Demographics and the Evolution of Global Imbalances

Michael Sposi[*]

March 1, 2021

Abstract

The age distribution evolves asymmetrically across countries, influencing capital flows through differences in aggregate saving rates and labor supply. I build a general equilibrium model featuring overlapping generations and international trade where dynamics are driven by capital accumulation and borrowing and lending. The equilibrium can be replicated by a model in which every country is inhabited by a representative household that experiences an endogenous, time-varying discount factor reflecting the co-evolution of the entire age distribution and relevant prices. This equivalence affords computation of the exact transitional dynamics. I calibrate the model to match national accounts and bilateral trade data and quantify how demographic forces affected capital flows between 28 countries since 1970. On average, increasing a country's mean age by one year boosts its current account by 0.4 percent of GDP. Observed bilateral trade patterns dictate the cross-country dispersion and magnitude of capital flows in response to changes in any individual country's demographics.

JEL codes: F11, F21, J11

Keywords: Demographics; Global imbalances; Dynamics; International trade

1 Introduction

Trade and current account imbalances have been a focal point of high-level policy debates in recent years. While current accounts fluctuate at a fairly high frequency, they also display persistent, low-frequency temporal trends: The United States has run a deficit for almost four decades, and China a surplus for over two decades. At the same time, countries have undergone persistent, low-frequency changes in their age distribution. While most countries' populations have aged over the past half century, there have been, and still are, sizable differences in demographics across countries, inducing differences in saving demand, labor supply, and ultimately capital flows between countries.

The current account reflects the net present value of past and future net exports. In turn, two distinct strands of literature offer perspectives on the determinants of trade imbalances. One is an international macro/finance perspective emphasizing intertemporal considerations—saving minus investment—where imbalances stem from frictions on cross-border financial transactions and investment returns, as well as natural forces such as demographics. Much of this literature uses models with one good focusing on *net* trade flows and has been silent on the *gross* trade flows between countries by assuming frictionless intratemporal trade. The other is an international trade perspective emphasizing the pattern of trade between countries—exports minus imports—where comparative advantage and trade costs determine trade flows. With a few exceptions, multicountry trade models have been silent on aggregate dynamics.[1] These different perspectives yield disparate implications for whether demographic-induced changes in saving end up in investment or net exports.

This paper bridges the two perspectives and examines how demographics interact with bilateral trade and investment to determine capital flows between 28 countries since 1970.[2] The model features (i) overlapping generations (OLG), (ii) endogenous trade imbalances, (iii) bilateral trade between countries, (iv) capital accumulation, and (v) many countries in a general equilibrium environment. To my knowledge, this paper is the first to integrate all of these features. The results offer both methodological and quantitative insights.

Under certain assumptions regarding capital ownership across cohorts, I demonstrate how

---

[1]Some papers incorporate trade imbalances as exogenous transfers in the context of static models (Caliendo, Parro, Rossi-Hansberg, and Sarte, 2017; Dekle, Eaton, and Kortum, 2007). Only a few papers model trade imbalances between many countries via international borrowing and lending in a dynamic setting (Eaton, Kortum, and Neiman, 2016; Eaton, Kortum, Neiman, and Romalis, 2016; Reyes-Heroles, 2016; Ravikumar, Santacreu, and Sposi, 2019; Sposi, 2012).

[2]Papers that incorporate both perspectives in general equilibrium generally use two-country models and build on Backus, Kehoe, and Kydland (1992).

the aggregate equilibrium dynamics of the OLG model can be replicated by a model where each country is inhabited by representative household that experiences an appropriately chosen, endogenous, time-varying *preference shifter*. Changes in the preference shifter are isomorphic to an endogenous, time-varying discount factor that summarizes the dynamic interplay between the entire age distribution and relevant prices and distortions.[3] This equivalence allows me to restrict attention to the representative household setting and employ recently developed methods to compute the transitional dynamics.

