Demographics and the Evolution of Trade Imbalances

Michael Sposi*

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Abstract

The age distribution evolves asymmetrically across countries, influencing relative saving rates and labor supply. Using a dynamic, multicountry trade model I quantify how demographic changes affected trade imbalances across 28 countries since 1970. Counterfactually holding demographics constant reduces net exports in emerging economies that experienced rising working age shares, and boosts them in advanced economies that experienced flatter, or declining, working age shares. This helps alleviate the allocation puzzle. On average, a one percentage point increase in a country's working age share, relative to the world, increased its ratio of net exports to GDP by one-third of a percentage point.

JEL codes: F11, F21, J11

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*Address: Southern Methodist University, 3300 Dyer Street, Dallas, TX 75275. Phone: 214-768-2806; Email: msposi@smu.edu. I am grateful to George Alessandria, Mario Crucini, Klaus Desmet, Mick Devereux, Jonathan Eaton, Andy Glover, Karen Lewis, Kanda Naknoi, Kim Ruhl, Jake Short, Joe Steinberg, Mark Wynne, Kei-Mu Yi, Jing Zhang, and Min Qiang Zhao for helpful comments. This paper also benefited from audiences at the Bank of Canada, Bank of Mexico, Dallas Fed, Penn State, Southern Methodist University, University of Toronto, and UConn as well as conference participants at the Institute of New Structural Economics, Midwest Macroeconomics Meetings, Society for Economic Dynamics, and System Committee for International Economic Analysis. Kelvin Virdi provided excellent research assistance.

1 Introduction

Recently, trade imbalances have taken center stage in many high-level policy debates. Imbalances can arise for a variety of reasons such as asymmetric trade costs, frictions on cross-border financial transactions, distortions on investment returns, and because of natural forces such as demographic-induced differences in saving behavior. Yet, existing research has yet to systematically disentangle the effects of various forces across many countries.

Three characteristics of trade imbalances stand out. First, absent business cycle fluctuations, trade imbalances demonstrate highly persistent and low frequency trends. The United States has run a trade deficit for four decades, and China a surplus for over two decades.

Second, a somewhat more obvious observation is that trade imbalances vary across countries – one country's liability is another country's asset. Nonetheless, it is important to understand why some countries produce more than they spend, while others do the opposite, since imbalances reflect how resources are allocated across countries and over time.

Third, the pattern of trade imbalances are more-or-less uncorrelated with productivity growth over the long run and, if anything, the correlation is slightly positive. This third fact, known as the allocation puzzle, poses a challenge of sorts since neoclassical economic theory predicts a negative correlation between the two variables: countries that experience faster productivity growth should borrow and run deficits.[1]

Demographic forces offer a possible explanation. First, countries have undergone persistent and low frequency changes in their age distributions, defined as the young share (age 14 and below), the working age share (age 15-64), and the retired share (age 65 and above). These moments influence a country's propensity to save and to supply labor. Thus, differential demographic trends across countries exert pressure on the balance sheets of large pensions, on returns to capital, and ultimately on trade imbalances.

Second, changes in the age distribution vary across countries. From 1970 to 2014, China's working age share increased at double the rate (from 0.56 to 0.73) of the entire world's (from 0.57 to 0.66). Conversely, the United States' working age share increased at half the rate (from 0.62 to 0.66) of the entire world's. More generally, emerging economies experienced faster increases in their working age shares than advanced economies did.

Third, changes in the age distribution correlate positively with productivity growth. On average, emerging economies have grown faster than advanced economies have.

[1] The allocation puzzle is typically stated in terms of the current account (see Gourinchas and Jeanne, 2013; Prasad, Rajan, and Subramanian, 2007). Trade imbalances display similar characteristics as current account imbalances in this context.

This paper incorporates demographic forces into a dynamic, multicountry, general equilibrium model to study the consequences for trade imbalances across 28 countries since 1970. The main finding is that the mechanism called into question by the allocation puzzle is indeed present, but partly masked by demographic-induced saving behavior.

The main counterfactual freezes every country's age distribution at its 1970 level and explores the resulting equilibrium dynamics. On average, a one percentage point increase in a country's working age share from 1970 to 2014, relative to that of the world, induces a one-third percentage point increase in its net exports to GDP over the same time. Specifically, observed demographics contributed positively to net exports in emerging economies with quickly rising working age shares, and contributed negatively to net exports in advanced economies with slowly rising, or decreasing, working age shares. The correlation between economic growth and changes in the age distribution induces resources to flow from fast-growing emerging economies to slower-growing advanced economies, helping alleviate the allocation puzzle. Counterfactually, the elasticity of the ratio of net exports to GDP with respect to contemporaneous TFP growth is -19.4%, compared to 7.5% in the data.

In the main counterfactual China runs a trade deficit, in contrast to its observed surplus, while the U.S. deficit shrinks relative to the data. These contrasts emerge most prominently after 1990 as China enters its demographic window.[2] This provides an demographic-based explanation for the global savings glut hypothesis proposed by Bernanke (2005).

I also explore asymmetric population growth as a separate demographic characteristic. A fast growing population lowers the real exchange rate, as foreign import penetration accounts for a declining share of gross absorption. In turn, the real interest rate declines inducing capital to flow out in search of higher rates of return, i.e., increasing net exports.

The primary measure of trade imbalances is the ratio of net exports to GDP. Mechanically, this ratio equals the product of (i) the ratio of net exports to total gross trade and (ii) the ratio of total gross trade to GDP. Distinct literatures have emerged offering different perspectives. One is an international macro/finance perspective – saving minus investment. Much of this literature has used small open economy models with one good focusing on *net* trade flows, and has been silent on the *gross* trade flows by assuming frictionless intratemporal trade.[3] My results indicate that the assumption of frictionless trade is not innocuous when studying trade imbalances. The other is an international trade perspective – exports minus imports.

[2]The United Nations defines a demographic window as a period when "the fraction of the population under age 15 falls below 30 percent and the fraction 65 years and older is still below 15 percent."

[3]Papers that incorporate both perspectives in general equilibrium are primarily based in two-country settings and build on Backus, Kehoe, and Kydland (1992).

With a few exceptions, multicountry models of trade have been silent on trade imbalances, and have focused on the intratemporal pattern and volume of gross trade.[4] Alessandria and Choi (2019) and Reyes-Heroles (2016) demonstrate that broad trends in the ratio of net exports to GDP are accounted for primarily by the trends in gross trade flows, i.e., the ratio trade to GDP rather than the ratio of net exports to trade.

These two different perspectives lead to different implications for the effects of demographic-induced changes in saving. That is, whether saving ends up in investment versus net exports is of first order importance when studying external imbalances, so care must be taken to simultaneously discipline both margins. My paper embraces both perspectives by incorporating several relevant features: (i) endogenous trade imbalances through trade in one-period bonds, (ii) gross bilateral trade between countries across a continuum of varieties, (iii) physical capital accumulation, (iv) endogenous labor supply and (v) many countries in a general equilibrium environment. To my knowledge, this paper is the first to combine all of these features and compute exact transitional dynamics.

The first feature of the model is trade one period bonds. This allows for borrowing and lending between countries and, hence, serves as a means to finance trade imbalances.

The second feature is gross bilateral trade between countries. International trade is determined by productivity differences along a continuum of varieties and bilateral trade costs as in Eaton and Kortum (2002). Countries specialize in different subsets of the variety space inducing gross intraindustry trade flows. These trade flows are disciplined by the magnitude of trade costs and govern how fundamentals in one country shape prices in another; a channel that is absent in models that consider only net trade flows. In particular, since borrowing and lending is ultimately offset by trade imbalances, the magnitude trade costs affect rates of return on bonds.[5] Large trade costs, for instance, imply that a demographic-induced increase in saving shows up as higher investment rather than higher net exports.

The third feature is investment and capital accumulation as in the neoclassical growth

[4]Some papers incorporate trade imbalances as exogenous transfers in the context of static models (Caliendo, Parro, Rossi-Hansberg, and Sarte, 2017; Dekle, Eaton, and Kortum, 2007). Only a few papers model trade imbalances between many countries via international borrowing and lending in a dynamic setting (Eaton, Kortum, and Neiman, 2016; Eaton, Kortum, Neiman, and Romalis, 2016; Reyes-Heroles, 2016; Ravikumar, Santacreu, and Sposi, 2019; Sposi, 2012).

[5]This intuition is articulated by Obstfeld and Rogoff (2001) who postulate that trade costs have the potential to help reconcile six different puzzles in international macroeconomics (they do not discuss the allocation puzzle). Eaton, Kortum, and Neiman (2016) find that trade costs do indeed quantitatively help account for a few of those puzzles, including two directly related to international capital flows. Alessandria and Choi (2019) and Reyes-Heroles (2016) argue that declining trade costs over time generate larger gross trade volumes, thereby amplifying the net trade positions as a share of GDP.

model.[6] Differential productivity growth along with capital market distortions and intertemporal adjustment costs are key determinants of the net return to investment. Differences in rates of return to investment across countries provides an incentive for countries to borrow and lend as capital seeks higher rates of return. In the presence of either large investment distortions or large adjustment costs, for instance, a demographic-induced increase in saving shows up as higher net exports rather than higher investment.

The fourth feature is endogenous labor supply. As in Ohanian, Restrepo-Echavarria, and Wright (2018), changes in labor supply influence returns to investment through transitory effects on the capital-labor ratio. In my model, a demographic-induced increase in labor supply would lower the capital-labor ratio in the short run, increase the rate of return to investment, trigger capital inflows and, hence, lower the trade balance. There is also a competing effect whereby greater labor supply increases a country's ability to fund its existing liabilities, thus lowering demand for borrowing and raising the trade balance.

Finally, my framework encapsulates all of these features in a unified, multicountry, general equilibrium setting. Incorporating many countries generates new insights on the systematic relationships, rather than focusing on idiosyncrasies particular to certain countries or time periods. This is particularly important in the context of the allocation puzzle. Doing the analysis in general equilibrium disciplines how prices and interest rates respond to counterfactual demographic trajectories and ensures that world saving is always balanced.

Incorporating all of these features introduces computational challenges, and the degree of complexity depends on the demand structure. If one models the age distribution explicitly using an overlapping generations (OLG) framework where agents live for, say, 9 periods (ages $0, 10, \dots, 80$), then there are $2 \times 28 \times 9 = 504$ state variables: the distribution of both capital and assets across 28 countries and 50 age groups (this would be conservative since it does not accommodate annual data). In addition, the presence of trade costs complicates matters since the mapping from the world distribution of state variables to prices differs across countries; this is the not case in a model of frictionless trade or in a model of net trade flows.[7] In order to make progress, I reduce the dimensionality to $2 \times 28 = 56$ state variables and make

[6] Other papers that feature capital accumulation in a multicountry trade model include (Alvarez, 2017; Anderson, Larch, and Yotov, 2019; Eaton, Kortum, Neiman, and Romalis, 2016; Eaton, Kortum, and Neiman, 2016; Ravikumar, Santacreu, and Sposi, 2019; Zylkin, 2016).

[7] For example, the absence of trade costs results in price equalization across countries and the mapping from the world distribution of state variables into each country's prices would be identical. If trade costs between India and the United States were large, then the state of India's economy would have an indirect and negligible influence on U.S. prices, whereas if trade costs between India and Japan were small, then the state of India's economy would have a direct and meaningful influence on Japanese prices.

use of annual data by modeling intertemporal decisions using a representative household framework. As a result, I can efficiently compute transitional dynamics by extending the methods developed in Ravikumar, Santacreu, and Sposi (2019).

Sacrificing the explicit OLG structure and embracing the trade structure between many countries is a notable departure from the existing literature that studies capital flows and external imbalances in general equilibrium settings. Ferrero (2010) uses a two-country OLG model, Backus, Cooley, and Henriksen (2014) use a three-country OLG model, and Krueger and Ludwig (2007) use a four-country OLG model. Domeij and Flodén (2006) study capital flows between 18 OECD countries, but abstract from emerging economies as well as gross trade flows. Bárány, Coeurdacier, and Guibaud (2019) study fertility and longevity risk as drivers capital flows from emerging to advanced economies in a model with many countries. I do not confront fertility or longevity risk, but instead I focus on the projected differences in age distributions across countries. All of these papers abstract from gross trade flows.

Since heterogeneity in age is not an explicit feature of the representative household framework, life-cycle forces are captured through *preference shifters* – time varying changes in the household's discount factor and in its marginal utility of leisure.[8] Time-varying discount factors are familiar to dynamic trade models.[9] The representative household in each country faces standard consumption-saving and labor-leisure trade-offs. In addition to preference shifters, the household faces distortions to net-foreign income and to labor income, so that the product of each preference shifter and the corresponding distortion appears as a wedge in an Euler equation. These wedges are recovered using data on some prices and quantities.[10]

I decompose the wedges into a preference component and a distortionary component by projecting the calibrated wedges onto observed moments of the age distribution across countries and over time. The preference shifters are recovered as the component that covaries with these moments. The distortionary component captures non-demographic forces, including, for instance, financial market distortions and labor market distortions. The sources of these distortions is not important for my analysis, so long as they are orthogonal to the age distribution. The decomposition provides a means to study counterfactuals in which alterations to the age distribution are manifested in alterations to the preference shifters.

In order to discipline the counterfactual analysis, I embed the household into a general

[8]Consumption-saving decisions arising from an OLG framework can be replicated in a representative household framework with appropriate changes in the discount factor (see Blanchard, 1985; Yaari, 1965).

[9]See Eaton, Kortum, Neiman, and Romalis (2016); Kehoe, Ruhl, and Steinberg (2018); Steinberg (2019).

[10]Whether the wedges appear as one versus the other is immaterial for allocations since they enter the Euler equations multiplicatively.

equilibrium environment with production, investment, and international trade. The supply side of the model is saturated with *wedges* – exogenous forces including distortions to capital returns, bilateral trade costs, and productivity. The wedges are calibrated so that the model reproduces each variable in the household's two key Euler equations (consumer prices, real wages, consumption levels, and employment levels) rendering the preference wedges consistent with both the model and the data. In turn, the model also replicates the observed bilateral trade flows, investment levels, and investment prices.[11]

I also quantify the importance of non-demographic forces in the context of the allocation puzzle. Neither changes in bilateral trade costs nor changes in labor market distortions help reconcile the puzzle, but exaggerate it. Changes in saving distortions account for the most, followed by changes in the age distribution, and then changes in investment distortions.

Gourinchas and Jeanne (2013) compute saving wedges in a similar manner as in my paper and show that these wedges are highly correlated with economic growth rates across countries. My analysis reduces the burden of the overall saving wedge by extracting a *predictable* demographic component. They also argue that the allocation puzzle is one about saving and not investment, and in particular, public saving.

Alfaro, Kalemli-Ozcan, and Volosovych (2008) use detailed data to separate cross-border capital flows into private flows and government flows and demonstrate that government saving is indeed the culprit for the allocation puzzle. I abstract from distinguishing between private and public saving by appealing to Ricardian equivalence. Still, my findings point to demographics as a strong candidate explanation. That is, in many countries both the assets (income taxes and bond revenue) and liabilities (pensions and medical coverage) of government budgets are greatly influenced by demographics. Börsch-Supan, Ludwig, and Winter (2006) reinforce this point by demonstrating the importance of demographic driven pension reform as a determinant of global imbalances.

2 Life-cycle forces in a representative household

This section describes how life-cycle forces are be built into a representative household using preference shifters. There are $i = 1, \ldots, I$ countries, each populated by a representative household, and time runs from $t = 1, \ldots, \infty$. Each household has population N_{it} and values both consumption and leisure per capita. The intertemporal discount factor between periods

[11]This wedge accounting is similar to that used in Eaton, Kortum, Neiman, and Romalis (2016) and has its roots in business cycle accounting (see Chari, Kehoe, and McGrattan, 2007).

t and $t+1$ is given by $\beta\left(\frac{\psi_{it+1}}{\psi_{it}}\right)$, where the country-specific and time-varying preference shifter, ψ_{it}, captures life-cycle influences on saving rates. The marginal utility of leisure, ζ_{it}, is the second preference shifter that captures life-cycle forces on labor supply.

Households face country-specific consumer prices, P_{cit}, country-specific wages, w_{it}, and a common world interest rate, q_t, all of which vary over time. In each period the household chooses consumption, C_{it}, labor supply, L_{it} and holdings of one-period bonds, $\mathcal{A}_{it+1}$. Returns on bonds and labor income are subject to time-varying and country-specific distortions, $\tau^{\mathcal{A}}_{it}$ and τ^{L}_{it}, which are modeled as taxes for simplicity. Tax revenue is rebated back to households through lump sum transfers, T_{it}. The household maximizes lifetime utility:

$$\mathcal{U}_i = \max_{\{C_{it},\mathcal{A}_{it+1},L_{it}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1}\psi_{it}N_{it}\left(\ln\left(\frac{C_{it}}{N_{it}}\right) + \zeta_{it}\frac{\left(1-\frac{L_{it}}{N_{it}}\right)^{1-1/\phi}}{1-1/\phi}\right) \tag{1}$$

$$\text{s.t. } P^c_{it}C_{it} + \mathcal{A}_{it+1} = w_{it}L_{it}(1-\tau^L_{it}) + (1+q_t)\mathcal{A}_{it}(1-\tau^{\mathcal{A}}_{it}) + T_{it}$$

Two key Euler equations characterize the household's decisions:

$$\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}} = \underbrace{\beta\left(\frac{\psi_{it+1}}{\psi_{it}}\right)\left(1-\tau^{\mathcal{A}}_{it+1}\right)}_{\omega^{\mathcal{A}}_{it}}\left(\frac{1+q_{t+1}}{P^c_{it+1}/P^c_{it}}\right), \tag{2}$$

$$1-\frac{L_{it}}{N_{it}} = \underbrace{\left(\zeta_{it}\right)^{\phi}\left(1-\tau^L_{it}\right)^{-\phi}}_{\omega^L_{it}}\left(\frac{w_{it}}{P^c_{it}}\right)^{-\phi}\left(\frac{C_{it}}{N_{it}}\right)^{\phi}. \tag{3}$$

Equation (2) demonstrates that the presence of preference shifters and distortions drive a wedge between per-capita consumption growth and the discounted real interest rate. The *saving wedge*, defined as $\omega^{\mathcal{A}}_{it}$, multiplicatively captures the both the effects of demographic forces (through preference shifters) and of distortions on net-foreign income from bonds.[12] Equation (3) demonstrates that preference shifters and distortions drive a wedge between how labor supply relates to the real wage and the level of consumption. The *labor wedge*, defined as ω^L_{it}, multiplicatively captures both the effects of demographics (again, through preference shifters) and of distortions on labor income.

