summing up (A-58) across agents, the equilibrium total effort X must solve

$$\widehat{h}(X, r, \mathbf{a}) \equiv \sum_i \widehat{f}(X, r, a_i) - X = 0.$$

Clearly, since $\widehat{f}(\underline{X}_i, a_i, r) - \underline{X}_i = 0$,

$$\widehat{h}(\min_i \underline{X}_i, r, \mathbf{a}) > 0 \text{ and } \widehat{h}(\max_i c'^{-1}(a_i), r, \mathbf{a}) < 0.$$

Hence, by continuity, there is a solution to

$$\widehat{h}(X, \mathbf{a}, r) = 0,$$

which constitutes an equilibrium by (A-58).

To prove the rest of the proposition, we first implicitly differentiate (A-57), namely $\Lambda(\widehat{f}(X, r, a_i), X, r, a_i) = r$, and find that

$$\widehat{f}_{a_i}(., a_i) = -\frac{\Lambda_{a_i}}{\Lambda_{x_i}} > 0, \tag{A-60}$$

since $\Lambda_{a_i} = -\frac{c_i'(x_i)(r+X) - c_i(x_i)}{a_i} < 0$ and $\Lambda_{x_i} = c_i''(x_i)(r + X) + 1 - c_i'(x_i) > 0$.

Next let $a_1 > a_2$ for some $i = 1, 2$. Then, (A-60) implies

$$x_1^L = \widehat{f}(X^L, r, a_1) > \widehat{f}(X^L, r, a_2) = x_2^L.$$

Finally, to show $\widehat{u}_1^L > \widehat{u}_2^L$, note that

$$\begin{aligned} \widehat{u}_1^L &= \max_{x_1} \frac{x_1 - c_1(x_1)}{r + x_1 + X_{-1}^L} \\ &> \frac{x_2^L - c_1(x_2^L)}{r + x_2^L + X_{-1}^L} \\ &> \frac{x_2^L - c_2(x_2^L)}{r + x_2^L + X_{-2}^L} \\ &= \widehat{u}_2^L, \end{aligned}$$

where the second line follows by the optimality of x_1^L and the third line follows because (1) $X_{-1}^L < X_{-2}^L$ given $x_1^L > x_2^L$ and (2) $c_1(x) < c_2(x)$ given $a_1 > a_2$. ■

Proof of Proposition 7. Suppose $a_i = a$ for all i. Under both team- and solo work, the (interior) equilibrium must be symmetric. That is, in teamwork, $x_i^* = x^* > 0$ and $q_i^* = \frac{1}{n}$, reducing the first-order condition in (9) to

$$c'(x^*)(r + nx^*) - c(x^*) = \frac{ra}{n}.$$

Let $\lim_{r \to 0} x^* = x_\ell^*$, which must satisfy

$$c'(x_\ell^*)nx_\ell^* - c(x_\ell^*) = 0. \tag{A-61}$$

If $x_\ell^* > 0$, then the left-hand side of (A-61) would be strictly positive since $c'' > 0$, yielding a contradiction. Hence, $x_\ell^* = 0$ and in turn, by (12),

$$\lim_{r \to 0} u^* = \frac{1}{n}.$$

Next, consider the solo work. By symmetry, $x_i^L = x^L > 0$, and the first-order condition in (17) becomes

$$c'(x^L)(r + nx^L) - c(x^L) = ra + (n-1)ax^L$$

As $r \to 0$, it further becomes

$$c'(x_\ell^L)nx_\ell^L - c(x_\ell^L) = (n-1)ax_\ell^L. \tag{A-62}$$

Clearly, $x_\ell^L = 0$ is a solution to (A-62) since $c'(0) = c(0) = 0$, but it would not constitute an equilibrium because $\frac{\partial \widehat{u}_i}{\partial x_i}\Big|_{x^L=0,r=0} = \frac{1}{r+X^L}\Big|_{x^L=0,r=0} = \infty$. Hence, we look for a positive solution, $x_\ell^L > 0$. To this end, divide both sides of (A-62) by x_ℓ^L:

$$c'(x_\ell^L)n - \frac{c(x_\ell^L)}{x_\ell^L} = (n-1)a. \tag{A-63}$$

As $x_\ell^L \to 0$, the left-hand side of (A-63) approaches 0 (since $c'(0) = 0$) whereas the right-hand side remains $(n-1)a > 0$. Moreover, as $x_\ell^L \to \infty$, the left-hand side of (A-63) grows unbounded because of the cost assumption (C3) above, i.e., $\lim_{x\to\infty} \frac{c(x)}{xc'(x)} < 1$, and the fact that $c'' > 0$, which, again, are satisfied by the iso-elastic cost. Hence, there is a positive solution to (A-63), i.e., $x_\ell^L > 0$. Finally, note from (A-63) that $c'(x_\ell^L)/a = \frac{n-1}{n} + \frac{c(x_\ell^L)/a}{nx_\ell^L}$, and from (A-56), we observe that

$$\begin{aligned}
\lim_{r\to 0} \widehat{u}^L &= 1 - c'(x_\ell^L)/a \\
&= 1 - \left[\frac{n-1}{n} + \frac{c(x_\ell^L)/a}{nx_\ell^L}\right] \\
&= \frac{1}{n} - \frac{c(x_\ell^L)/a}{nx_\ell^L} \\
&< \lim_{r\to 0} u^*.
\end{aligned}$$

■

Proof of Proposition 8 . (21) is obtained by solving (14) and (20) for q_i^{SL}. For part (a), note that

$$q_i^{SL} < \frac{r + \sum_{\ell \neq i} x_\ell^S c_\ell'(x_\ell^S)}{r + \sum_{\ell \neq i} x_\ell^S} = \frac{r + z^S X_{-i}^S}{r + X_{-i}^S} < 1,$$

where the first inequality follows since $c''(x) > 0$; the second equality and the third inequality follow since $c_\ell'(x_\ell^S) = z^S \in (0,1)$ for all $\ell = 1, ..., n$ from the proof of Lemma 1 and $X_{-i} = \sum_{\ell \neq i} x_\ell$.

For part (b), take the iso-elastic specification: $c(x) = \frac{x^k}{k}$. Then, $c_\ell'(x_\ell^S) = \frac{x_\ell^S}{k} c_\ell'(x_\ell^S)$ and, in turn, $c_\ell'(x_\ell^S) = \frac{x_\ell^S}{k} z^S$. From here, (21) simplifies to

$$q_i^{SL} = \frac{r + \frac{z^S}{k} X_{-i}^S}{r + X_{-i}^S}.$$

Note that $\frac{z^S}{k} \in (0,1)$ since $k > 1$ and $z^S \in (0,1)$. Thus, q_i^{SL} is strictly decreasing in X_{-i}^S. Next, note that for $a_i > a_j$, we have $X_{-i}^S < X_{-j}^S$ since $x_i^S > x_j^S$ by Lemma 1. Hence, $q_i^{SL} > q_j^{SL}$ for $a_i > a_j$. Finally, employing (20), (19) implies

$$\widehat{u}_i^{SL} = q_i^{SL} - c_i'(x_i^S) = q_i^{SL} - z^S,$$

which, in turn, implies that $\widehat{u}_i^{SL} > \widehat{u}_j^{SL}$ for $a_i > a_j$. ■

Proof of Proposition 9 . The result is immediate from (12) and (19) since $q_i^{SL} < 1 < q_i^{ST}$ and $\mathbf{x}^* = \mathbf{x}^S = \mathbf{x}^L$. ■