The demand side of the model is disciplined using annual data on the age distribution across countries. The remainder of the model is saturated with *wedges*—exogenous forces including distortions to saving and investment, bilateral trade costs, and productivity—that are calibrated to match annual national accounts and bilateral trade data.[4]

The main counterfactual imposes a common (world average), time-varying age distribution across countries and explores the resulting equilibrium dynamics. For the average country, the absolute value of the change in capital flows as a share of GDP, relative to the baseline scenario, is 3%. U.S. capital inflows in the counterfactual are considerably smaller than in the baseline (0.2% of GDP compared to 2% in the baseline), as are China's capital outflows (0.5% of GDP compared to 2.7% in the baseline). On average, increasing a country's mean age by one year boosts its current account by 0.4 percent of GDP. The bilateral incidence of demographic-induced changes in a country's saving is distributed across all other countries in a manner that correlates with observed trade patterns.

The central model feature is an OLG structure whereby demographic forces operate through two distinct channels. The primary channel is aggregate saving propensities vis-à-vis the endogenous discount factor. Countries with younger populations tend to have a lower discount factor and, thus, lower saving demand and a lower current account. The other channel is aggregate labor supply. Each cohort supplies labor inelastically during working age life so that demographic change alters the size of the labor force. As in Rodriguez (2019), younger populations tend to have higher aggregate labor supply which temporarily raises the capital-labor ratio, lowers the rate of return to investment, and raises the current account.

The second model feature is trade in one-period bonds. This allows for borrowing and lending between countries, enabling cross-country capital flows and current account dynam-

---

[3] *Exogenous* time-varying discount factors are familiar to dynamic trade models as a means to generate current account dynamics (see Eaton, Kortum, Neiman, and Romalis, 2016; Kehoe, Ruhl, and Steinberg, 2018; Steinberg, 2019).

[4] This wedge accounting is similar to that used in Eaton, Kortum, Neiman, and Romalis (2016) and has its roots in business cycle accounting (see Chari, Kehoe, and McGrattan, 2007).

ics. All countries face a common world interest rate, while capital flows are subject to country-specific distortions that capture high-frequency forces unrelated to demographics. When the distortion is low, the effective return to saving is high, as is the current account.

The third feature is gross bilateral trade between countries. International trade is determined by productivity differences along a continuum of varieties and bilateral trade costs as in Eaton and Kortum (2002). Since borrowing and lending is ultimately offset by trade imbalances, the magnitude of trade costs affects rates of return to bonds.[5] Intuitively, the ratio of net exports to GDP equals the product of (i) the ratio of net exports to total gross trade and (ii) the ratio of total gross trade to GDP. Alessandria and Choi (2019) and Reyes-Heroles (2016) demonstrate that broad trends in the ratio of net exports to GDP are accounted for primarily by the trends in gross trade flows, i.e., the ratio trade to GDP rather than the ratio of net exports to trade. In turn, large trade costs imply that a demographic-induced increase in saving shows up as higher investment rather than higher net exports.

The bilateral structure of gravity further governs how fundamentals in one country shape real exchange rates in another; a channel that is absent in models that consider only net trade flows. While any two countries can borrow and lend independently of their bilateral trade costs, it turns out that changes in the dispersion of capital flows in response to changes in one country's demographics is highly correlated with the pattern of trade. I demonstrate this by unilaterally changing the age distribution in a few countries. For example, by unilaterally making the United States younger, U.S. borrowing increases. This entails either lower U.S. gross exports or higher U.S. gross imports, the incidence of which is born by countries for which the United States occupies a large share of trade, such as Canada. Such countries experience higher net exports and realize the greatest increase in lending. In other words, Canada's real exchange rate is sensitive to U.S. economic conditions because of its bilateral trade relationship. Since changes in the real exchange rate are closely related to interest rate differentials, Canada's saving and current account respond to U.S. economic conditions.

The fourth feature is investment and capital accumulation as in the neoclassical growth model.[6] Differential productivity growth along with capital market distortions are key determinants of the net return to investment. Differences in rates of return to investment across

---

[5]This intuition is articulated by Obstfeld and Rogoff (2001) who postulate that trade costs have the potential to help reconcile six different puzzles in international macroeconomics. Eaton, Kortum, and Neiman (2016) find that trade costs do indeed quantitatively help account for a few of those puzzles, including two directly related to international capital flows.