While the distortions are modeled as ad valorem taxes, they are meant to be interpreted

[12] This is essentially the same wedge definition as in Gourinchas and Jeanne (2013). The difference is that, in my model, this wedge is time-varying and is the product or a preference component and a distortionary component. Preference shifters are also commonly used in trade models in order to account for observed trade imbalances (see Kehoe, Ruhl, and Steinberg, 2018; Eaton, Kortum, Neiman, and Romalis, 2016).

more broadly to capture any non-age-related force that is not explicitly modeled, so that the theory can *fit* the data. For example, saving distortions capture to exchange rate risk, monetary policy, pension systems, and capital controls. Labor distortions capture trends in female labor force participation, unemployment insurance programs, and labor income taxes.

Two remarks are in order in regarding the saving distortion. First, since the model tracks cross-country financial flows in *net* terms, the distortion symmetrically captures taxes on both inflows and outflows. Second, since the distortion is applied to both the stock, $\mathcal{A}$, and the flow payment, $q\mathcal{A}$, it captures both the capital outflow (purchase of foreign bond) and ensuing capital inflow (receipt from bond maturity).

Backing out saving and labor wedges The two wedges can be recovered from readily available data, given a couple of preference parameters. The discount rate is set to $\beta = 0.96$, in line with a long-run average real interest rate of about 4 percent. The Frisch elasticity of labor supply is set to $\phi = 2$, based on Peterman (2016).

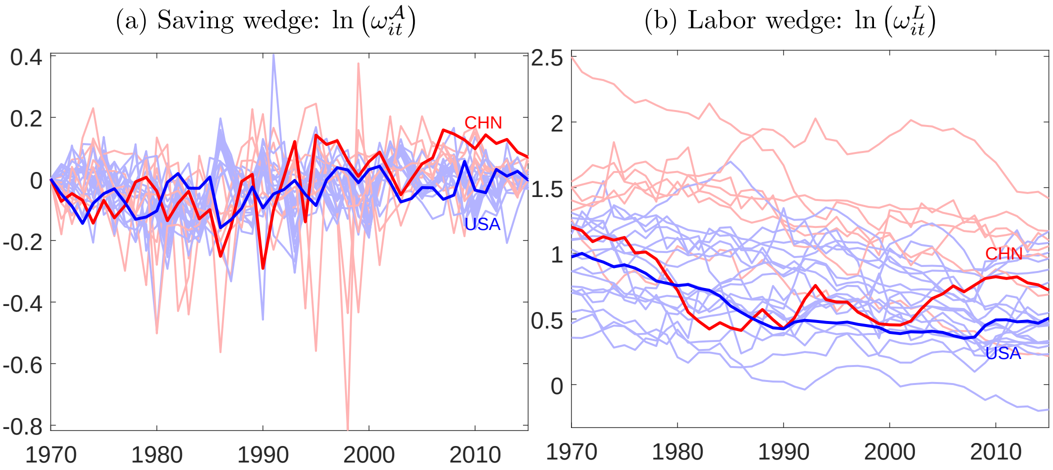
The saving wedge is backed out as a residual from equation (2) by using data on a risk-free interest rate, consumer price growth, and per-capita consumption growth. The interest rate is the annual, nominal yield on the 10-year U.S. Treasury note. Consumption—sum of both private and public—is measured as GDP minus investment expenditures minus net exports, all divided by the consumer price level. The consumer price level is measured in 2011 U.S. dollars at current purchasing power parities (PPP).

The labor wedge is backed out as a residual from equation (3) using data on wages, consumer prices, employment, and consumption. The wage rate is defined as GDP times labor's share in GDP divided by the number of workers engaged. Appendix A provides more details on the data sources and manipulation.

Figure 1 illsutrates the calibrated wedges for 27 countries plus a rest-of-world aggregate from 1970-2014, which is the same sample of countries used in the quantitative analysis later in the paper. Clearly, saving wedges fluctuate at a much higher frequency than labor wedges do, while both wedges also exhibit lower frequency trends, which are more noticeable in the declining labor wedges.

This is a similar measurement approach as that employed by Gourinchas and Jeanne (2013), who treat wedges as preference shifters, and Ohanian, Restrepo-Echavarria, and Wright (2018), who treat wedges as distortions. Absent any additional assumptions, the two components are indistinguishable through the lens of the Euler equations. My view is that some of the variation in the wedges is "predictable and efficient" in the sense that it is driven

Figure 1: Calibrated wedges



Notes: The log-saving wedge is defined as $\ln\left(\omega_{it}^{\mathcal{A}}\right) = \ln\left(\frac{\psi_{it+1}}{\psi_{it}}\right) + \ln\left(1 - \tau_{it+1}^{\mathcal{A}}\right)$ and the log-labor wedge is defined as $\ln\left(\omega_{it}^{L}\right) = \phi \ln\left(\zeta_{it}\right) - \phi \ln\left(1 - \tau_{it}^{L}\right)$. Emerging economies in red include Brazil, China, India, Indonesia, South Korea, Mexico, Turkey, and the rest-of-world aggregate. Advanced economies in blue include the remaining 20 countries.

by demographic forces, and is thus modeled as changes in demand forces through preference shifters. The remainder of the wedges are due non-demographic forces. Next I decompose the calibrated wedges into a demographic component and a distortionary component.

Decomposing wedges Equations (2) and (3) characterize each wedge as the product of a demographic component (preference shifter) and a distortionary component. To separately identify these two components, I impose that each preference shifter is a function of the age distribution: $\ln\left(\frac{\psi_{it+1}}{\psi_{it}}\right) = \mu^{\mathcal{A}} \times \mathbf{s}_{it}$ and $\ln\left(\left(\zeta_{it}\right)^{\phi}\right) = \mu^{L} \times \mathbf{s}_{it}$. The variable $\mathbf{s}_{it} = \left(s_{it}^{15-64}, s_{it}^{65+}\right)$ includes the share of the population aged 15-64 (*working age share*, using the World Bank's definition) and the share of the population aged 65 and up (*retired age share*).[13] The retired age share informs about saving behavior beyond the working age share alone, since the working age share can decrease for two reasons. One being an increase in the share of individuals under 15 years old, indicating a more youthful population. The other being an increase in the retired age share, indicating an aging population. Armed with these

[13] Higgins (1998) offers evidence that increases in both youth and old-age dependency ratios individually correlate with lower saving rates.

assumptions, I decompose each wedge using OLS:

$$\ln\left(\omega_{it}^{\mathcal{A}}\right) = \underbrace{\mu^{\mathcal{A}} \times \mathbf{s}_{it}}_{\ln\left(\frac{\psi_{it+1}}{\psi_{it}}\right)} + \underbrace{\gamma_i^{\mathcal{A}} + \kappa_t^{\mathcal{A}} + \varepsilon_{it}^{\mathcal{A}}}_{\ln\left(1-\tau_{it+1}^{\mathcal{A}}\right)}, \tag{4}$$

$$\ln\left(\omega_{it}^{L}\right) = \underbrace{\mu^{L} \times \mathbf{s}_{it}}_{\phi\ln(\zeta_{it})} + \underbrace{\gamma_i^{L} + \kappa_t^{L} + \varepsilon_{it}^{L}}_{-\phi\ln\left(1-\tau_{it}^{L}\right)}. \tag{5}$$

The demographic component, or preference shifter, is identified as the component that covaries with the age distribution. The distortionary component of each wedge includes country and time fixed effects as well as an error term. Country fixed effects capture persistent policy differences across countries and differences in social norms that influence saving behavior or labor-force participation. Time fixed effects capture changes in global economic conditions that influence saving behavior and labor supply in all countries. The error term includes other idiosyncratic influences that are orthogonal to the age distribution.

Table 1 reports estimates from equations (4) and (5). These estimates make use of unbalanced panels of 166 countries from 1960-2013 for the saving wedge (8,964 observations), and 127 countries from 1950-2014 for the labor wedge (8,255 observations).

Table 1: Estimated elasticity of wedges w.r.t. the age distribution

	Right-hand side variable			
Left-hand side variable	s_{it}^{15-64}	s_{it}^{65+}	R^2	# Obs.
Saving wedge: $\omega_{it}^{\mathcal{A}}$	$0.089^{\star\star\star}$	0.003	0.22	8,964
	(0.030)	(0.010)		
Labor wedge: ω_{it}^{L}	$-2.234^{\star\star\star}$	$-0.196^{\star\star\star}$	0.80	8,255
	(0.091)	(0.028)		

Notes: The estimates are based on OLS regressions using equation (4) and (5) with $\mathbf{s}_{it} = \left(s_{it}^{15-64}, s_{it}^{65+}\right)$. Standard errors are in parentheses and *** indicates statistical significance at the 95% level. Data availability limit the estimation to unbalanced panels of 166 countries from 1960-2013 for the saving wedge, and 127 countries from 1950-2014 for the labor wedge.

The effectiveness of this reduced-form specification is its ability to disentangle the low frequency demographic forces distinctly from non-demographic forces. For the saving wedge, the coefficient on the working age share is significantly positive, reflecting the fact that working age individuals tend to save more than young individuals (under the age of 15). The coefficient on the retired age share is statistically indistinguishable from zero. The age distribution, combined with country and time fixed effects, accounts for almost one quarter of the overall variation in the saving wedge. This is not trivial because saving wedges fluctuate at a high frequency, whereas demographics fluctuate at low frequency.

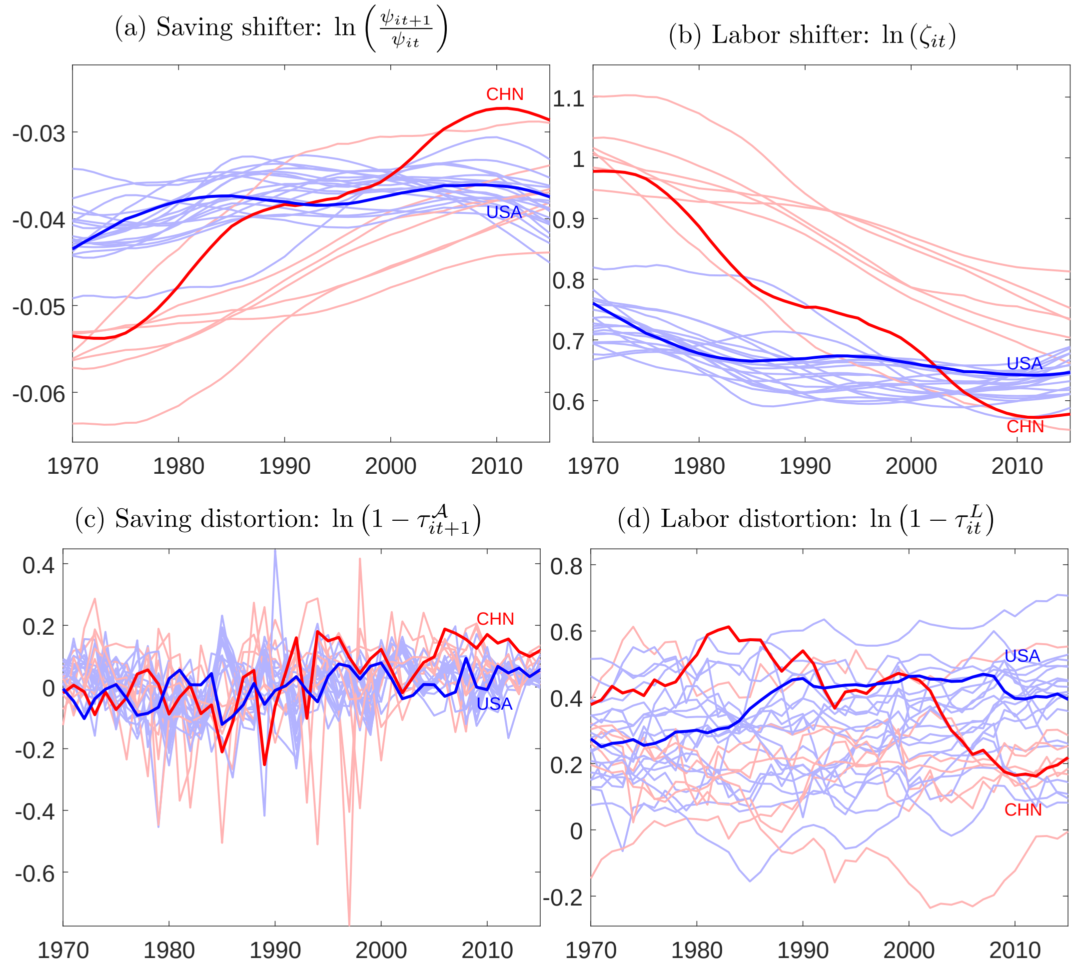
For the labor wedge, the coefficient on the working age share is significantly negative, reflecting the fact that working age individuals tend to work more (have lower marginal utility of leisure) than young individuals. The coefficient on the retired age share is also negative, reflecting a non-trivial share of retired age individuals engaged in the workforce, compared to the share of young individuals that are engaged.

Figure 2 displays the demographic and distortionary components of each wedge. The upward trend in the saving shifter in most countries (Figure 2a) reflects population aging as youth enter working age life and begin to save. Notably, the saving shifter in China is quite low compared to that of the U.S. in 1970, but increases much more rapidly over time and surpasses that of the United States in the early 1990s, coinciding with the beginning of China's demographic window. The saving distortion (Figure 2c) does not exhibit any common time trend across countries.

The downward trend in the labor wedge in most countries (Figure 2b) reflects population aging as youth enter working age and supply labor. Again, the distortionary component exhibits no common trend across countries (Figure 2d). The labor distortions do exhibit medium frequency changes in some countries. For instance, in China there is a sharp decline from 1980-1990 and then again from 2000-2010. These may capture market reform policies that raised employment levels by more than they raised wages.

The lack of low-frequency trends in the distortionary component of each wedge is reassuring, as is the lack of high-frequency variation in the demographic component. From this perspective, the preferences shifters appear to plausibly capture demographic forces. The next section of the paper develops the supply side of the model so that the effects of demographic forces can be studied in a general equilibrium setting.

Figure 2: Decomposition of calibrated wedges



Notes: The log-saving wedge is defined as $\ln\left(\omega^{\mathcal{A}}_{it}\right) = \ln\left(\frac{\psi_{it+1}}{\psi_{it}}\right) + \ln\left(1-\tau^{\mathcal{A}}_{it+1}\right)$ and the log-labor wedge is defined as $\ln\left(\omega^{L}_{it}\right) = \phi \ln\left(\zeta_{it}\right) - \phi \ln\left(1-\tau^{L}_{it}\right)$. Emerging economies in red include Brazil, China, India, Indonesia, South Korea, Mexico, Turkey, and the rest-of-world aggregate. Advanced economies in blue include the remaining 20 countries.

3 General equilibrium model

The general equilibrium model incorporates a rich supply side in which prices are determined globally and saving is allocated between investment and net exports. Relative to section 2, the model features physical capital accumulation through investment, production, and bilateral trade between countries.

Technology There is a unit interval of potentially tradable varieties, $v \in [0,1]$. All varieties are combined symmetrically to construct a composite good:

$$Q_{it} = \left[\int_0^1 q_{it}(v)^{1-1/\eta} dv \right]^{\eta/(\eta-1)},$$

where η is the elasticity of substitution between any two varieties. The term $q_{it}(v)$ is the quantity of variety v used to construct the composite good in country i at time t.

The composite good is nontradable, and is used to satisfy final consumption demand, final investment demand, and intermediate input demand. One unit of the composite good can be transformed into χ_{it}^c units of consumption, into χ_{it}^x units of investment, or into one unit of the intermediate input. These transformation costs pin down relative prices, which influence rates of return to investment and, hence, capital flows.[14]

Country i can produce variety v using capital, labor, and intermediates:

$$Y_{it}(v) = z_{it}(v) \left(A_{it} K_{it}(v)^{\alpha} L_{it}(v)^{1-\alpha} \right)^{\nu_{it}} M_{it}(v)^{1-\nu_{it}}.$$

The term $M_{it}(v)$ is the quantity of the composite good used as an input to produce $Y_{it}(v)$ units of variety v, while $K_{it}(v)$ and $L_{it}(v)$ are the quantities of capital and labor used.

The parameter ν_{it} denotes the share of value added in total output in country i at time t, providing flexibility for the model to simultaneously reconcile *gross* trade flows and *value added* output. The parameter α denotes capital's share in value added.

The term A_{it} denotes country i's value-added productivity at time t while the term $z_{it}(v)$ is country i's idiosyncratic productivity draw for producing variety v at time t, which scales gross output. Idiosyncratic productivity in each country is drawn independently from a Fréchet distribution with c.d.f. $F(z) = \exp(-z^{-\theta})$.

Trade Each country sources each variety from its least-cost supplier. International trade is subject to physical iceberg costs. At time t, country i must purchase $d_{ijt} \geq 1$ units of any intermediate variety from country j in order for one unit to arrive; $d_{ijt} - 1$ units *melt* away in transit. As a normalization I assume that $d_{iit} = 1$ for all i and t.