[6]Other papers that feature capital accumulation in a multicountry trade model include Alvarez (2017); Anderson, Larch, and Yotov (2020); Eaton, Kortum, Neiman, and Romalis (2016); Eaton, Kortum, and Neiman (2016); Ravikumar, Santacreu, and Sposi (2019); Zylkin (2016).

countries provide an incentive for countries to borrow and lend as capital seeks higher rates of return. In the presence of large investment distortions, a demographic-induced increase in saving shows up as higher net exports rather than higher investment.

The last feature is the unification of 28 countries in a general equilibrium setting. Including many countries provides a window to explore how demographics in certain countries impact the global dispersion of capital flows. Doing the analysis in general equilibrium encapsulates the interaction between demographics and bilateral trade costs in determining real exchange rates and capital flows so that the model's structure discplines the split of savings between investment and net exports. Moreover, it disciplines how prices respond to counterfactual demographic trajectories and ensures that world saving is balanced.

Incorporating all of these features introduces computational challenges. In the model there are $2 \times 28 \times 85 = 4,760$ state variables: the distribution of both capital and bonds across 28 countries and 85 age groups.[7] Trade costs complicate matters since the mapping from the world distribution of state variables to prices differs across countries; This is the not case in a model of frictionless trade.[8] To make progress I impose three assumptions on the ownership of capital across cohorts so that one only needs to keep track of the aggregate capital stock in each country rather than the distribution. In addition, the distribution of asset holdings across cohorts can be analytically summarized by interacting the age distribution with some prices and exogenous forces so that one only needs to keep track of the aggregate net-foreign asset position in each country. With this aggregation property I prove that the equilibrium dynamics can be replicated by a model where each country is inhabited by a representative household that experiences an appropriately chosen, endogenous discount factor. As a result, the dimensionality is reduced to $2 \times 28 = 56$ state variables. I employ the representative-household framework to compute the exact transitional dynamics by extending the methods developed in Ravikumar, Santacreu, and Sposi (2019).

This paper builds on existing literature that studies demographic-driven capital flows in general equilibrium settings. Ferrero (2010) uses a two-country OLG model, Backus, Cooley, and Henriksen (2014) use a three-country OLG model, and Krueger and Ludwig (2007) use

---

[7]One approach is to focus on stationary equilibria as in Bai and Zhang (2010) and Chang, Kim, and Lee (2013). However, given the non-stationary characteristics of demographics and current accounts for countries like the U.S. and China, imposing stationarity is less appropriate for my analysis.

[8]The absence of trade costs results in price equalization across countries and the mapping from the world distribution of state variables into each country's prices would be identical. If trade costs between Poland and the United States were large, then the state of the U.S. economy would have an indirect and negligible influence on Poland's prices, whereas if trade costs between Canada and the United States were small, then the state of the U.S. economy would have a direct and meaningful influence on Canada's prices.

a four-country OLG model. Domeij and Flodén (2006) study capital flows between 18 OECD countries and abstract from emerging economies. Papetti (2020) argues that an aging population induces borrowing through an increased demand for nontradables. Bárány, Coeurdacier, and Guibaud (2019) study fertility and longevity risk as drivers of capital flows from emerging to advanced economies in a model with many countries. I do not confront fertility or longevity risk; I instead focus on the differences in observed age distributions.

Incorporating bilateral trade between a large number of countries constitutes a notable departure from the above papers that abstract from gross trade flows. To demonstrate the relevance of trade patterns, I consider an alternative specification with frictionless trade and a high trade elasticity to approximate a one-good model. This alternative specification generates counterfactualy large volumes of gross trade flows and sharply different predictions for how capital flows respond to counterfactual demographic scenarios. Furthermore, in the absence of gravity, the bilateral incidence of changes in capital flows in response to changes in one country's demographics is uncorrelated with observed bilateral trade patterns.

Explaining the direction of capital flows across countries has long been a challenge for economists. Neoclassical theory predicts that capital should flow from countries with low productivity growth to countries with high productivity growth. However, the data do not reveal such a pattern, an observation referred to as the allocation puzzle. I find that demographic trends in emerging economies partially offset productivity-induced incentives to borrow. Neither bilateral trade costs nor labor market distortions help reconcile the allocation puzzle, but exaggerate it. Saving distortions account for the most, followed by demographic patterns in emerging economies, and then investment distortions.

2   Model

This section begins with a simple, two-period OLG model to illustrate how the age distribution shapes aggregate consumption growth. I show how the equilibrium can be replicated by an otherwise identical economy where each country is inhabited by a representative household with an endogenous, time-varying discount factor. I then extend the model to a richer setting and characterize the general equilibrium dynamics.

2.1   An illustrative example

Agents live for two periods. At time t a cohort arrives with $N_{w,t}$ workers that coexist with $N_{r,t}$ retirees. At time $t+1$, workers from the previous period become retirees, and retirees from

the previous period disappear. Workers supply one unit of labor ($\ell_w = 1$), choose how much to consume ($c_{w,t}$), and choose how much to borrow/lend in one-period international bonds. Retirees consume their assets ($c_{r,t+1}$) and do not work ($\ell_r = 0$). Productivity is constant over time and is such that one unit of labor generates one unit of output. I normalize the wage to 1, implying that the price of consumption is also 1. The world interest rate on bonds, q, is constant over time. Future consumption is discounted at the rate β. Agents born at time t solve

$$\max_{c_{w,t}, c_{r,t+1}} \ln\left(c_{w,t}\right) + \beta \ln\left(c_{r,t+1}\right), \ \text{ s.t. } \ c_{w,t} + \frac{c_{r,t+1}}{1+q} = \ell_w$$

The solution for every cohort is characterized by $c_r = \beta(1+q)c_w$. While each cohort's decision is constant over time, aggregate consumption, $C_t = N_{w,t}c_w + N_{r,t}c_r$, and aggregate labor supply, $L_t = N_{w,t}\ell_w$, vary over time with demographics.

Let $\mu_{w,t} = N_{w,t}/N_t$ denote the working age share in the population at time t. Then, per-capita consumption growth and the employment-population ratio are

$$\frac{C_{t+1}/N_{t+1}}{C_t/N_t} = \left(\frac{\beta + \left(\frac{1-\beta(1+q)}{(1+q)}\right)\mu_{w,t+1}}{\beta(1+q) + (1-\beta(1+q))\mu_{w,t}}\right)(1+q) \ \text{ and } \ \frac{L_t}{N_t} = \mu_{w,t}.$$

Now consider a similar economy to the one above, except there is an infinitely-lived representative household of size N_t with labor force L_t. Each period the household decides how much to consume, C_t, and how many assets to carry over into the next period, $\mathcal{A}_{t+1}$. The household is subject to time-varying preference shifter, ψ_t, which can be thought of as a shock to the discount factor. This household's optimization problem is given by

$$\max_{\{C_t, \mathcal{A}_{t+1}\}} \psi_t N_t \ln\left(C_t/N_t\right), \ \text{ s.t. } \ C_t + \mathcal{A}_{t+1} = L_t + (1+q)\mathcal{A}_t,$$

with solution characterized by a standard intertemporal Euler equation:

$$\frac{C_{t+1}/N_{t+1}}{C_t/N_t} = \left(\frac{\psi_{t+1}}{\psi_t}\right)(1+q)$$

The representative household solution clearly coincides with that of the OLG economy with an appropriately chosen preference shifter and labor force:

$$\frac{\psi_{t+1}}{\psi_t} = \frac{\beta + \left(\frac{1-\beta(1+q)}{(1+q)}\right)\mu_{w,t+1}}{\beta(1+q) + (1-\beta(1+q))\mu_{w,t}} \ \text{ and } \ L_t = \mu_{w,t}N_t.$$

The change in the preference shifter (discount factor) depends both on level and change in the age distribution, and its interaction with the real interest rate.[9] A younger population with an increasing working age share has a lower discount factor and tends to borrow.