[14]Caselli and Feyrer (2007) argue that differences in the relative price of investment help rationalize the Lucas (1990) paradox – why capital does not flow from rich to poor countries. This puzzle is related to, but different from, the allocation puzzle since the allocation puzzle relates capital flows to growth rates rather than to levels of income per capita. I incorporate these relative prices exogenously since there is no obvious reason why they should depend on the age distribution.

Endowments and demographics Initially each country is endowed with a capital stock, K_{i1}, and NFA position, $\mathcal{A}_{i1}$. In each period population is given exogenously by N_{it}. The model does not explicitly include heterogeneity in the population with respect to age. Instead, the age distribution is manifested in "preference shifters" described in section 2.

Preferences Lifetime utility is defined exactly as in equation (1) in section 2.

Net-foreign asset accumulation The representative household enters period t with NFA position $\mathcal{A}_{it}$. If $\mathcal{A}_{it} < 0$ then the household has a net debt position. It is augmented by net purchases of one-period bonds (the current account balance), $\mathcal{B}_{it}$. With $\mathcal{A}_{i1}$ given,

$$\mathcal{A}_{it+1} = \mathcal{A}_{it} + \mathcal{B}_{it}.$$

Capital accumulation The household enters period t with K_{it} units of capital. Investment, X_{it}, adds to the stock of capital subject to an adjustment cost and depreciation. With $K_{i1} > 0$ given, capital accumulates according to

$$K_{it+1} = (1-\delta)K_{it} + \delta^{1-\lambda} X_{it}^{\lambda} K_{it}^{1-\lambda}.$$

The depreciation rate, δ, and the adjustment cost elasticity, λ, are constant both across countries and over time. The term $\delta^{1-\lambda}$ ensures that there are no adjustment costs to replace depreciated capital; for instance, in a steady state, $X^\star = \delta K^\star$. The adjustment cost implies that the return to capital investment depends on the quantity of investment. This guarantees a unique, optimal portfolio choice with respect to capital investment and bond purchases, and also ensures that gross capital formation is positive.

Budget constraint Capital and labor are compensated at the rates r_{it} and w_{it}, respectively, and are subject to distortionary taxes, τ_{it}^{L} and τ_{it}^{K}. Current investment expenditures are deductible from current capital gains taxes.[15] The world interest rate on bonds at time t is denoted by q_t and cross-border financial flows are subject to a distortionary tax, $\tau_{it}^{\mathcal{A}}$. All tax revenue is returned in lump sum to the household, T_{it}.

With consumption and investment prices denoted by P_{it}^{c} and P_{it}^{x}, respectively, the budget

[15]This assumption brings tractability by making the distortion separable from the gross return on capital.

constraint in each period is given by

$$P_{it}^c C_{it} + \mathcal{A}_{it+1} = (r_{it}K_{it} - P_{it}^x X_{it})\left(1-\tau_{it}^K\right) + w_{it}L_i\left(1-\tau_{it}^L\right) + (1+q_t)\mathcal{A}_{it}\left(1-\tau_{it}^{\mathcal{A}}\right) + T_{it}.$$

3.1 Equilibrium

A competitive equilibrium satisfies the following conditions: (i) taking prices as given, the representative household in each country maximizes its lifetime utility subject to its budget constraint and technologies for accumulating physical capital and assets, (ii) taking prices as given, firms maximize profits subject to the available technologies, (iii) intermediate varieties are purchased from their lowest-cost provider, and (iv) markets clear. At each point in time world GDP is the numéraire: $\sum_{i=1}^{I} r_{it}K_{it} + w_{it}L_i = 1$. That is, all prices are expressed in units of current world GDP. Appendix B describes the equilibrium conditions in more detail.

Remark on general equilibrium effects Demographics, operating through preference shifters, influence external imbalances (i) directly through the consumption-saving trade-off and (ii) indirectly through labor supply as a country's ability to finance existing liabilities depends on its productive capacity. Indeed, the extent that saving is allocated toward investment versus net exports depends on the magnitude of trade barriers and investment distortions.

For instance, the overall price level in a country depends on technologies across the world, subject to the trade costs with said countries:

$$P_{it} = \gamma\left(\sum_{j=1}^{I}\left((A_{jt})^{-\nu_{jt}}u_{jt}d_{ijt}\right)^{-\theta}\right)^{-\frac{1}{\theta}},$$

where $u_{jt} = \left(\frac{r_{jt}}{\alpha\nu_{jt}}\right)^{\alpha\nu_{jt}}\left(\frac{w_{jt}}{(1-\alpha)\nu_{jt}}\right)^{(1-\alpha)\nu_{it}}\left(\frac{P_{jt}}{1-\nu_{jt}}\right)^{1-\nu_{jt}}$ is the unit cost of production. Productivity growth or declining trade costs result in a lower price level, thereby raising the real interest rate, $\frac{1+q_{t+1}}{P_{it+1}^c/P_{it}^c}$. In turn, a country with faster declining prices due to technology or trade costs would attract foreign investment and experience faster consumption growth, growing real wages, and increased labor supply.

In addition, investment distortions affect the real interest rate through a no arbitrage condition: Households must be indifferent between purchasing a unit of investment and purchasing a bond, so that the after-tax returns are equalized:

$$\left(1-\tau_{it+1}^{\mathcal{A}}\right)\left(1+q_{t+1}\right)=\left(\frac{\frac{r_{it+1}}{P_{it+1}^{x}}-\Phi_{2}(K_{it+2},K_{it+1})}{\Phi_{1}(K_{it+1},K_{it})}\right)\left(\frac{P_{it+1}^{x}}{P_{it}^{x}}\right)\left(\frac{1-\tau_{it+1}^{K}}{1-\tau_{it}^{K}}\right),$$

where $X \equiv \Phi(K', K) = \delta^{1-1/\lambda}\left(\frac{K'}{K}-(1-\delta)\right)^{1/\lambda} K$ denotes investment, and Φ_1 and Φ_2 denote the derivatives with respect to the first and second arguments, respectively.[16] The conversion costs, χ_{it}^{c} and χ_{it}^{x}, also impact imbalances by governing the rate of transforming tradables into consumption and investment.

As investment distortions fall, the real rate of return on capital investment rises implying a higher real interest rate thus attracting foreign financial flows. The rental rate of capital also depends on the labor supply relative to the stock of capital, and therefore on the age distribution, and thus influences financial flows and external imbalances across countries.

4 Calibration

The model is applied to 28 countries (27 individual countries plus a rest-of-world aggregate) from 1970-2060. Data from 1970-2014 are realized, while data from 2015-2060 are based on projections. Incorporating projections serves two purposes. First, it imposes the terminal conditions as of 2060, far enough beyond 1970-2014, which is the period of interest. Second, it provides external discipline to the expectations of households in formulating saving decisions prior to 2014. Appendix A describes the data and lists countries with their 3-digit ISO codes.

The calibration involves two parts. The first part assigns values to the common parameters: $(\beta, \phi, \alpha, \delta, \lambda, \theta, \eta)$. These are taken from existing literature.

The second part assigns values to the country-specific and time-varying parameters: $\left(K_{i1}, \mathcal{A}_{i1}, \{N_{it}, \chi_{it}^{c}, \chi_{it}^{x}, \nu_{it}, \psi_{it}, \zeta_{it}, \tau_{it}^{K}, \tau_{it}^{L}, \tau_{it}^{\mathcal{A}}, d_{ijt}, A_{it}\}_{t=1}^{T}\right)$ for all (i, j). Some of the country-specific parameters are observable. For the ones that are not observable, I invert structural equations from the model and link them with data. That is, these parameters are inferred to rationalize both the observed and projected data as a solution to a perfect foresight equilibrium. The data targets, roughly in order of how they map into the model parameters, are (i) initial stock of capital, (ii) initial NFA position, (iii) population, (iv) price level of consumption using PPP exchange rates, (v) price level of investment using PPP exchange rates, (vi) ratio of value added to gross output, (vii) real consumption growth, (viii) ratio

[16] With $\lambda = 1$, there is no adjustment cost and capital depreciates linearly: $\Phi_1 = 1$ and $\Phi_2 = (\delta - 1)$. With $\lambda = 0$, the adjustment cost is infinite and capital never changes.

of employment to population, (ix) real investment, (x) bilateral trade flows, and (xi) price level of tradables using PPP exchange rates. The calibration ensures internal consistency with national accounts in every country-year.[17]

Common parameters Table 2 reports values for the common parameters. As in section 2, I set $\beta = 0.96$ and $\phi = 2$.

Table 2: Common parameters

β	Annual discount factor	0.96
ϕ	Elasticity of labor supply	2
α	Capital's share in value added	0.33
δ	Annual depreciation rate for stock of capital	0.06
λ	Adjustment cost elasticity	0.76
θ	Trade elasticity	4
η	Elasticity of substitution between varieties	2

Capital's share in value added is set to 0.33, based on evidence in Gollin (2002). In line with the literature, I set the depreciation rate for capital to $\delta = 0.06$. The adjustment cost elasticity is set to $\lambda = 0.76$, which is the midpoint between 0.52 and 1. The value 0.52 corresponds to the median value used by Eaton, Kortum, Neiman, and Romalis (2016) who work with quarterly data ($\lambda = 1$ corresponds to no adjustment costs).

I set the trade elasticity to $\theta = 4$ as in Simonovska and Waugh (2014). The parameter η plays no role in the model other than satisfying $1 + \frac{1}{\theta}(1 - \eta) > 0$; I set $\eta = 2$.

Initial conditions For each country the initial stock of capital, K_{i1}, is taken directly from the data in 1970, while the initial net-foreign asset position is set to $\mathcal{A}_{i1} = 0$.

Population, value added shares, and relative prices Population, N_{it}, is observable. The parameter ν_{it} is defined as the ratio of aggregate value added to gross production, and is constructed as such. The (inverse) relative price of consumption, χ^c_{it}, is computed as the price of intermediates relative to consumption. Similarly, χ^x_{it} is the price of intermediates

[17] Note that there are 13 time-varying parameters and only 11 independent data series. The national accounts data identifies the saving wedge, which is the product of the saving shifter and the saving distortion, as well as the labor wedge, which is the product of the labor shifter and the labor distortion. Section 2 separately identifies the preference shifters from the distortions by utilizing information on demographics.

relative to investment. Prices of intermediates, consumption, and investment are observable.

$$\nu_{it} = \frac{r_{it}K_{it} + w_{it}L_{it}}{r_{it}K_{it} + w_{it}L_{it} + P_{it}M_{it}} = \frac{\text{GDP}_{it}}{\text{Gross production}_{it}}, \qquad \chi^c_{it} = \frac{P_{it}}{P^c_{it}}, \qquad \chi^x_{it} = \frac{P_{it}}{P^x_{it}}$$

Preference shifters, saving distortions, and labor distortions The wedges, $\omega^{\mathcal{A}}_{it} = \left(\frac{\psi_{it+1}}{\psi_{it}}\right)\left(1 - \tau^{\mathcal{A}}_{it+1}\right)$ and $\omega^L_{it} = (\zeta_{it})^{\phi}\left(1 - \tau^L_{it}\right)^{-\phi}$, are exactly those computed from equations (2) and (3) in section 2. From these wedges, the preference shifters, $\left(\frac{\psi_{it+1}}{\psi_{it}}, \zeta_{it}\right)$, and the distortions, $\left(\tau^{\mathcal{A}}_{it+1}, \tau^L_{it}\right)$, are those recovered from regressions (4) and (5).

Investment distortions Without loss of generality, I initialize $\tau^K_{i1} = 0$. The remaining investment distortions require measurements of the capital stock in every period. Given capital stocks in period 1, K_{i1}, and data on investment in physical capital, X_{it}, I construct the sequence of capital stocks iteratively using $K_{it+1} = (1-\delta)K_{it} + \delta^{1-\lambda}X^{\lambda}_{it}K^{1-\lambda}_{it}$.

To ease notation, define $\Phi(K', K) \equiv X \equiv \delta^{\frac{\lambda-1}{\lambda}}\left(\frac{K'}{K} - (1-\delta)\right)^{\frac{1}{\lambda}}K$, and let Φ_1 and Φ_2 denote the partial derivatives with respect to the first and second arguments respectively. Given the constructed sequence of capital stocks, I recover τ^K_{it+1} iteratively using the Euler equation for investment in physical capital:

$$\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}} = \beta\left(\frac{\psi_{it+1}}{\psi_{it}}\right)\left(\frac{\frac{r_{it+1}}{P^c_{it+1}} - \left(\frac{P^x_{it+1}}{P^c_{it+1}}\right)\Phi_2(K_{it+2}, K_{it+1})}{\left(\frac{P^x_{it}}{P^c_{it}}\right)\Phi_1(K_{it+1}, K_{it})}\right)\left(\frac{1-\tau^K_{it+1}}{1-\tau^K_{it}}\right).$$

Prices of consumption and investment come from the national accounts data, as does per-capita consumption. The rental rate for capital, defined according to the theory, is constructed as: $r_{it} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_{it}L_{it}}{K_{it}}\right)$.

Trade costs The trade cost for any given country pair is computed using data on prices and bilateral trade shares using the following structural equation:

$$\frac{\pi_{ijt}}{\pi_{iit}} = \left(\frac{P_{jt}}{P_{it}}\right)^{-\theta} d^{-\theta}_{ijt}, \tag{6}$$

where π_{ijt} is the share of country i's absorption that is sourced from country j and P_{it} is the price of tradables in country i. I set $d_{ijt} = 10^8$ for observations in which $\pi_{ijt} = 0$ and set $d_{ijt} = 1$ if the inferred value is less than 1. As a normalization, $d_{iit} = 1$.

Productivity I back out productivity, A_{it}, using price data and home trade shares,

$$P_{it} = \left(\frac{\gamma\left(\pi_{iit}\right)^{\frac{1}{\theta}}}{\left(A_{it}\right)^{\nu_{it}}}\right)\left(\frac{r_{it}}{\alpha\nu_{it}}\right)^{\alpha\nu_{it}}\left(\frac{w_{it}}{(1-\alpha)\nu_{it}}\right)^{(1-\alpha)\nu_{it}}\left(\frac{P_{it}}{1-\nu_{it}}\right)^{1-\nu_{it}}. \tag{7}$$

More simply, $P_{it} = \frac{u_{it}}{Z_{it}}$, where u_{it} is the unit cost of an input bundle and $Z_{it} = \left(\frac{(A_{it})^{\nu_{it}}}{\gamma(\pi_{iit})^{\frac{1}{\theta}}}\right)$ is measured productivity of gross output.[18]

4.1 Key drivers of gross bilateral trade flows

As is well known, the Eaton-Kortum model of trade gives rise to a gravity structure whereby productivity and trade costs are the primary determinants of *gross* international trade patterns. These forces therefore have direct implications for *net* bilateral trade flows and, ultimately, aggregate trade imbalances. Figure 3 shows the paths for the calibrated fundamental productivity and bilateral trade costs, (A_{it}, d_{it}).

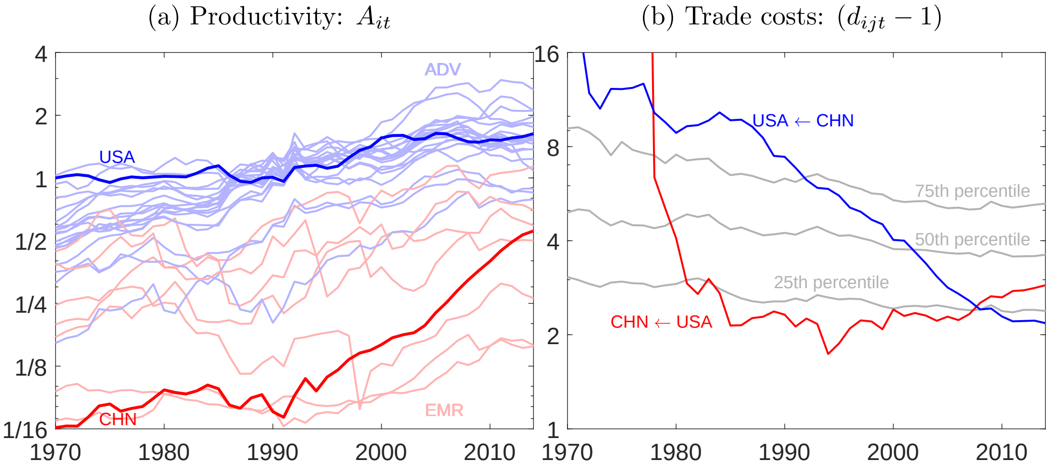
Throughout most of the sample, most countries experienced growth in fundamental productivity (see Figure 3a). Note that this measure of productivity is not the same as TFP, which can also grow due to an expanding import share.[19] Still, there are differences in both the levels and rates of productivity growth across countries. For instance, China's rapid growth beginning in early 1990s is captured to a large extent by its catch up in productivity. The United States exhibited slightly lower productivity growth than other high-income countries. All else equal, these features give rise to relatively fast growing exports in China and relatively fast growing imports in the United States.

The levels of the calibrated trade costs are in line with most estimates in the literature that use gravity-based models (see Figure 3b). The trade costs decline over time, reflecting growing international trade volumes, partly due to tariff cuts and free trade agreements, in addition to improvements in shipping technology. Beyond a common trend of declining trade costs, there is substantial heterogeneity across countries. In particular, there is rapid decline in bilateral trade costs between China and the United States following China's economic reforms in 1978 under Deng Xiaoping. China's trade costs prior to 1978 are too large to include on the same scale. The trade cost from China to the United States gradually declined ever since the 1980s. On the other hand, the trade cost from the United States to China declined rapidly for a few years after 1978, but remained fairly flat thereafter and,

[18]The parameter $\gamma = \Gamma(1 + (1-\eta)/\theta)^{1/(1-\eta)}$ is a constant, where $\Gamma(\cdot)$ is the Gamma function.

[19]Measured TFP also accounts for an endogenous change in the home trade share: $\text{TFP}_{it} \propto A_{it}\left(\pi_{iit}\right)^{-\frac{1}{\theta\nu_{it}}}$.