2.2 General equilibrium model

The world consists of $i = 1, \ldots, I$ countries and time runs from $t = 1 \ldots, \infty$. Each country is populated by overlapping generations and production is carried out by competitive firms.

2.2.1 Firms

A unit interval of potentially tradable varieties, $v \in [0, 1]$, combine to form a composite good:

$$Q_{i,t} = \left[\int_0^1 q_{i,t}(v)^{1-1/\eta} dv \right]^{\eta/(\eta-1)},$$

where η is the elasticity of substitution between varieties. The term $q_{i,t}(v)$ is the quantity of variety v used to construct the composite good in country i at time t. Varieties can be sourced from anywhere, while the composite good is sold domestically to households (for consumption and investment) and to firms (for intermediate inputs).

Country i can produce variety v using domestic capital, labor, and intermediates:

$$Y_{i,t}(v) = z_i(v) \left(A_{i,t} K_{i,t}(v)^\alpha L_{,it}(v)^{1-\alpha} \right)^{\nu_{i,t}} M_{i,t}(v)^{1-\nu_{i,t}}.$$

The term $M_{i,t}(v)$ is the quantity of the composite good used as an input to produce $Y_{i,t}(v)$ units of variety v, while $K_{i,t}(v)$ and $L_{i,t}(v)$ are the quantities of capital and labor used.

The parameter $\nu_{i,t}$ denotes the share of value added in total output in country i at time t. It varies across countries and over time, enabling the model to reconcile both *gross* trade flows and *value added* output. The parameter α denotes capital's share in value added.

$A_{i,t}$ denotes country i's value-added productivity at time t. Country i's idiosyncratic efficiency draw for variety v is independently drawn from a Fréchet distribution, as in Eaton and Kortum (2002): $z_i(v) \sim \exp(-z^{-\theta})$. The parameter θ governs the trade elasticity.

Prices and trade flows   Each country sources each variety from its least-cost supplier, subject to physical iceberg costs. At time t, country i must purchase $d_{i,j,t} \geq 1$ units of any

---

variety from country j in order for one unit to arrive, wth $d_{i,i,t} = 1$. Well-known properties of the Fréchet distirbution yield expressions for composite prices and bilateral trade shares:

$$P_{i,t} = \gamma \left(\sum_{j=1}^{I} \left((A_{j,t})^{-\nu_{j,t}} u_{j,t} d_{i,j,t} \right)^{-\theta} \right)^{-\frac{1}{\theta}} , \quad \pi_{i,j,t} = \frac{\left((A_{j,t})^{-\nu_{j,t}} u_{j,t} d_{i,j,t} \right)^{-\theta}}{\sum_{j'=1}^{I} \left((A_{j',t})^{-\nu_{j',t}} u_{j',t} d_{i,j',t} \right)^{-\theta}},$$

where γ is a constant and $u_{j,t}$ is the marginal input cost faced by producers in country j. The variable $\pi_{i,j,t}$ is the fraction of country i's total absorption sourced from country j.

One unit of the composite good can be transformed into $\chi_{i,t}^{c}$ units of consumption, into $\chi_{i,t}^{x}$ units of investment, or into one unit of the intermediate input. These transformation costs pin down relative prices, which influence rates of return to investment and, hence, capital flows.[10] The respective prices for consumption and investment are

$$P_{i,t}^{c} = \frac{P_{i,t}}{\chi_{i,t}^{c}}, \quad P_{i,t}^{x} = \frac{P_{i,t}}{\chi_{i,t}^{x}}.$$

Next I define total factor usage (K, L, M) and output (Y) by summing over varieties.

$$K_{i,t} = \int_0^1 K_{i,t}(v) dv, \quad L_{i,t} = \int_0^1 L_{i,t}(v) dv, \quad M_{i,t} = \int_0^1 M_{i,t}(v) dv, \quad Y_{i,t} = \int_0^1 Y_{i,t}(v) dv.$$

The term $K_{i,t}(v)$ denotes the quantity of capital employed in the production of variety v at time t. If country i imports variety v at time t, then $K_{i,t}(v) = 0$. Hence, $K_{i,t}$ is the total quantity of capital employed in country i at time t. Similarly, $L_{i,t}$ is the amount of labor employed, $M_{i,t}$ is the quantity of the composite good used as an intermediate input, and $Y_{i,t}$ is the quantity of output produced. Capital is rented at the rate $r_{i,t}$, and labor at the rate $w_{i,t}$. Due to constant returns to scale in production, input costs engross total revenue:

$$r_{i,t} K_{i,t} = \alpha \nu_{i,t} P_{i,t} Y_{i,t}, \quad w_{i,t} L_{i,t} = (1-\alpha) \nu_{i,t} P_{i,t} Y_{i,t}, \quad P_{i,t} M_{i,t} = (1-\nu_{i,t}) P_{i,t} Y_{i,t}.$$

2.2.2 Households

Households own the primary factors of production—capital and labor—and supply these factors to domestic firms. Factor income is split between domestic consumption, domestic investment, and net purchases (sales) of one-period, internationally traded bonds.

---

[10]Caselli and Feyrer (2007) argue that differences in the relative price of investment help rationalize the Lucas (1990) paradox – why capital does not flow from rich to poor countries.

Each country is populated by $N_{i,t}$ households. Each household lives for $g = 1 \ldots, G$ periods and generations overlap. The share of age g households in country i at time t is denoted by $\mu_{i,g,t}$. Lifetime utility for a household born at time t is

$$\mathcal{U}_i[t] = \sum_{g=1}^{G} \beta^{g-1} \ln(c_{i,g,t+g-1}),$$

where $c_{i,g,t+g-1}$ denotes consumption of an age g household at time $t + g - 1$ and β is the constant discount factor.

Every generation in every country supplies labor over the life cycle based on their age. Let ℓ_g indicate whether or not a household of age g supplies labor:

$$\ell_g = 1, \text{ if } g \in \mathcal{G}_w, \text{ and } 0 \text{ otherwise.}$$

$\mathcal{G}_w$ denotes the age range for working age households.[11] Among those of working age, a fraction $\tau_{i,t}^L$ are not employed, either due to business cycle conditions or non-age related factors such as female labor force participation and un-modeled labor market distortions, etc. From now on I refer to τ^L as a labor market distortion.

Each period households earn income from supplying capital and labor to domestic firms. They face a consumption-saving decision and can access two channels for saving: one-period internationally traded bonds and investment in the domestic physical capital stock. One unit of consumption has price $P_{i,t}^c$, one unit of investment has price $P_{i,t}^x$, while bonds are denominated in units of world GDP – the numéraire.

Households of age g, born at time t, enter period $t+g-1$ with a net asset position $a_{i,g,t+g-1}$. Each household is born with zero assets and dies with zero assets: $a_{i,1,t} = a_{i,G+1,t+G} = 0$. At every other age the asset position is unrestricted, up to a lifetime budget constraint. The net asset (debt) position reflects lending (borrowing) from the previous period and accrues an interest rate q_t at time t. The entire value of asset holdings, including interest, is taxed at the rate $\tau_{i,t}^A$. From now on I refer to τ^A as a saving distortion. Taxes are rebated in lump sum back to the household in the same period. The bonds expire immediately thereafter and households can trade newly floated one-period bonds.

All households in country i at time t realize an "excess return" on international bond markets of $\varepsilon_{i,t} = \mathcal{E}_{i,t}/N_{i,t}$. Since the model is restricted to have a common world interest rate, the aggregate excess returns, $\mathcal{E}_{i,t}$, help square the difference between the current account

---

[11]In the quantitative application later on, $\mathcal{G}_w$ consists of ages 15-64.

balance and trade balance with that of the data. Mechanically, these can be interpreted as exogenous, cross-country transfers that are exogenous to the model such as debt forgiveness, disaster relief or various types of aid; They sum to zero across countries in each period.