Figure 3: Productivity and trade costs



Notes: Productivity is shown relative to that of the United States in 1970. Trade cost percentiles are computed from the entire matrix of bilateral trade costs in each year, excluding the diagonal. Prior to 1978, China's are too large to include on the same scale, reflecting the negligible volume of international trade, particularly imports, taking place. Emerging economies (EMR) in red include Brazil, China, India, Indonesia, South Korea, Mexico, Turkey, and the rest-of-world aggregate. Advanced economies (ADV) in blue include the remaining 20 countries.

subsequently, mildly increased since the mid-1990s. Mechanically, the asymmetry in the bilateral cost between China and the United States since the 1990s captures the widening bilateral trade imbalance between the two countries. Economically, the asymmetry might partially capture Chinese policies aimed at keeping the Yuan "devalued" against the U.S. dollar, or other policies that favor exports by Chinese state-owned enterprises.

5 Quantitative analysis

This section quantifies the importance of demographics, and other forces, in shaping the evolution of trade imbalances. This is done by evaluating counterfactual equilibria arising from counterfactual processes for preference shifters and other exogenous forces. In each case, I compute the dynamic equilibrium under perfect foresight from 1970-2060. Appendix C provides the details of the algorithm for solving for the transition, which builds on Ravikumar, Santacreu, and Sposi (2019).

5.1 The role of changes in the age distribution across the world

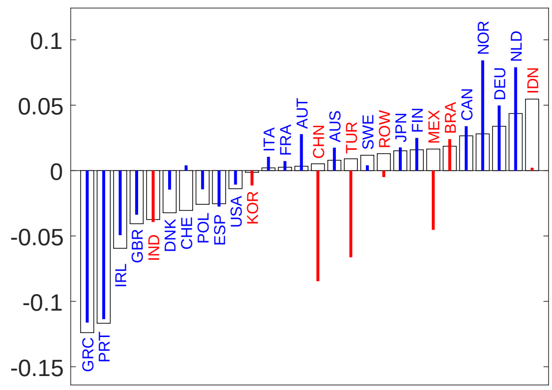
In order to quantify the importance of differential changes in the age distribution across countries, this counterfactual assumes that every country's age distribution is constant from 1970-2060. I construct counterfactual preference shifters as

$$\frac{\widetilde{\psi}_{it+1}}{\widetilde{\psi}_{it}} = \frac{\psi_{i1971}}{\psi_{i1970}}, \qquad \widetilde{\zeta}_{it} = \zeta_{i1970}, \tag{8}$$

with $\widetilde{\psi}_{i1970} = \psi_{i1970} = 1$, and hold all other parameters at their calibrated values.

Figure 4 illustrates, for every country, how trade imbalances compare in the counterfactual relative to the baseline. Specifically, the figure reports, for each country, the average ratio of net exports to GDP taken over 1970-2014: $\left(\frac{1}{45}\right)\sum_{1970}^{2014}\frac{NX_{it}}{GDP_{it}}$. There is a distinct difference between advanced and emerging economies. In advanced economies, where the working age share only slightly increased, or in some cases decreased, the ratio of net exports to GDP is higher in the counterfactual than in the baseline. The opposite is true in emerging economies where the working age share increased at much higher rates, and the ratios of net exports to GDP in the counterfactual are lower compared to the baseline.

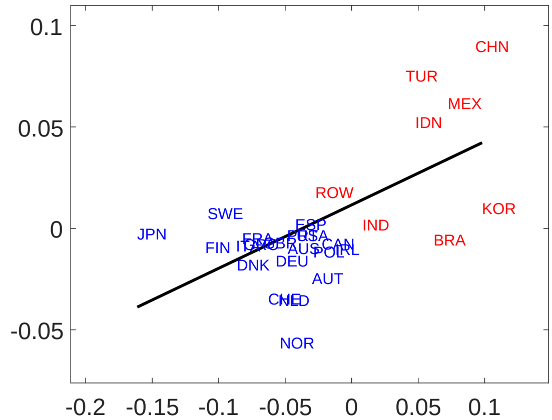
Figure 4: Average ratio of net exports to GDP from 1970-2014



Notes: Average ratio of net exports to GDP from 1970-2014: $\left(\frac{1}{45}\right)\sum_{t=1970}^{2014}\frac{NX_{it}}{GDP_{it}}$. Bars refer to the baseline model. Lines refer to the counterfactual with every country's age distribution simultaneously fixed at 1970 levels. Emerging economies are in red and advanced economies, in blue.

Figure 5 illustrates the systematic relationship between changes in each country's working age share and the difference in its trade imbalances in the baseline relative to the counterfactual. The vertical axis is the difference in each country's average ratio of net exports to GDP in the baseline relative to the counterfactual: $\left(\frac{1}{45}\right)\sum_{t=1970}^{2014}\frac{NX_{it}^{base}}{GDP_{it}^{base}}-\left(\frac{1}{45}\right)\sum_{t=1970}^{2014}\frac{NX_{it}^{cf}}{GDP_{it}^{cf}}$. The horizontal axis is the change in each country's working age share relative to that of the entire world: $(s_{i2014}-s_{i1970})-(s_{W2014}-s_{W1970})$. The slope of the best fit line is 0.31, implying that a one percentage point increase in a country's working age share, relative to that of the world, generates about one-third percentage point increase in its net exports to GDP.

Figure 5: Difference in average ratio of net exports to GDP from 1970-2014

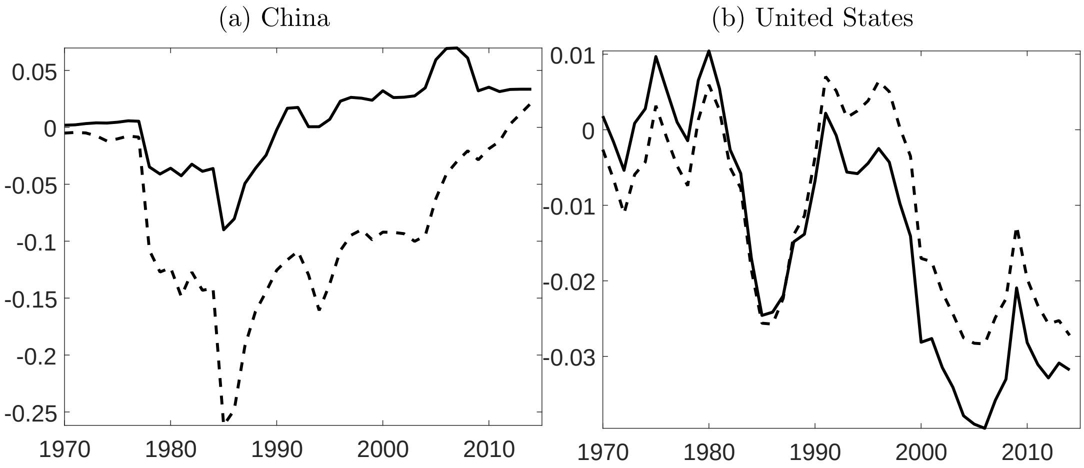


Notes: Vertical axis – Change in average ratio of net exports to GDP from 1970-2014: $\left(\frac{1}{45}\right)\sum_{t=1970}^{2014}\frac{NX_{it}^{base}}{GDP_{it}^{base}}-\left(\frac{1}{45}\right)\sum_{t=1970}^{2014}\frac{NX_{it}^{cf}}{GDP_{it}^{cf}}$, where cf denotes the counterfactual with every country's age distribution simultaneously fixed at 1970 levels and $base$ refers to the baseline. Horizontal axis – Change in working age share relative to the world: $(s_{i2014}-s_{i1970})-(s_{W2014}-s_{W1970})$. Emerging economies are in red and advanced economies, in blue.

The main takeaway is that differences in demographic trends between advanced and emerging economies induced a growing appetite for net-saving by emerging economies, resulting in lower net exports in advanced economies. As an example, consider a contrast between China and the United States. China experienced a rapid increase in its working age share from 1970 to 2014, while the United States experienced only a mild increase. Figure 6a shows that if age distributions were held constant in all countries, then China would have run a large trade deficit. Note that the counterfactual trade imbalance in China returns to surplus, and is almost the same as in the baseline by 2014. This reflects the sharp shift in China's working age share going from a fast increase leading up to 2014, followed by a rapid projected decline after 2014.

Figure 6b illustrates the path for the ratio of net exports to GDP in the United States. Not only is the ratio higher on average in the counterfactual compared to the baseline, but

Figure 6: Ratio of net exports to GDP from 1970-2014



Notes: Solid lines refer to the baseline model. Dashed lines refer to the counterfactual with age distributions simultaneously held fixed at 1970 levels.

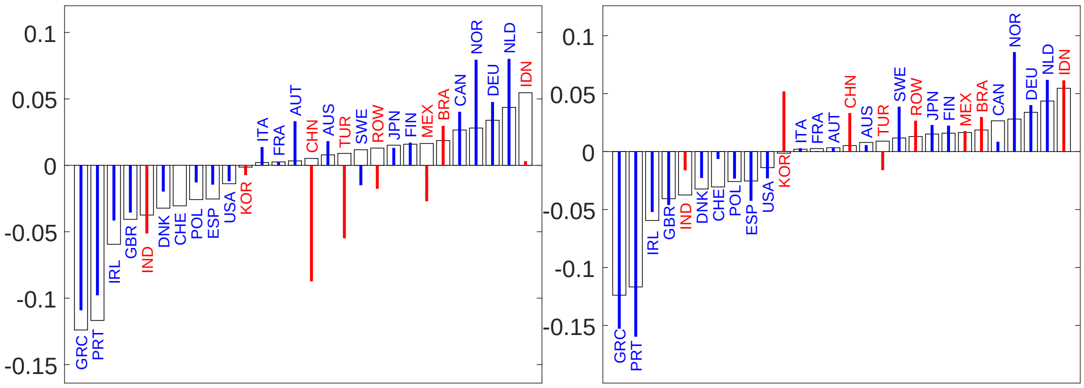
the ratio is also *flatter* over time. In particular, the counterfactual ratio is actually slightly lower than the baseline ratio from 1970-1985, but after 1990, the counterfactual ratio is permanently higher the baseline ratio. Put differently, the counterfactual ratio falls by only 2.4 percentage points between 1970 and 2014 (from -0.3% to -2.7%) instead of falling by 3.4 percentage points (from 0.2% to -3.2%) in the baseline.

Figure 10 also illustrates that demographic-induced forces give rise to a persistent and gradual influence on trade imbalances. Higher frequency fluctuations are still present in the counterfactual as in the baseline. The baseline and counterfactual trade imbalances for all countries are in Figure D.4 in Appendix D.

Demographic effects through two distinct channels Brooks (2003) argues a that a relatively fast-aging region will tend to save more in preparation for retirement, but at the same time investment demand will fall as the workforce shrinks. The following two counterfactuals isolate the individual importance of the saving shifter and the labor shifter induced by the same underlying demographic trends. The first imposes the counterfactual path for saving shifters, $\{\widetilde{\psi}_{it}\}$ from equation (8), holding all other parameters at their baseline values. The second imposes the counterfactual path for labor shifters, $\{\widetilde{\zeta}_{it}\}$ from equation (8), holding all other parameters at their baseline values.

Figure 7 plots the average ratio of net exports to GDP from 1970-2014, for both the baseline and each counterfactual. In particular, Figure 7a illsutrates the comparison for the counterfactual saving shifters, while Figure 7b illustrates that for the counterfactual labor shifters. There are notable differences in each counterfactual with respect to the baseline, and also with respect to each other.

Figure 7: Average ratio of net exports to GDP from 1970-2014



Notes: Average ratio of net exports to GDP from 1970-2014: $\left(\frac{1}{45}\right)\sum_{t=1970}^{2014}\frac{NX_{it}}{GDP_{it}}$. Bars refer to the baseline model. Lines refer to the counterfactual with preference shifters in every country simultaneously frozen at 1970 levels. Emerging economies are in red and advanced economies, in blue.

With counterfactual saving shifters, the United States accumulates a slightly smaller trade deficit compared to the data, while China accumulates a larger deficit. More generally, advanced economies that experienced relatively slow increases, or even decreases, in the working age share from 1970 to 2014 (most of Europe and former British colonies) all demonstrate reduced trade deficits or greater trade surpluses, in the counterfactual relative to the baseline. Conversely, emerging economies that experienced relatively sharp increases in the working age share from 1970-2014 (China, India, Indonesia, Mexico, South Korea, Turkey) demonstrate increased trade deficits or reduced trade surpluses, in the counterfactual relative to the baseline. The reason is that, by freezing saving shifters, the demographic-induced demand for saving by emerging economies is eliminated.

Labor shifters generally have the opposite impact on trade imbalances than saving shifters do. With counterfactual labor shifters, the United States accumulates a slightly larger trade deficit compared to the data, whereas China's trade surplus is magnified. Again, this result is fairly systematic with respect to advanced versus emerging economies. Advanced economies with relative declines in the working age share generally experience lower ratios of net exports to GDP, compared to the baseline, whereas emerging economies with relative increases in the working age share generally experience higher ratios. The reason is that a growing workforce raises the return to capital investment, thereby attracting financial resources from abroad resulting in more borrowing and lower net exports; a mechanism similar to the distortion-induced capital flows though labor supply discussed in Ohanian, Restrepo-Echavarria, and Wright (2018).

In sum, relatively fast increases in the working age share induce higher propensity to save, while at the same time generate higher propensity to supply labor thereby triggering capital inflows (borrowing). On net, the saving channel is quantitatively more prominent, so relatively fast increases in the working age share induce a greater demand for saving and a trade surplus, all else equal.

5.2 Population growth as a driver of imbalances

Aside from the age distribution, population size itself is an important demographic characteristic. In particular, countries exhibit noticeable differences in population growth rates. For instance, from 1970 to 2014 Germany's population grew by less than one-tenth of a percent per year, on average, whereas Mexico's population grew by over two percent. (Figure D.2 in Appendix D shows the observed and projected population growth rates for all countries.)

In a standard closed economy neoclassical growth model, changes in the population growth rate have a similar effect of the consumption-saving tradeoff as a change in the discount factor. That is, in anticipation of higher population in the future, households invest (save) more in order to smooth consumption per capita. This incentive is present in the open economy model, but the general equilibrium effects occur through the real exchange rate instead, since saving can be achieved via borrowing and lending instead of through investment. Moreover, in an open economy model, borrowing and lending is impacted by *relative* differences in population growth across countries.

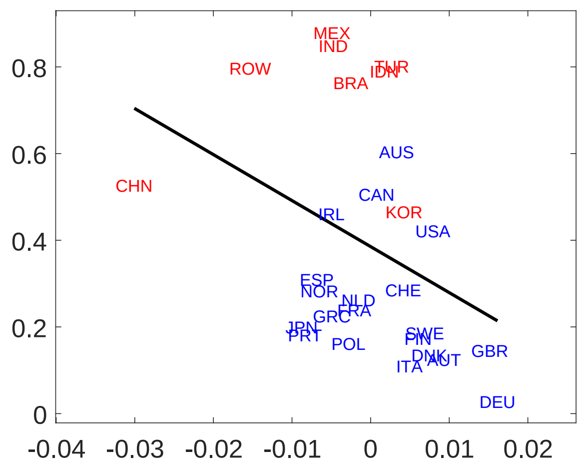
Recall the expression for measured productivity: $(A_{it})^{\nu_{it}} (\pi_{iit})^{-1/\theta}$. All else equal, higher population implies higher home trade share and, hence, lower measured productivity. So if a country expects fast population growth, it also expects to have a deterioration in its measured productivity and in its real exchange rate (RER).

To make this point more concrete, define country i's RER as the trade-weighted average of all of its bilateral RERs, $\frac{P_i}{P_j}$, for all $j \neq i$:

$$RER_{it} = \frac{\sum\limits_{j=1, j\neq i}^{I} \left(\frac{P_{it}}{P_{jt}}\right)(Trd_{ijt} + Trd_{jit})}{\sum\limits_{j=1, j\neq i}^{I} (Trd_{ijt} + Trd_{jit})}. \tag{9}$$

On the vertical axis of Figure 8 is the log-difference between RER appreication in the baseline model and that in the counterfactual with population fixed over time. Countries near the bottom of the vertical axis realize a depreciation in their RER in the baseline compared to the counterfactual. On the horizontal axis is the log-change in observed population from 1970 to 2014. Countries on the far right of the horizontal axis had high observed population growth from 1970 to 2014. The negative relationship indicates that higher population growth induces a real devaluation. The intuition is that countries that have high population growth want to save more to smooth consumption per capita. In equilibrium, these countries will face relatively rising consumer prices and a declining real exchange rate, all else equal. Emerging economies experienced faster population growth than advanced economies, on average, partly boosting their saving demand, all else equal.

Figure 8: Difference in real exchange appreciation against observed population growth



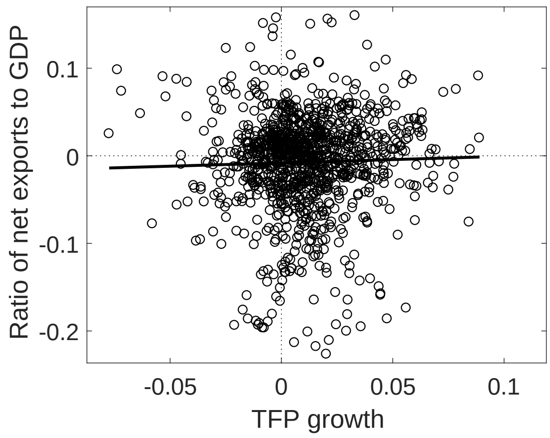
Notes: Vertical axis is $\ln\left(RER^{base}_{i2014}/RER^{base}_{i1970}\right) - \ln\left(RER^{cf}_{i2014}/RER^{cf}_{i1970}\right)$. RER denotes the real exchange rate given by equation (9). Superscript cf refers to the counterfactual with population simultaneously fixed in every country at 1970 levels. Superscript $base$ refers to the baseline model (data). Horizontal axis is $\ln\left(N_{i2014}/N_{i1970}\right)$. Emerging economies are in red and advanced economies, in blue.