Households can also save by purchasing physical capital. The quantity of capital owned by an individual of age g, born at time t, is denoted by $k_{i,g,t+g-1}$. The capital stock is managed by an intermediary that brokers the demand by firms with the supply by households. The capital stock under management by the intermediary at the beginning of period t is $K_{i,t} = \sum_{g=1}^{G} k_{i,g,t}\mu_{i,g,t}N_{i,t}$. The intermediary collects the rental rate from firms and passes along those payments to the household at zero fee. A capital gains tax rate $\tau_{i,t}^{K}$ is applied to capital income earned by households at time t. I refer to τ^{K} as an investment distortion.

Age g households born at time t can purchase new investment, $x_{i,g,t+g-1}$. The new investment expenditures are tax deductible at the current capital gains tax rate.

Aggregate investment is thus $X_{i,t} = \sum_{g=1}^{G} x_{i,g,t}\mu_{i,g,t}N_{i,t}$. During the period a fraction δ of the capital stock depreciates. The aggregate capital stock accumulates according to

$$K_{i,t+1} = (1 - \delta)K_{i,t} + X_{i,t}.$$

To facilitate aggregation, I make three assumptions regarding the capital market.

Assumption 1. *Within a county-period, all working age households own equal claims to the capital stock and non-working age households have no claims to it.*

Assumption 2. *Within a county-period, the share of factor income spent on investment is equalized across working age households, and is zero for non-working age households.*

Assumption 3. *The intermediary ensures that there are no arbitrage opportunities by restricting access to the capital stock, as needed, so that the after-tax rate of return is equal to that from the bond market.*

Assumptions 1 and 2 ensure that (i) capital-labor ratios are equalized across cohorts and (ii) there is a clean separation between saving in bonds versus investment. The former assumption facilitates aggregation, while the latter implies that differences in saving rates over the life cycle manifest in differences in bond saving rates. Thus, even though capital ownership is restrictive, there is plenty of scope for life-cycle forces to influence total saving.

Assumption 3 ensures that the after-tax rates of return equalize between the two saving instruments so the household is indifferent between any make-up of the saving portfolio

thereby mitigating the restrictions imposed by Assumptions 1 and 2. This assumption arises naturally from an unrestricted portfolio choice problem faced by a representative household.

Given the sources of income and spending described above, a household born at time t faces the following sequence of budget constraints over their life cycle:

$$
\begin{aligned}
P^c_{i,t+g-1}c_{i,g,t+g-1} + a_{i,g+1,t+g} =\ & w_{i,t+g-1}\ell_g(1 - \tau^L_{i,t+g-1}) \\
& + \left(r_{i,t+g-1}k_{i,g,t+g-1} - P^x_{i,t+g-1}x_{i,g,t+g-1}\right)(1 - \tau^K_{i,t+g-1}) \\
& + (1 + q_t)a_{i,g,t+g-1}(1 - \tau^{\mathcal{A}}_{i,t+g-1}) + \varepsilon_{i,t+g-1} \\
& + t_{i,g,t+g-1},
\end{aligned}
\tag{1}
$$

where $t_{i,g,t+g-1}$ is the tax rebate afforded to an age g household at time t.

2.2.3 Government

Tax revenue is rebated to each household with no inter-generational transfers:

$$
t_{i,g,t} = (1 + q_t)a_{i,g,t}\tau^{\mathcal{A}}_{i,t} + (r_{i,t}k_{i,g,t} - P^x_{i,t}x_{i,g,t})\tau^K_{i,t}.
$$

2.2.4 Aggregation and equilibrium dynamics

A competitive equilibrium satisfies the following conditions: (i) taking prices as given, households maximize lifetime utility subject to their budget constraint, (ii) taking prices as given, firms maximize profits subject to the available technologies, (iii) varieties are purchased from their lowest-cost provider, and (iv) markets clear. At each point in time, world GDP is the numéraire.[12] Table A.1 in Appendix A summarizes the equilibrium conditions.