Since the model is deterministic, a realized change in any bilateral RER is equal to the change in real interest rate differential between the two countries. An obvious corollary is that a net increase in immigration would induce more saving. Taylor and Williamson (1994) take a historical perspective along these lines and argue that capital flows from Britain to the New World were effectively an intergenerational transfer from old savers to young savers.

5.3 Demographics as a partial resolution to the allocation puzzle

Figure 9 depicts the allocation puzzle. Specifically, there is clearly no negative correlation between the ratio of net exports to GDP and TFP growth. The slope of the best fit curve between (elasticity of) the ratio of net exports to GDP and TFP growth is 7.5%.

Figure 9: Ratio of net exports to GDP against labor TFP growth



Notes: Horizontal axis is the average annual growth in labor productivity during five year windows. Vertical axis is the average ratio of net exports to GDP during five year windows. Windows run from [1970,1974]-[2010,2014]. The line corresponds to the best fit curve using OLS.

This section considers how various forces contribute to the allocation puzzle by simulating counterfactuals in which particular exogenous forces are held constant over time. In each counterfactual, I recompute the elasticity between the ratio of net exports to GDP and TFP growth. Table 3 reports the elasticity for the baseline and counterfactual specifications.

Freezing the age distribution In this counterfactual I freeze the demographic forces—saving shifters and labor shifters—at their initial levels, as in equation 8.

Removing changes in the age distribution unveils a strong negative relationship between net exports and TFP growth. The elasticity is significantly negative, at -19.4%, which is

Table 3: Elasticity between ratio of net exports to GDP and TFP growth

Specification	Elasticity	(S.E.)
Baseline model (data)	7.5%	(7.5%)
Counterfactual: Freezing the age distribution	−19.4%	(9.8%)
Counterfactual: Freezing saving distortions	−96.3%	(50.5%)
Counterfactual: Freezing labor distortions	40.0%	(12.4%)
Counterfactual: Freezing investment distortions	−2.7%	(13.7%)
Counterfactual: Freezing bilateral trade distortions	102.7%	(15.9%)

Notes: The elasticity is the slope of the line corresponding to the best fit curve using OLS (see Figure 9 for the case of the baseline model/data). The y-variable in the regression is the average annual growth in TFP during five year windows. The x-variable in the regression is the average ratio of net exports to GDP during five year windows. Windows run from [1970,1974]-[2010,2014].

much lower than in the baseline.

This finding indicates an intimate relationship between changes in the age distribution and the rate of TFP growth. Countries that experienced fast TFP growth, but did not run trade deficits, also tend to be countries that experienced relatively fast increases in their working age shares. That is, the allocation puzzle is partly due to the fact that fast growing emerging economies have an incentive to save because of demographics.

Freezing saving distortions In this counterfactual I freeze the saving distortion in each country at its initial level:

$$\widetilde{\tau}^{\mathcal{A}}_{it+1} = \tau^{\mathcal{A}}_{i1971},$$

with $\widetilde{\tau}^{\mathcal{A}}_{i1970} = \tau^{\mathcal{A}}_{i1970} = 0$. The relationship between trade imbalances and TFP growth becomes strongly negative, with an elasticity of -96%. This finding suggests that the saving distortion encompasses important frictions that give rise to the allocation puzzle.

This is no surprise, given the findings in Gourinchas and Jeanne (2013). However, there is a subtle difference here: The saving distortion in my model is only one part of the overall saving wedge explored by Gourinchas and Jeanne (2013). The other component is the demographic-induced saving shifter. Demographics chip away at saving wedge in this sense, but a sizable chunk remains to be explained.

Therefore, there is plenty of room for more analysis to open up the saving wedge further, perhaps along the lines of public saving, suggested by Alfaro, Kalemli-Ozcan, and Volosovych (2008). While large portions of government saving are influenced by demographics through pensions, government saving is likely affected by non-demographic forces such as un-modeled political economy features. Another posssibility is that the consumption-saving trade-off depends on past levels of consumption. That is, Carroll, Overland, and Weil (2000) posit that habits can account for the fact that a boom income is not immediately met with a boom in consumption. Such mechanisms are beyond the scope of this paper.

Freezing labor distortions In this countrfactual I freeze the labor distortion in each country at its initial level:

$$\widetilde{\tau}_{it}^{L} = \tau_{i1970}^{L}.$$

The elasticity of the ratio of net exports to GDP with respect to TFP growth is 40%. This elasticity is greater than that in the baseline, implying that labor distortions do not systematically explain the allocation puzzle, but actually make it more puzzling.

Ohanian, Restrepo-Echavarria, and Wright (2018) argue that the labor wedge explains why capital flew out of Latin America and into Asia from 1950-1970. In their model, the labor wedge is computed similarly to mine, but the distortionary component is not separated from the demographic component (preference shifter). I do not rule out the possibility that the labor distortion is important for certain regional flows over certain time periods, but I instead study the period 1970-2014 across a much larger set of countries.

Freezing investment distortions In this countrfactual I freeze the investment distortion in each country at its initial level:

$$\frac{1-\widetilde{\tau}_{it+1}^{K}}{1-\widetilde{\tau}_{it}^{K}} = \frac{1-\tau_{i1971}^{K}}{1-\tau_{i1970}^{K}},$$

with $\widetilde{\tau}_{i1970}^{K} = \tau_{i1970}^{K}$=0. The elasticity of the ratio of net exports to GDP with respect to TFP growth is -2.7%. This elasticity is slightly negative, indicating that investment distortions systematically explain some of the allocation puzzle. These distortions capture frictions pertaining to investment, such as those in Buera and Shin (2017), as well as institutional problems that discourage investment, such as those discussed in Aguiar and Amador (2011).

Aguiar and Amador (2011) put forward a theoretical justification that highly indebted developing countries have little incentive to invest due to the risk of being expropriated by their government. As a result, saving shows up in net exports; This is an example of an investment distortion in my model. Buera and Shin (2017) argue that financial frictions restrict the extent that investment can respond to fast GDP growth, implying that a fast growing country will tend to run a current account surplus in the short run.

Freezing bilateral trade distortions I decompose the bilateral trade costs (the portion that melts away, $d-1$) into a trend component and a bilateral distortionary component:

$$(d_{ijt}-1)=\mathcal{D}_t\times\varepsilon^d_{ijt}.$$

The estimated trend component is then

$$\widehat{\mathcal{D}}_t=\exp\left(\frac{1}{I\times(I-1)}\sum_{i=1}^{I}\sum_{\substack{j=1\\j\neq i}}^{I}\ln(d_{ijt}-1)\right),$$

which steadily declines from 7.7 in 1970 to 3.6 in 2014, capturing reductions in shipping costs as well as large-scale tariff reductions implemented under the General Agreement on Tariffs and Trade and subsequently the World Trade Organization. The distortionary component captures changes in bilateral trade costs, such as bilateral trade agreements, for instance, or other policies implemented unilaterally.

In this countrfactual I freeze the bilateral trade distortions between each country at their initial levels and construct counterfactual trade costs:

$$\widetilde{d}_{ijt}=1+\widehat{\mathcal{D}}_t\times\widehat{\varepsilon}^d_{ij1970}.$$

The elasticity of the ratio of net exports to GDP with respect to TFP growth is 103%, indicating that asymmetries in trade distortions don't reconcile the allocation puzzle.

This result does not mean that asymmetries is trade costs are not important for explaining specific bilateral trade imbalances. For instance, much has been made in the political arena of unequal trade costs, specifically between China and the United States, both in terms of levels and changes. Indeed, in this counterfactual the U.S. bilateral trade deficit with China disappears as of 2014, compared to being 1.8 percent of U.S. GDP in the data. This finding is consistent with Alessandria and Choi (2019), who argue that asymmetric changes in trade costs faced by the U.S. influenced the U.S. trade deficit.

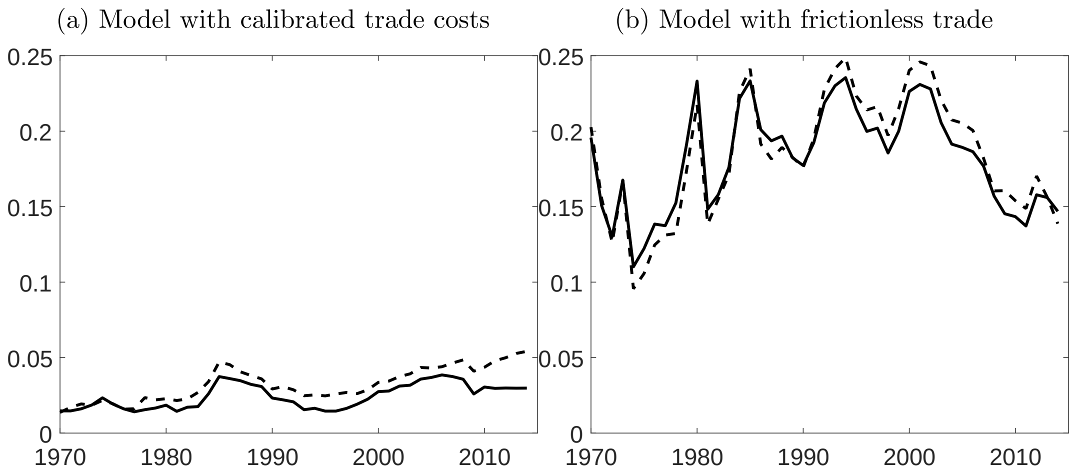
5.4 Gross trade volumes and trade imbalances

This section illustrates the importance of disciplining gross trade flows when studying trade imbalances. The thought experiment is to compare the baseline model, in which the trade costs are calibrated to match the observed gross trade flows, to an alternative model in which trade is frictionless: $\tilde{d}_{ijt}=1$.

In each model specification, I decompose the ratio of net exports to GDP into two pieces: the ratio of trade to GDP (intratemporal margin) times the ratio of net exports to trade (intertemporal margin). The first piece is a focal point of the international trade literature, and the second piece, a focal point of the international finance literature. For brevity, I present results for the world aggregate. I define *global trade imbalances* as the sum of the absolute value of net exports across countries, *global trade flows* as the sum of bilateral trade flows across countries, and *world GDP* as the sum of GDP across countries.

Figure 10 illustrates the ratio of global trade imbalances to world GDP in both model specifications. In each panel, the solid line corresponds to a world with time-varying demographics. Notably, the ratio of global trade imbalances to global trade flows is much higher under frictionless trade than in the baseline model, mostly because of the higher ratio of global trade flows to world GDP. In the baseline model (which matches the data), the ratio of global trade flows to world GDP, averaged from 1970-2014, is 0.13, compared to 1.87 in the alternative model. Meanwhile, the ratio of global trade imbalances to global trade flows, averaged from 1970-2014, is 0.18 in the baseline model compared to 0.14 in the alternative model. While the volume of gross trade significantly differs between the two models, the extent of net trade flows are remarkably similar.

Figure 10: Ratio of global trade imbalances to world GDP from 1970-2014



Notes: Global trade imbalance is the sum of the absolute net trade position across countries: $\sum_i |NX_{it}|$. World GDP is $\sum_i GDP_{it}$. Solid lines refer to the model with observed demographics. Dashed lines refer to the counterfactual with age distributions simultaneously held fixed at 1970 levels. The left panel is the baseline model with trade costs calibrated to match observed bilateral trade flows. The right panel is the model with frictionless trade.

Indeed, it is not surprising that the alternative model generates implausibly large global trade flows as a share of world GDP. Nonetheless, the relevant concern is whether the demographic-induced change in trade imbalances differs between the baseline model and the frictionless trade model. To explore this I consider two scenarios. One scenario holds the age distribution constant in the baseline model, while the other scenario holds the age distribution constant in the alternative model. In both scenarios, the ratio of global trade imbalances to world GDP is depicted by the dashed lines in Figure 10.

In the baseline model with the age distribution frozen, the average ratio of global trade flows to world GDP remains at 0.13 (compared to 0.13 the baseline model with time-varying demographics). In the alternative model with the age distribution frozen, the same ratio barely changes to 1.87 (compared to 1.86 in the alternative model with time-varying demographics). That is to say, in both model specifications, demographics do not have any meaningful quantitative effect on the ratio of global trade flows to world GDP.

In the baseline model with the age distribution frozen, the ratio of global imbalances to global trade shoots up to 0.70 (compared to 0.18 in the baseline model with time-varying demographics). In the alternative model with the age distribution frozen, the same ratio slightly decreases to 0.11 (compared to 0.14 in the alternative model with time-varying demographics).

This result implies that, in the baseline model, demographic forces suppressed the ratio of global trade imbalances to world GDP, and the suppression effect gradually strengthened over time, as indicated by the widening gap between the dashed and solid lines in Figure 10a. The effect of demographics are fundamentally different in the alternative model. On average, demographic forces inflated the ratio, but the effects are not even qualitatively consistent over time. Figure 10b illustrates that demographics inflated the ratio from 1970-1990, then suppressed the ratio from 1990 to 2010, and by 2014 again inflated the ratio.

The takeaway is that it is not a harmless assumption to impose frictionless trade, or to abstract from disciplining gross trade volumes, when studying the effect of demographics on trade imbalances.

5.5 China

My paper also speaks to the large and growing body of work on China, which has become the poster child for the allocation puzzle due to its rapid growth and large trade surplus. The literature has found evidence that both demographics and market distortions have been at work. Wei and Zhang (2011) argue that male-biased gender ratios encourage men to save by purchasing real estate to attract scarce female partners. Yang, Zhang, and Zhou (2012) argue that successive cohorts of young Chinese workers face increasingly flat life-cycle earnings profiles, thereby reducing household borrowing in the face of higher future aggregate productivity. İmorohoğlu and Zhao (2018a) argue that, due to China's one-child policy, the elderly rely more on personal savings to replace lacking family support during retirement. İmorohoğlu and Zhao (2018b) argue that financial constraints faced by firms are equally important as the one-child policy in accounting for the rise in China's saving rate. In Song, Storesletten, and Zilibotti (2011), financial market imperfections imply that private firms in China finance the adaptation of technology through internal savings. My findings do not rule out any of these theories but, instead, offer a systematic assessment of the role of demographic-induced saving behavior across many countries over a long time period. In addition, my findings suggest that the age distribution played a quantitatively important role in contributing to China's trade surplus.

6 Conclusion

The paper builds a dynamic, multicountry, Ricardian model of trade, where dynamics are driven by international borrowing and lending and capital accumulation. Trade imbalances arise endogenously as the result of relative shifts in technologies, trade costs, factor market distortions, and demographics. All of the exogenous forces are calibrated using a wedge accounting procedure so that the model rationalizes past and projected national accounts data and bilateral trade flows across 28 countries from 1970-2060.

Cross-country differences in changes in the age distribution systematically drive both the direction and magnitude of trade imbalances. On average, a one percentage point increase in a country's working age share, relative to that of the entire world, implies a one-third percentage point increase in its net exports to GDP. Demographic trends since 1970 contributed positively to net exports in emerging economies with quickly rising working age shares, and contributed negatively to net exports in advanced economies with slowly rising, or even falling, working age shares. The systematic relationship between economic growth rates and changes in the age distribution provides a channel in which resources flow from fast-growing emerging economies to slower-growing advanced economies, thereby alleviating the allocation puzzle.

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A Data

This section of the Appendix describes the sources of data as well as adjustments made to the data. Sources include the 2016 release of the World Input-Output Database (Timmer, Dietzenbacher, Los, Stehrer, and de Vries, 2015, (WIOD)), version 9.0 of the Penn World Table (Feenstra, Inklaar, and Timmer, 2015, (PWT)), Organization for Economic Cooperation and Development (2014) Long-Term Projections Database (OECD), 2015 revision of the United Nations (2015) World Population Prospects (UN), the International Monetary Fund Direction of Trade Statistics (IMFDOTS), and Federal Reserve Economic Data (FRED). Table A.1 summarizes the data raw data.

Table A.1: Model variables and corresponding data sources

Variable description	Model counterpart	Data source 1970-2014	Data source 2015-2060
Age distribution	s_{it}	UN	UN
Population	N_{it}	PWT	UN
Employment	L_{it}	PWT	OECD
Value added*	$w_{it}L_{it} + r_{it}K_{it}$	PWT & WIOD	OECD
Price of consumption**	P^c_{it}	PWT	Imputed
Price of investment**	P^x_{it}	PWT	Imputed
Price of composite intermediate**	P_{it}	PWT	Imputed
Risk free nominal interest rate	q_t	FRED	Imputed
Consumption***	C_{it}	PWT	Imputed
Investment***	X_{it}	PWT	OECD
Initial capital stock***	K_{i1}	PWT	N/A
Gross output*	$P_{it}Y_{it}$	Imputed & WIOD	Imputed
Bilateral trade flow*	$P_{it}Q_{it}\pi_{ijt}$	IMFDOTS & WIOD	Imputed
Absorption*	$P_{it}Q_{it}$	Imputed & WIOD	Imputed

Notes: The age distribution tracks the working age share and the retired share of the population: $\mathrm{s}_{it} = \left(s^{15-64}_{it}, s^{65+}_{it}\right)$. *Values are measured in current prices using market exchange rates. **Prices are measured using PPP exchange rates. ***Quantities are measured as values deflated by prices.

Selection of countries is based on constructing a panel with data spanning 1970-2060. The countries (3-digit isocodes) are: Australia (AUS), Austria (AUT), Brazil (BRA), Canada (CAN), China (CHN), Denmark (DNK), Finland (FIN), France (FRA), Germany (DEU), Greece (GRC), India (IND), Indonesia (IDN), Ireland (IRL), Italy (ITA), Japan (JPN), South Korea (KOR), Mexico (MEX), Netherlands (NLD), Norway (NOR), Poland (POL), Portugal (PRT), Spain (ESP), Sweden (SWE), Switzerland (CHE), Turkey (TUR), United Kingdom (GBR), United States (USA), and Rest-of-world (ROW). Below I provide descriptions of how the data from 1970-2014 are constructed, and separately for 2015-2060.

Constructing realized data from 1970-2014

- Age distribution data from 1970-2014 come from the UN. For the ROW aggregate I take the age distribution data for the "world" aggregate that the UN reports, and subtract the sum of the data for the countries in my sample.
- Population data from 1970-2014 come directly from PWT. For the ROW aggregate, the population is computed as the sum of the entire population across all countries in PWT, minus the sum of the population across countries in my sample.
- Employment data from 1970-2014 come directly from PWT. For the ROW aggregate, employment is computed as the sum of across all countries in PWT, minus the sum across countries in my sample.
- Value added in current U.S. dollars is taken from various sources. From 2000-2014, these data are obtained from WIOD and are computed as the sum of all value added across every industry in each country-year. From 1970-2000 these data are obtained from PWT and computed as output-side real GDP at current PPP times the price level of GDP at current PPP exchange rate (relative to the U.S.). The data from PWT are multiplicatively spliced to WIOD as of the year 2000.
- Price of consumption from 1970-2014 is computed directly from PWT. For ROW, it is computed as the ratio of consumption in current prices relative to consumption in PPP prices. Consumption in ROW is computed as the sum across all countries in PWT minus the sum across countries in my sample.
- Price of investment is computed analogously to the price of consumption.
- Price of the composite intermediate good from 1970-2014 is constructed using various data in PWT. I take a weighted average of the price level of imports and of exports, each of which come directly from PWT. The weight applied to imports is the country's import share in total absorption, and the weight applied to exports is the country's home trade share in total absorption. Trade and absorption data are described below. For ROW, I compute the price of imports as the ratio of ROW imports in current

prices divided by ROW imports in PPP prices. ROW imports is computed as the sum of imports across all countries in PWT minus the sum across countries in my sample.

- The risk free nominal rate of interest is defined as the annual yield on the 10-year U.S. Treasury note, taken from Federal Reserve Economic Data (FRED).

- Consumption quantities are calculated in two steps. The first step computes total consumption expenditures as total value added minus investment expenditures, minus net exports. The second step deflates the consumption expenditure by the price of consumption. This way, consumption includes that of both households and government, and necessarily ensures that the national income identity holds in both the data and in the model.

- Investment quantities from 1970-2014 are computed from various data in the PWT. I begin by computing the nominal investment rate (ratio of expenditures on investment as a share of GDP in current prices). I then multiply the nominal investment rate by GDP in current U.S. dollars to arrive at total investment spending in current U.S. dollars. Finally, I deflate the current investment expenditures by the price of investment.

- Initial capital stock is taken directly from PWT. Capital stock in ROW is computed as the sum across all countries in PWT minus the sum across countries in my sample.

- Gross output in current U.S. dollars from 2000-2014 is obtained from WIOD and is computed as the sum of all gross output across every industry in each country-year. Prior to 2000, I impute these data using the ratio of value added to gross output in 2000, and applying that ratio to scale value added in each year prior to 2000.

- Bilateral trade in current U.S dollars from 2000-2014 is computed directly from WIOD as the sum of all trade flows (intermediate usage and final usage) across all industries. Prior to 2000, bilateral trade flows are obtained from the IMFDOTS. Bilateral trade flows with ROW are computed as imports (exports) to (from) the world minus the sum of imports (exports) to (from) the countries in my sample. These data are multiplicatively spliced to the WIOD data as of the year 2000.

- Absorption in current U.S. dollars from 2000-2014 is computed using WIOD data as gross output minus net exports, summed across all industries.

Constructing projected data from 2015-2060

- Age distribution data from 2015-2060 come from the UN. For the ROW aggregate I take the age distribution data for the "world" aggregate that the UN reports, and subtract the sum of the data for the countries in my sample.

- Population data from 2015-2060 are taken directly from UN and spliced to the PWT levels as of the year 2014.

- Employment data from 2015-2060 are taken directly from OECD and spliced to the PWT levels as of the year 2014.

- From 2015-2060, data on value added in current U.S. dollars are obtained from OECD projections and computed as real GDP per capita (in constant, local currency units) times the population, times the price level (in local currency units), times the PPP level (relative to the U.S.), times the nominal exchange rate (local currency per U.S. dollar at current market prices). The data from OECD are multiplicatively spliced to WIOD as of the year 2014.

- Price of consumption from 2015-2060 is imputed by assuming equal growth rates to the price deflator for aggregate GDP, where the price deflator for aggregate GDP is directly computed using data in the OECD projections.

- Price of investment from 2015-2060 is imputed using information on its co-movement with the price of consumption. In particular, I estimate the relationship between growth in the relative price against a constant and a one-year lag in the relative price growth, for the years 1972-2014.

$$\ln\left(\frac{P^c_{it}/P^x_{it}}{P^c_{it-1}/P^x_{it-1}}\right) = \beta_0 + \beta_1 \ln\left(\frac{P^c_{it-1}/P^x_{it-1}}{P^c_{it-2}/P^x_{it-2}}\right) + \varepsilon_{it}. \tag{A.1}$$

  I use the estimates from equation (A.1) to impute the sequence of prices for investment from 2015-2060, given the already imputed data for the price of consumption during these years.

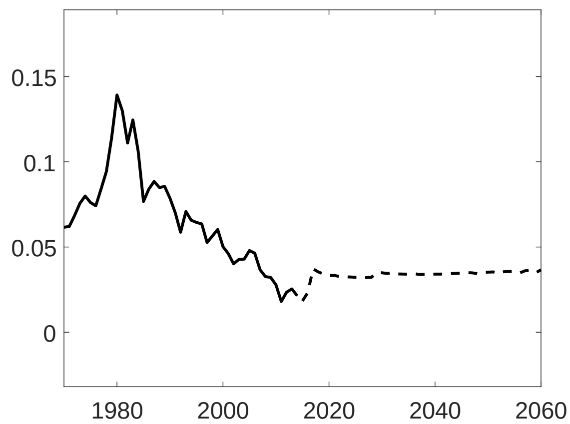
- Price of the composite intermediate good from 2015-2060 is imputed by first constructing data for the price of imports and exports. Prices of imports and exports are each computed analogously to the price of investment by estimating equation (A.1) for each series. I then take a weighted average of the price levels of imports and of exports to determine the price of the composite good. The weight applied to imports is the country's import share in total absorption, and the weight applied to exports is the country's home trade share in total absorption. Trade and absorption data are described below.

- The risk free nominal rate of interest is recovered using an intertemporal Euler equation together with projections for consumption, population, and the price level of consumption for the United States only. Specifically, I normalize the preference wedge in the United States to $\omega^A_{Ut} = 1$, for all years $t \geq 2015$ and recover the world interest rate as

$$\frac{C_{Ut+1}/N_{Ut+1}}{C_{Ut}/N_{Ut}} = \beta\omega^A_{Ut}\left(\frac{1+q_{t+1}}{P^c_{Ut+1}/P^c_{Ut}}\right), \tag{A.2}$$

for all years $t \geq 2014$. This implies an average world interest rate of 3.4 percent from 2015-2060.

Figure A.1: World interest rate



Notes: Solid lines – observed annual yield on 10-year U.S. Treasury. Dashed lines – Imputed using equation (A.2).

- Consumption quantities from 2015-2060 are calculated in the same exact way as done for 1970-2014.

- Investment quantities from 2015-2060 are computed from various variables in the OECD projections. I begin by imputing the nominal investment rate (ratio of expenditures on investment to GDP in current prices) using information on its co-movement with the relative prices. I estimate the relationship between the investment rate against a country-fixed effect, the lagged investment rate, the contemporaneous and lagged relative price of investment, and the contemporaneous and lagged real GDP per capita for the years 1971-2014. Letting $\rho_{it} = \frac{P_{xit}X_{it}}{GDP_{it}}$ denote the investment rate,

$$\ln\left(\frac{\rho_{it}}{1-\rho_{it}}\right) = \alpha_i + \beta_1 \ln\left(\frac{\rho_{it-1}}{1-\rho_{it-1}}\right) + \beta_2 \ln\left(\frac{P^x_{it}}{P^c_{it}}\right) + \beta_3 \ln\left(\frac{P^x_{it-1}}{P^c_{it-1}}\right) + \beta_4 \ln(y_{it}) + \beta_5 \ln(y_{it-1}) + \varepsilon_{it}. \quad \text{(A.3)}$$

Using $\ln(\rho/(1-\rho))$ to ensures that the imputed values of ρ are bounded between 0 and 1. I use the estimated coefficients from equation (A.3) together with projections on the relative price and income per capita to construct projections for the investment rate. I then multiply the nominal investment rate by GDP in current U.S. dollars (available in the OECD projections) to arrive at total investment spending in current U.S. dollars. Finally, I deflate the investment expenditures by the price of investment.

- Gross output in current U.S. dollars from 2015-2060 is imputed using the ratio of value added to gross output in 2014, and applying that ratio to scale value added in each year after 2014. The value added data (GDP in current U.S. dollars) after 2014 is obtained directly from OECD projections.

- Bilateral trade in current U.S dollars from 2015-2060 are constructed in multiple steps. First, let $x_{ijt} = \frac{X_{ijt}}{GO_{jt}-EXP_{jt}}$ be the ratio of country j's exports to country i, relative to country j's gross output net of its total exports. I then estimate how changes in this trade share co-moves with changes in the importer's aggregate import price index, changes in the exporter's aggregate export price index, and changes in both the importer's and exporter's levels of GDP:

$$\ln\left(\frac{x_{ijt}}{x_{ijt-1}}\right) = \beta_1 \ln\left(\frac{P^m_{it}}{P^m_{it-1}}\right) + \beta_2 \ln\left(\frac{P^x_{jt}}{P^x_{jt-1}}\right) + \beta_3 \ln\left(\frac{GDP_{it}}{GDP_{it-1}}\right) + \beta_4 \ln\left(\frac{GDP_{jt}}{GDP_{jt-1}}\right) + \varepsilon_{it}. \quad \text{(A.4)}$$

I use the estimated coefficients from equation (A.4) together with projections of prices of imports and of exports and levels of GDP to construct trade shares from 2015-2060.

In the second step, I use the fact that country i's domestic sales is determined by

$$X_{iit} = \frac{GO_{it}}{1 + \sum_{j \neq i} x_{jit}},$$

where gross output and trade shares from 2015-2060 have already been constructed.

Finally, given projected domestic sales and trade shares, the bilateral trade flow is

$$X_{ijt} = X_{jjt} x_{ijt}.$$

- Absorption in current U.S. dollars from 2015-2060 is gross output minus net exports.

B Equilibrium conditions

This section describes the solution to a perfect foresight equilibrium and allows for a more general CES utility function, $\frac{(C/N)^{1-1/\sigma}}{1-1/\sigma}$, with intertemporal elasticity of substitution, $\sigma \neq 1$.

Household optimization The representative household's optimal path for consumption satisfies two Euler equations:

$$\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}} = \beta^{\sigma} \left(\frac{\psi_{it+1}}{\psi_{it}} \right)^{\sigma} \left(1 - \tau_{it}^{\mathcal{A}}\right)^{\sigma} \left(\frac{1 + q_{t+1}}{P_{it+1}^{c}/P_{it}^{c}} \right)^{\sigma}, \tag{B.1}$$

$$\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}} = \beta^{\sigma} \left(\frac{\psi_{it+1}}{\psi_{it}} \right)^{\sigma} \left(\frac{\frac{r_{it+1}}{P_{it+1}^{c}} - \left(\frac{P_{it+1}^{c}}{P_{it+1}^{c}} \right) \Phi_2(K_{it+2}, K_{it+1})}{\left(\frac{P_{it}^{x}}{P_{it}^{c}} \right) \Phi_1(K_{it+1}, K_{it})} \right)^{\sigma} \left(\frac{1 - \tau_{it+1}^{K}}{1 - \tau_{it}^{K}} \right)^{\sigma}. \tag{B.2}$$

The first Euler equation describes the trade-off between consumption and saving in one-period bonds, while the second Euler equation describes the trade-off between consumption and investment in physical capital. Investment is written as $X_{it} \equiv \Phi(K_{it+1}, K_{it})$, where Φ_1 and Φ_2 are the derivatives with respect to the first and second arguments, respectively.

Labor supply in each period is chosen to satisfy

$$1 - \frac{L_{it}}{N_{it}} = (\zeta_{it})^{\phi} \left(1 - \tau_{it}^{L}\right)^{-\phi} \left(\frac{w_{it}}{P_{it}^{c}} \right)^{-\phi} \left(\frac{C_{it}}{N_{it}} \right)^{\frac{\phi}{\sigma}}. \tag{B.3}$$

Given K_{i1} and $\mathcal{A}_{i1}$, the paths of consumption, net-purchases of bonds, investment in physical capital, and labor supply must satisfy the budget constraint and the accumulation technologies for capital and net-foreign assets:

$$P_{it}^c C_{it} + \mathcal{A}_{it+1} = \left(r_{it}K_{it} - P_{it}^x X_{it}\right)\left(1-\tau_{it}^K\right) + w_{it}L_{it}\left(1-\tau_{it}^L\right) + (1+q_t)\mathcal{A}_{it}\left(1-\tau_{it}^{\mathcal{A}}\right) + T_{it}. \quad \text{(B.4)}$$

Firm optimization Markets are perfectly competitive so firms set prices equal to marginal costs. Omitting time subscripts for now, denote the price of variety v, produced in country j and purchased by country i, as $p_{ij}(v)$. Then $p_{ij}(v) = p_{jj}(v)d_{ij}$, where $p_{jj}(v)$ is the marginal cost of producing variety v in country j. Since country i purchases each variety from the country that can deliver it at the lowest price, the price in country i is $p_i(v) = \min_{j=1,\ldots,I}[p_{jj}(v)d_{ij}]$. The price of the composite intermediate good in country i at time t is then

$$P_{it} = \gamma \left[\sum_{j=1}^{I} \left(\left(A_{jt}\right)^{-\nu_{jt}} u_{jt} d_{ijt}\right)^{-\theta}\right]^{-\frac{1}{\theta}}, \quad \text{(B.5)}$$

where $u_{jt} = \left(\frac{r_{jt}}{\alpha\nu_{jt}}\right)^{\alpha\nu_{jt}} \left(\frac{w_{jt}}{(1-\alpha)\nu_{jt}}\right)^{(1-\alpha)\nu_{jt}} \left(\frac{P_{jt}}{1-\nu_{jt}}\right)^{1-\nu_{jt}}$ is the unit cost for a bundle of inputs for intermediate-goods producers in country j at time t.

One unit of the composite good can be converted into χ_{it}^c consumption or into χ^x investment. Hence, price of consumption and investment are given by

$$P_{it}^c = \frac{P_{it}}{\chi_{it}^c}, \qquad P_{it}^x = \frac{P_{it}}{\chi_{it}^x}.$$

Next I define total factor usage (K, L, M) and output (Y) by summing over varieties.

$$K_{it} = \int_0^1 K_{it}(v)dv, \qquad L_{it} = \int_0^1 L_{it}(v)dv,$$
$$M_{it} = \int_0^1 M_{it}(v)dv, \qquad Y_{it} = \int_0^1 Y_{it}(v)dv.$$

The term $L_{it}(v)$ denotes the quantity of labor employed in the production of variety v at time t. If country i imports variety v at time t, then $L_{it}(v) = 0$. Hence, L_{it} is the total quantity of labor employed in country i at time t. Similarly, K_{it} is the total quantity of capital used, M_{it} is the total quantity of the composite good used as an intermediate input in production, and Y_{it} is the total quantity of output produced.

Cost minimization by firms implies that factor expenses exhaust the value of production:

$$r_{it}K_{it} = \alpha\nu_{it}P_{it}Y_{it}, \qquad w_{it}L_{it} = (1-\alpha)\nu_{it}P_{it}Y_{it}, \qquad P_{it}M_{it} = (1-\nu_{it})P_{it}Y_{it}.$$

That is, $\alpha\nu_{it}$ is the fraction of the gross production that compensates capital services, $(1-\alpha)\nu_{it}$ compensates labor services, and $1-\nu_{it}$ covers the cost of intermediate inputs.

Trade flows The fraction of country i's expenditures allocated to varieties produced by country j is given by

$$\pi_{ijt} = \frac{\left((A_{jt})^{-\nu_{jt}}\, u_{jt} d_{ijt}\right)^{-\theta}}{\sum_{\ell=1}^{I}\left((A_{\ell t})^{-\nu_{\ell t}}\, u_{\ell t} d_{i\ell t}\right)^{-\theta}}. \tag{B.6}$$

Market clearing conditions Tax revenue is returned in lump sum to the household:

$$\tau_{it}^{K}\left(r_{it}K_{it} - P_{it}^{x}X_{it}\right) + \tau_{it}^{L} w_{it} L_{it} + \tau_{it}^{\mathcal{A}}(1+q_{it})\mathcal{A}_{it} = T_{it}.$$

The supply of the composite good—an aggregate of all imported and domestic varieties—must equal the demand., which consists of consumption, investment, and intermediate input.

$$P_{it}^{c}C_{it} + P_{it}^{x}X_{it} + P_{it}M_{it} = P_{it}Q_{it}.$$

The balance of payments must hold in each country: the current account equals net exports plus net-foreign income. With net exports equal to gross output less gross absorption, this condition implies

$$\mathcal{B}_{it} = P_{it}Y_{it} - P_{it}Q_{it} + q_t\mathcal{A}_{it}.$$

To close the model, the current account equals the change in the NFA position:

$$\mathcal{A}_{it+1} = \mathcal{A}_{it} + \mathcal{B}_{it}$$

Remark The world interest rate is strictly nominal. As such, the value plays essentially no role other than pinning down a numéraire. Since my choice of numéraire is world GDP in each period, the world interest rate reflects the relative valuation of world GDP at two points in time. This interpretation is useful in guiding the solution procedure and also makes for straightforward mapping between model and data. That is, in the model the prices map into current units, as opposed to constant units. In other words, the model can be rewritten so that all prices are quoted in time-1 units (like an Arrow-Debreu world) with the world interest rate of zero and the equilibrium would yield identical quantities.

C Solution algorithm

In this section of the Appendix I describe the algorithm for computing the equilibrium transition path. Before going further into the algorithm, I introduce some notation. I denote the cross-country vector of a given variable at a point in time using vector notation, i.e., $\vec{K}_t = \{K_{it}\}_{i=1}^{I}$ is the vector of capital stocks across countries at time t.

C.1 Computing the equilibrium transition path

Given the initial conditions—$(\vec{K}_1, \vec{\mathcal{A}}_1)$—the equilibrium transition path consists of 17 objects: $\{\vec{w}_t, \vec{r}_t, q_t, \vec{P}_t, \vec{P}_t^c, \vec{P}_t^x, \vec{Y}_t, \vec{Q}_t, \vec{C}_t, \vec{X}_t, \vec{K}_{t+1}, \vec{L}_t, \vec{M}_t, \vec{\mathcal{B}}_t, \vec{\mathcal{A}}_{t+1}, \vec{T}_t, \vec{\vec{\pi}}_t\}_{t=1}^{T}$ (the double-arrow notation on $\vec{\vec{\pi}}_t$ indicates that this is an $I \times I$ matrix in each period t). Table C.1 provides a list of 17 equilibrium conditions that these objects must satisfy.

The solution procedure is boils down to two loops, similar to the algorithm in Ravikumar, Santacreu, and Sposi (2019). The outer loop consists of iterating on the labor supply decision and the rate of investment in physical capital. The inner loop consists of iterating on a subset of prices as in Sposi (2012), given the guess for labor supply and investment rates.

First, for the outer loop, guess at the sequence of labor supply and investment rate for every country in every period. Given the labor supply and investment rate, within the inner loop start with an initial guess for the entire sequences of wage vectors and the world interest rate on bonds. Form these two objects, recover all remaining prices and quantities, across countries and throughout time, using optimality conditions and market clearing conditions, excluding the balance of payments condition. Then use departures from the the balance of payments condition to update the wages, and use deviations from intertemporal price relationships to update the world interest rate. Iterate on this until the wages and world interest rate satisfy the balance of payments condition and the intertemporal condition for prices. Then back up to the outer loop and check if the labor supply and investment rate decisions salsify optimality. If not, update the guess at labor supply and investment rate and solve the inner loop again. The details to this procedure follow, making reference to equilibrium conditions in Table C.1.

1. Guess at a sequence of labor supply choices in every country, $\{\vec{h}_t\}_{t=1}^{T}$ with

$$0 < h_{it} \equiv L_{it}/N_{it} < 1,$$

Table C.1: Equilibrium conditions

1	$r_{it}K_{it} = \alpha\nu_{it}P_{it}Y_{it}$	$\forall(i,t)$
2	$w_{it}L_{it} = (1-\alpha)\nu_{it}P_{it}Y_{it}$	$\forall(i,t)$
3	$P_{it}M_{it} = (1-\nu_{it})P_{it}Y_{it}$	$\forall(i,t)$
4	$P_{it}Q_{it} = P^c_{it}C_{it} + P^x_{it}X_{it} + P_{it}M_{it}$	$\forall(i,t)$
5	$P_{it}Y_{it} = \sum_{j=1}^{I} P_{jt}Q_{jt}\pi_{jit}$	$\forall(i,t)$
6	$P_{it} = \gamma\left(\sum_{j=1}^{I}\left((A_{jt})^{-\nu_{jt}}u_{jt}d_{ijt}\right)^{-\theta}\right)^{-\frac{1}{\theta}}$	$\forall(i,t)$
7	$P^c_{it} = \frac{P_{it}}{\chi^c_{it}}$	$\forall(i,t)$
8	$P^x_{it} = \frac{P_{it}}{\chi^x_{it}}$	$\forall(i,t)$
9	$\pi_{ijt} = \frac{\left((A_{jt})^{-\nu_{jt}}u_{jt}d_{ijt}\right)^{-\theta}}{\sum_{\ell=1}^{I}\left((A_{\ell t})^{-\nu_{\ell t}}u_{\ell t}d_{i\ell t}\right)^{-\theta}}$	$\forall(i,j,t)$
10	$P^c_{it}C_{it} + \mathcal{A}_{it+1} = (r_{it}K_{it} - P^x_{it}X_{it})\left(1-\tau^K_{it}\right) + w_{it}L_{it}\left(1-\tau^L_{it}\right)$ $+(1+q_t)\mathcal{A}_{it}\left(1-\tau^{\mathcal{A}}_{it}\right) + T_{it}$	$\forall(i,t)$
11	$\mathcal{A}_{it+1} = \mathcal{A}_{it} + \mathcal{B}_{it}$	$\forall(i,t)$
12	$K_{it+1} = (1-\delta)K_{it} + \delta^{1-\lambda}\left(X_{it}\right)^{\lambda}K^{1-\lambda}_{it}$	$\forall(i,t)$
13	$\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}} = \beta^{\sigma}\left(\frac{\psi_{it+1}}{\psi_{it}}\right)^{\sigma}\left(1-\tau^{\mathcal{A}}_{it+1}\right)^{\sigma}\left(\frac{1+q_{t+1}}{P^c_{it+1}/P^c_{it}}\right)^{\sigma}$	$\forall(i,t)$
14	$\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}} = \beta^{\sigma}\left(\frac{\psi_{it+1}}{\psi_{it}}\right)^{\sigma}\left(\frac{\frac{r_{it+1}}{P^c_{it+1}} - \left(\frac{P^x_{it+1}}{P^c_{it+1}}\right)\Phi_2(K_{it+2},K_{it+1})}{\left(\frac{P^x_{it}}{P^c_{it}}\right)\Phi_1(K_{it+1},K_{it})}\right)^{\sigma}\left(\frac{1-\tau^K_{it+1}}{1-\tau^K_{it}}\right)^{\sigma}$	$\forall(i,t)$
15	$\frac{L_{it}}{N_{it}} = 1 - (\zeta_{it})^{\phi}\left(1-\tau^L_{it}\right)^{-\phi}\left(\frac{w_{it}}{P_{cit}}\right)^{-\phi}\left(\frac{C_{it}}{N_{cit}}\right)^{\phi/\sigma}$	$\forall(i,t)$
16	$\mathcal{B}_{it} = P_{it}Y_{it} - P_{it}Q_{it} + q_t\mathcal{A}_{it}$	$\forall(i,t)$
17	$T_{it} = (r_{it}K_{it} - P^x_{it}X_{it})\,\tau^K_{it} + w_{it}L_{it}\tau^L_{it} + (1+q_t)\mathcal{A}_{it}\tau^{\mathcal{A}}_{it}$	$\forall(i,t)$

Notes: $u_{jt} = \left(\frac{r_{jt}}{\alpha\nu_{jt}}\right)^{\alpha\nu_{jt}}\left(\frac{w_{jt}}{(1-\alpha)\nu_{jt}}\right)^{(1-\alpha)\nu_{jt}}\left(\frac{P_{jt}}{1-\nu_{jt}}\right)^{1-\nu_{jt}}$ and $\gamma = \Gamma(1+(1-\eta)/\theta)^{1/(1-\eta)}$, where $\Gamma(\cdot)$ is the Gamma function. $\Phi_1(K', K)$ and $\Phi_2(K', K)$ denote the derivatives of the function $\Phi(K', K) = \delta^{1-1/\lambda}\left(\frac{K'}{K} - (1-\delta)\right)^{1/\lambda}K$ w.r.t. the first and second arguments, respectively.

guess a sequence of nominal investment rates, $\{\vec{\rho}_t\}_{t=1}^{T}$ with

$$0 < \rho_{it} \equiv \frac{P^x_{it}X_{it}}{r_{it}K_{it} + w_{it}L_{it}} < 1,$$

and guess at a terminal NFA position in each country, $\mathcal{A}_{iT+1}$, with $\sum_{i=1}^{I}\mathcal{A}_{iT+1} = 0$. Take these as given for the next sequence of steps.

(a) Guess the entire paths for wages, $\{\vec{w}_t\}_{t=1}^{T}$, across countries, and the world interest rate, $\{q_t\}_{t=2}^{T}$, such that $\sum_i \frac{w_{it}L_{it}}{1-\alpha} = 1$, for all t (world GDP is the numéraire in each period.

(b) In period 1 use conditions 1-2 and set $\vec{r}_1 = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\vec{w}_1\vec{H}_1}{\vec{K}_1}\right)$, where the initial stock of capital is predetermined. Compute prices $\vec{P}_1, \vec{P}_1^c, \vec{P}_1^x$ using conditions 6-8.

(c) Solve for physical investment, $\vec{X}_1$, using the guess for the nominal investment rate together with prices: $\vec{X}_1 = \frac{\vec{\rho}_1\vec{w}_1\vec{L}_1}{(1-\alpha)\vec{P}_1^x}$. Given the investment, solve for the next-period capital stock, $\vec{K}_2$, using condition 10. Repeat this set of calculations for period 2, then for period 3, and all the way through period T to construct the entire sequence of investment and capital.

(d) Given prices, compute the bilateral trade shares $\{\vec{\vec{\pi}}_t\}_{t=1}^T$ using condition 9.

(e) This step is slightly more involved. I show how to compute the path for consumption and bond purchases by solving the intertemporal problem of the household. This is done in three parts. First I derive the lifetime budget constraint, second I derive the fraction of lifetime wealth allocated to consumption at each period t, and third I recover the sequences for bond purchases and the stock of NFAs.

Deriving the lifetime budget constraint To begin, compute the lifetime budget constraint for the representative household (omitting country subscripts for now). Begin with the period budget constraint from condition 10 and combine it with the NFA accumulation technology in condition 11 and the balanced tax revenue in condition 17:

$$\mathcal{A}_{t+1} = r_t K_t - P_t^x X_t + w_t L_t - P_t^c C_t + (1+q_t)\mathcal{A}_t.$$

Iterate the period budget constraint forward through time and derive a lifetime budget constraint. Given $A_{i1} > 0$, compute the NFA position at time $t = 2$:

$$\mathcal{A}_2 = r_1 K_1 - P_1^x X_1 + w_1 L_1 - P_1^c C_1 + (1+q_t)\mathcal{A}_1.$$

Similarly, compute the NFA position at time $t = 3$:

$$\begin{aligned} \mathcal{A}_3 &= r_2 K_2 - P_2^x X_2 + w_2 L_2 - P_2^c C_2 + (1+q_2)\mathcal{A}_2 \\ \Rightarrow \mathcal{A}_3 &= r_2 K_2 - P_2^x X_2 + w_2 L_2 + (1+q_2)\left(r_1 K_1 - P_1^x X_1 + w_1 L_1\right) \\ &\quad - P_2^c C_2 - (1+q_2)P_1^c C_1 + (1+q_2)(1+q_1)\mathcal{A}_{i1}. \end{aligned} \tag{C.1}$$

It is useful to define $(1+Q_t)=\prod_{n=1}^{t}(1+q_n)$. By induction, for any time t,

$$\begin{aligned}\mathcal{A}_{t+1} &= \sum_{n=1}^{t}\frac{(1+\mathcal{Q}_t)\left(r_nK_n-P_n^xX_n+w_nL_n\right)}{1+\mathcal{Q}_n} \\ &\quad -\sum_{n=1}^{t}\frac{(1+\mathcal{Q}_t)P_n^cC_n}{1+\mathcal{Q}_n}+(1+\mathcal{Q}_t)\mathcal{A}_1 \\ \Rightarrow \mathcal{A}_{t+1} &= (1+\mathcal{Q}_t)\left(\sum_{n=1}^{t}\frac{r_nK_n-P_n^xX_n+w_nL_n}{1+\mathcal{Q}_n}-\sum_{n=1}^{t}\frac{P_n^cC_n}{1+\mathcal{Q}_n}+\mathcal{A}_1\right).\end{aligned} \tag{C.2}$$

Observing the previous expression as of $t=T$ yields the lifetime budget constraint:

$$\sum_{n=1}^{T}\frac{P_n^cC_n}{1+\mathcal{Q}_n}=\underbrace{\sum_{n=1}^{T}\frac{r_nK_n-P_n^xX_n+w_nL_n}{1+\mathcal{Q}_n}+\mathcal{A}_1-\frac{\mathcal{A}_{T+1}}{1+\mathcal{Q}_T}}_{\mathcal{W}}. \tag{C.3}$$

In the lifetime budget constraint (C.3), $\mathcal{W}$ denotes the net present value of lifetime wealth, taking both the initial and terminal NFA positions as given.

Solving for the path of consumption Next, compute how the net-present value of lifetime wealth is optimally allocated throughout time. For this it is useful to define $(1-\mathcal{T}_t^A)=\prod_{n=1}^{t}(1-\tau_n^{\mathcal{A}})$. The Euler equation for bonds (condition 13) implies the following relationship between any two periods t and n:

$$\begin{aligned}C_n &= \left(\frac{N_n}{N_t}\right)\beta^{\sigma(n-t)}\left(\frac{\psi_n}{\psi_t}\right)^{\sigma}\left(\frac{1-\mathcal{T}_n^{\mathcal{A}}}{1-\mathcal{T}_t^{\mathcal{A}}}\right)^{\sigma}\left(\frac{1+\mathcal{Q}_n}{1+\mathcal{Q}_t}\right)^{\sigma}\left(\frac{P_t^c}{P_n^c}\right)^{\sigma}C_t \\ \Rightarrow \frac{P_n^cC_n}{1+\mathcal{Q}_n} &= \left(\frac{N_n}{N_t}\right)\beta^{\sigma(n-t)}\left(\frac{\psi_n}{\psi_t}\right)^{\sigma}\left(\frac{1-\mathcal{T}_n^{\mathcal{A}}}{1-\mathcal{T}_t^{\mathcal{A}}}\right)^{\sigma}\left(\frac{1+\mathcal{Q}_n}{1+\mathcal{Q}_t}\right)^{\sigma-1}\left(\frac{P_t^c}{P_n^c}\right)^{\sigma-1}\left(\frac{P_t^cC_t}{1+\mathcal{Q}_t}\right).\end{aligned}$$

Since equation (C.3) implies that $\sum_{n=1}^{T}\frac{P_{in}^cC_{in}}{1+\mathcal{Q}_n}=\mathcal{W}_i$, rearrange the previous expression (putting country subscripts back in) to obtain

$$\frac{P_{it}^cC_{it}}{1+\mathcal{Q}_{it}}=\underbrace{\left(\frac{N_{it}\beta^{\sigma t}(\psi_{it})^{\sigma}\left(1-\mathcal{T}_{it}^{\mathcal{A}}\right)^{\sigma}(1+\mathcal{Q}_{it})^{\sigma-1}\left(P_{it}^c\right)^{1-\sigma}}{\sum_{n=1}^{T}N_{in}\beta^{\sigma n}(\psi_{in})^{\sigma}\left(1-\mathcal{T}_{in}^{\mathcal{A}}\right)^{\sigma}(1+\mathcal{Q}_{in})^{\sigma-1}\left(P_{in}^c\right)^{1-\sigma}}\right)}_{\xi_{it}}\mathcal{W}_i. \tag{C.4}$$

That is, in period t, country i's net-present value of consumption spending is a fraction, ξ_{it}, of its lifetime wealth, with $\sum_{t=1}^{T}\xi_{it}=1$ for all i.

Computing bond purchases and the NFA positions In period 1 take as given consumption spending, investment spending, capital income, labor income, and net income from the initial NFA position; each of which have already been computed in previous steps. Then solve for next period's NFA position, $\vec{\mathcal{A}}_2$, using the period budget constraint in condition 10, then compute net bond purchases $\vec{B}_2$ using the period budget constraint in condition 11. Repeat this set of calculations iteratively for periods $2, \ldots, T$.

(f) Given final demand $\{\vec{C}_t, \vec{X}_t\}_{t=1}^{T}$ and prices, solve for gross absorption and gross production, $\{\vec{Q}_t, \vec{Y}_t\}_{t=1}^{T}$, and intermediate-input demand, $\{\vec{M}_t\}_{t=1}^{T}$ using conditions 3–5.

(g) Given gross national income, calculate the tax rebates, $\vec{T}_{it}$, using condition 17.

(h) Update the guesses for wages, the interest rate, and tax rebates.

Balance of payments condition I generalize Alvarez and Lucas (2007) and compute an excess demand equation by imposing that net exports equal the current account less net foreign income from assets.

$$Z_{it}^{w}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right) = \frac{P_{it}Y_{it} - P_{it}Q_{it} + q_t\mathcal{A}_{it} - \mathcal{B}_{it}}{w_{it}}.$$

Condition 16 requires that $Z_{it}^{w}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right) = 0$ for all (i,t). If this is different from zero in at some country at some point in time, update the wages:

$$1 + w_{it}^{new} = \Lambda_{it}^{w}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right) = \left(1 + \kappa \frac{Z_{it}^{w}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right)}{L_{it}}\right) w_{it}$$

where κ is chosen to be sufficiently small to ensure that $\Lambda^{w} > 0$.

Normalizing model units The last part of this step updates the world interest rate. Recall that the numéraire is defined to be world GDP at each point in time: $\sum_{i=1}^{I}(r_{it}K_{it} + w_{it}L_{it}) = 1$. For an arbitrary sequence of $\{q_{t+1}\}_{t=1}^{T}$, this condition need not hold. Update the the world interest rate:

$$q_t^{new} = \Lambda_{it}^{q}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right) = \frac{\sum_{i=1}^{I}(r_{it-1}K_{it-1} + \Lambda_{it-1}^{w}L_{it-1})}{\sum_{i=1}^{I}(r_{it}K_{it} + \Lambda_{it}^{w}L_{it})}(1 + q_t) - 1 \ (t \geq 2)$$

and $q_1^{new} = q_1$. The values for capital stock and the rental rate of capital are computed in step 2, while the values for wages are the updated values Λ^w above. I set $q_1 = \frac{1-\beta}{\beta}$ (the interest rate that prevails in a steady state) and chose A_{i1} so that $q_1 A_{i1}$ matches the desired initial NFA position in current prices.

Having updated the wages, the world interest rate, and tax rebates, repeat steps 1b-1g. Iterate through this procedure until the excess demand is sufficiently close to zero. In the computations I find that my preferred convergence metric:

$$\max_{t=1}^{T} \left\{ \max_{i=1}^{I} \left\{ \left| Z_{it}^{w} \left(\{\vec{w}_t, q_t\}_{t=1}^{T} \right) \right| \right\} \right\}$$

converges roughly monotonically towards zero.

2. The last step of the algorithm is to update the labor supply, investment rate, and terminal NFA position. Until now, the optimality condition 15 for the labor supply and condition 14 for the investment in physical capital have not been used. To this end, compute a "residual" from each of these first-order conditions as

$$Z_{it}^{h} \left(\{\vec{h}_t, \vec{\rho}_t\}_{t=1}^{T} \right) = \left(1 - \frac{L_{it}}{N_{it}} \right) - \zeta_{it}^{\phi} \left(1 - \tau_{it}^{L} \right)^{-\phi} \left(\frac{w_{it}}{P_{cit}} \right)^{-\phi} \left(\frac{C_{it}}{N_{cit}} \right)^{\frac{\phi}{\sigma}},$$

$$Z_{it}^{\rho} \left(\{\vec{h}_t, \vec{\rho}_t\}_{t=1}^{T} \right) = \beta \left(\frac{\psi_{it+1}}{\psi_{it}} \right) \left(1 - \tau_{it+1}^{\mathcal{A}} \right) \left(\frac{\frac{r_{it+1}}{P_{it+1}^{c}} - \left(\frac{P_{it+1}^{x}}{P_{it+1}^{c}} \right) \Phi_2(K_{it+2}, K_{it+1})}{\left(\frac{P_{it}^{x}}{P_{it}^{c}} \right) \Phi_1(K_{it+1}, K_{it})} \right) \left(\frac{1 - \tau_{it+1}^{K}}{1 - \tau_{it}^{K}} \right) - \left(\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}} \right)^{\frac{1}{\sigma}}.$$

Condition 15 requires that $Z_{it}^{h} \left(\{\vec{h}_t, \vec{\rho}_t\}_{t=1}^{T} \right) = 0$ for all (i, t), while condition 14 requires that $Z_{it}^{\rho} \left(\{\vec{h}_t, \vec{\rho}_t\}_{t=1}^{T} \right) = 0$. Update the labor supply and investment rate as

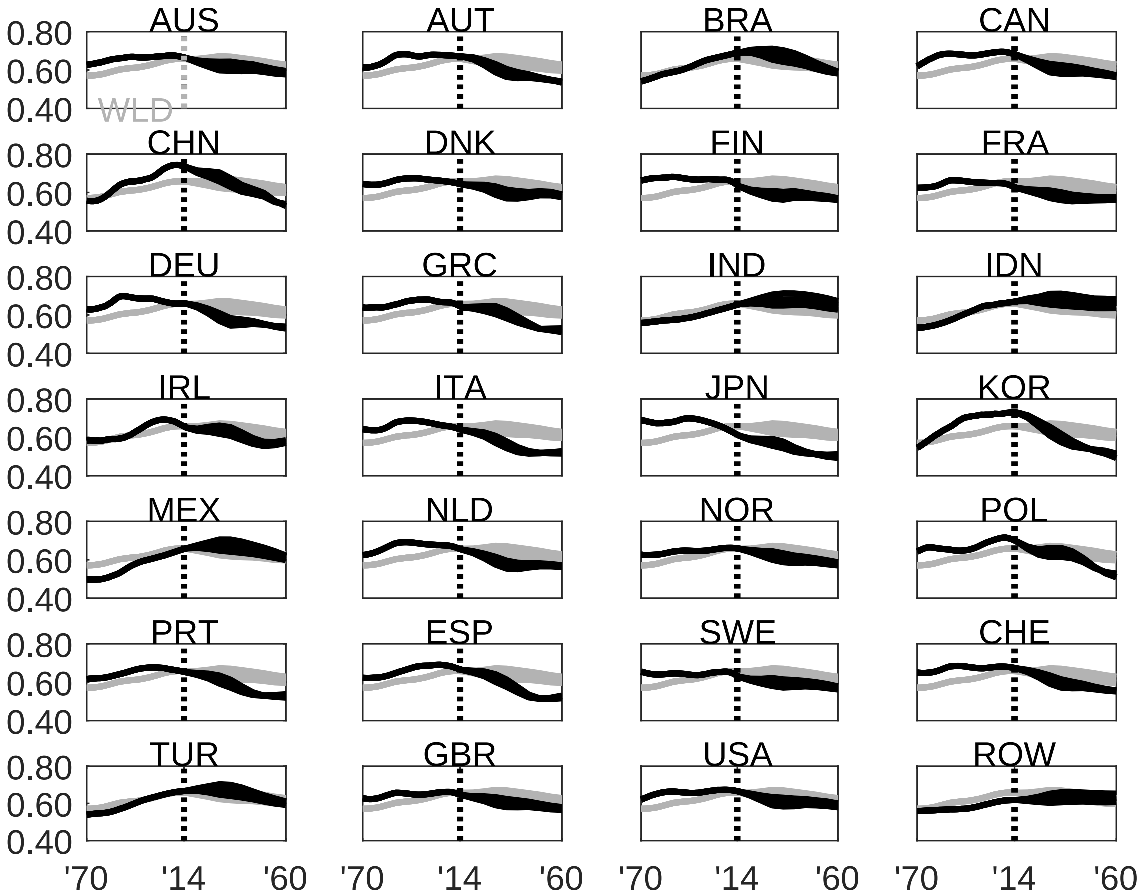
$$h_{it}^{new} = \Lambda_{it}^{h} \left(\{\vec{h}_t, \vec{\rho}_t\}_{t=1}^{T} \right) = \left(1 + \kappa Z_{it}^{h} \left(\{\vec{h}_t, \vec{\rho}_t\}_{t=1}^{T} \right) \right) h_{it},$$
$$\rho_{it}^{new} = \Lambda_{it}^{\rho} \left(\{\vec{h}_t, \vec{\rho}_t\}_{t=1}^{T} \right) = \left(1 + \kappa Z_{it}^{\rho} \left(\{\vec{h}_t, \vec{\rho}_t\}_{t=1}^{T} \right) \right) \rho_{it},$$

where κ is a constant value small enough to ensure that the updated guesses remain positive. Given the updated sequence of labor supply and investment rate, return to step 1 using the updated labor supply and investment rate as the "guess" for the next iteration. Iterate until conditions 14 and 15 hold.

With T chosen to be sufficiently large, the turnpike theorem implies that the terminal NFA position has minimal bearing on the transition path up to some time $t^\star < T$ (see Maliar, Maliar, Taylor, and Tsener, 2015).

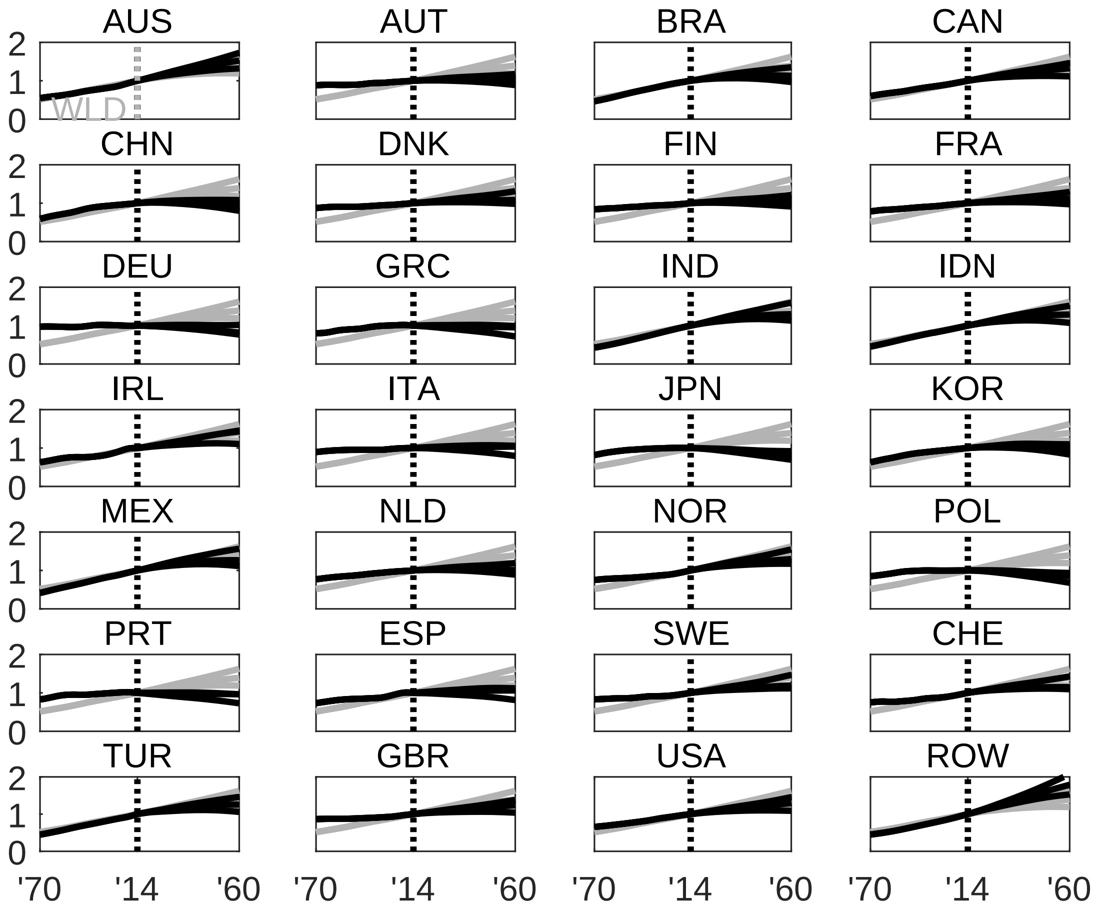
D Additional figures

Figure D.1: Working age share



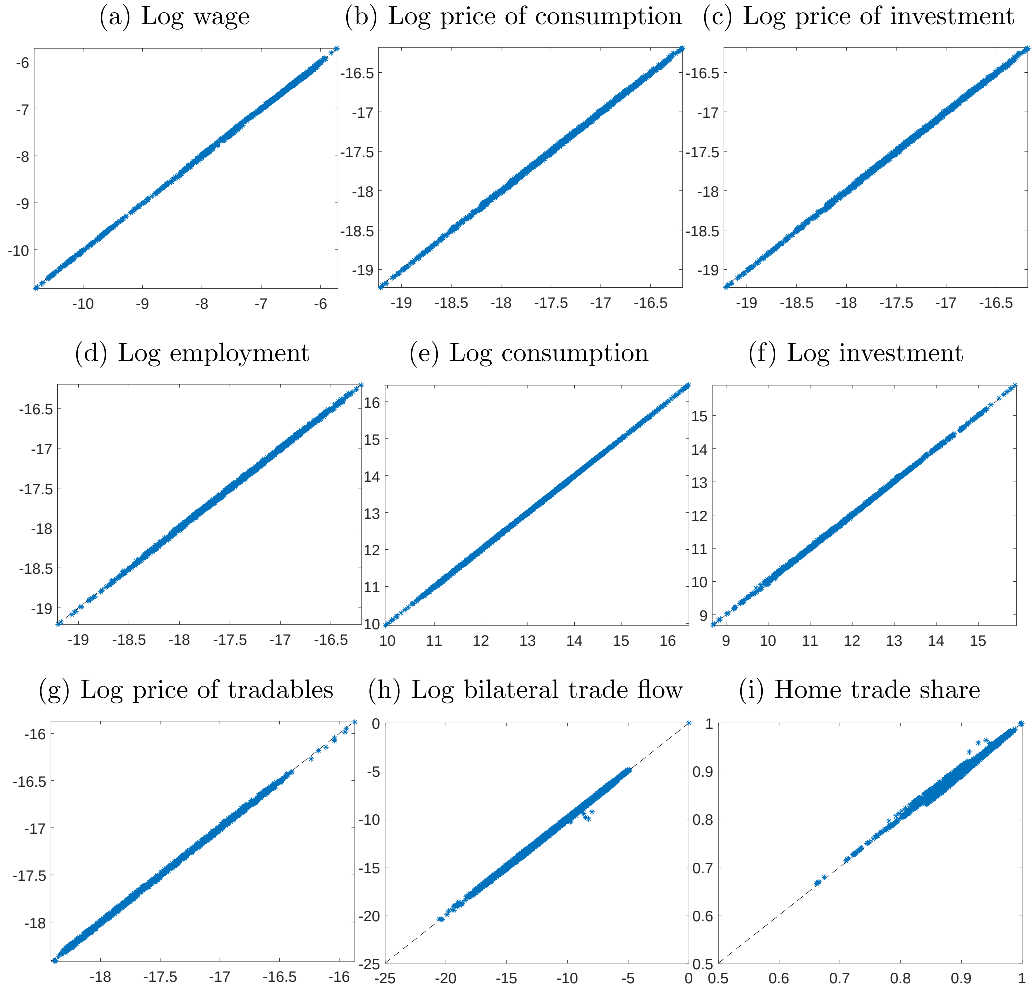
Notes: Black lines – country level. Gray lines – world aggregate. Data from 1970-2014 are observed. Data from 2015-2060 are based on projections: high-variant (upper), medium-variant (middle), and low-variant (lower). High- and low-variants assume plus and minus one-half child per woman, respectively, relative to the medium-variant.

Figure D.2: Population – indexed to 1 in 2014



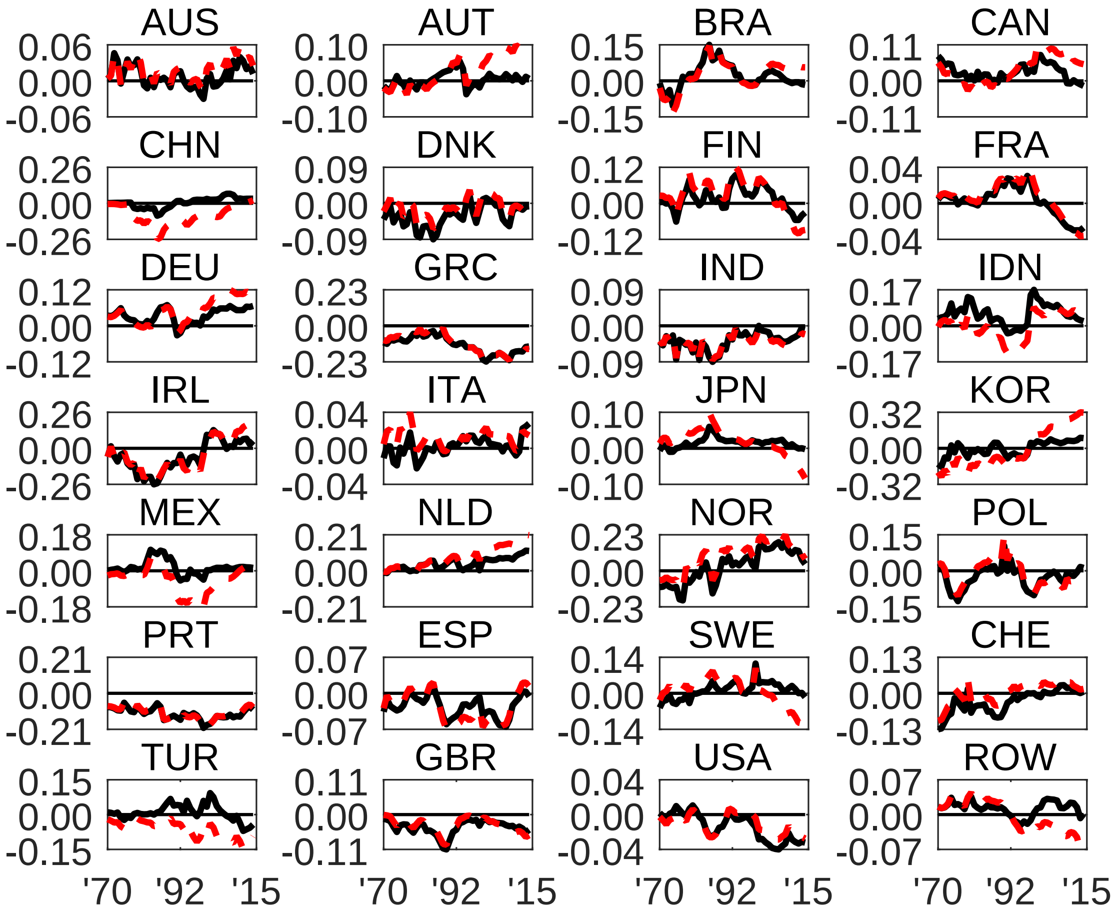
Notes: Black lines – country level. Gray lines – world aggregate. Data from 1970-2014 are observed. Data from 2015-2060 are based on projections: high-variant (upper), medium-variant (middle), and low-variant (lower). High- and low-variants assume plus and minus one-half child per woman, respectively, relative to the medium-variant.

Figure D.3: Model fit for targeted data: 1970-2014



Notes: Horizontal axis - data. Vertical axis - model. Each point corresponds to one country in one year. The dashed line represents the 45-degree line.

Figure D.4: Ratio of net exports to GDP, baseline and counterfactual



Notes: Solid black lines – baseline (data). Dashed red lines – counterfactual with age distributions simultaneously held constant over time in every country.