Consumption dynamics The optimal trade-off between consumption and saving in bonds gives rise to an intertemporal Euler equation for households of age g (born at time t):

$$
\frac{c_{i,g+1,t+g}}{c_{i,g,t+g-1}} = \beta \left(\frac{1 + q_{t+g}}{P^c_{i,t+g}/P^c_{i,t+g-1}} \right)(1 - \tau^{\mathcal{A}}_{i,t+g}),
\tag{2}
$$

Define "consumable income" as factor income net of investment spending, taxes and rebates:

$$
\begin{aligned}
y_{i,g,t+g-1} =\ & w_{i,t+g-1}\ell_g(1 - \tau^L_{i,t+g-1}) \\
& + \left(r_{i,t+g-1}k_{i,g,t+g-1} - P^x_{i,t+g-1}x_{i,g,t+g-1}\right)(1 - \tau^K_{i,t+g-1}) + \varepsilon_{i,t+g-1}.
\end{aligned}
\tag{3}
$$

---

[12]$\sum_{i=1}^{I} r_{i,t}K_{i,t} + w_{i,t}L_{i,t} = 1$. This implies that all prices are expressed in units of current world GDP.

Combining equations (1) and (3) and iterating over the entire life cycle implies that the net present value of consumption must equal the net present value of consumable income:

$$\sum_{g=1}^{G} \frac{P^c_{i,t+g-1} c_{i,g,t+g-1}}{(1+\mathcal{Q}_{t+g-1})/(1+\mathcal{Q}_t)} = \underbrace{\sum_{g=1}^{G} \frac{y_{i,g,t+g-1}}{(1+\mathcal{Q}_{t+g-1})/(1+\mathcal{Q}_t)}}_{\mathcal{W}_i[t]}, \tag{4}$$

where $1 + \mathcal{Q}_t = \prod_{t'=1}^{t}(1+q_{t'})$ is the compounded interest rate from period 1 through period t. This means that $(1+\mathcal{Q}_{t+g-1})/(1+\mathcal{Q}_t)$ is the compounded interest rate faced by an age g household born at t, starting from the period they were born. Similarly, define $1 - \mathcal{T}^{\mathcal{A}}_{i,t} = \prod_{t'=1}^{t}(1 - \tau^{\mathcal{A}}_{i,t'})$ to be the compounded, non-taxed portion of bond holdings. To simplify notation, I denote the net present value of consumable income for a household born at time t by $\mathcal{W}_i[t]$. Equations (2) and (4) imply that consumption at age g is given by

$$c_{i,g,t+g-1} = \left(\frac{\beta^{g-1}\left(1 - \mathcal{T}^{\mathcal{A}}_{i,t+g-1}\right)}{\sum_{g'=1}^{G} \beta^{g'-1}\left(1 - \mathcal{T}^{\mathcal{A}}_{i,t+g'-1}\right)} \right) \left(\frac{\mathcal{W}_i[t]}{1 + \mathcal{Q}_t} \right) \left(\frac{(1+\mathcal{Q}_{t+g-1})}{P^c_{i,t+g-1}} \right). \tag{5}$$

Investment and capital ownership Next consider the investment component of saving. Assumption 3 ensures that the after tax return to investment equals that of bonds:

$$(1 + q_{i,t+1})\left(1 - \tau^{\mathcal{A}}_{i,t+1}\right) = \left(\frac{r_{i,t+1}}{P^x_{i,t+1}} + 1 - \delta \right) \left(\frac{P^x_{i,t+1}}{P^x_{i,t}} \right) \left(\frac{1 - \tau^K_{i,t+1}}{1 - \tau^K_{i,t}} \right), \tag{6}$$

In other words, while households are restricted in their investment outlays, the no arbitrage condition ensures that they are indifferent between saving in bonds versus investment.

Aggregate income and expenditures Aggregate labor supply is characterized by:

$$L_{i,t} = (1 - \tau^L_{i,t}) \sum_{g=1}^{G} \ell_g \mu_{i,g,t} N_{i,t}. \tag{7}$$

All workers earn the same wage so aggregate labor income at time t is $w_{i,t} L_{i,t}$.

Assumption 1 implies that $k_{i,g,t} = \frac{K_{i,t}}{L_{i,t}}(1 - \tau^L_{i,t})\ell_g$, so aggregate capital income is

$$r_{i,t} K_{i,t} = r_{i,t} \sum_{g=1}^{G} k_{i,g,t} \mu_{i,g,t} N_{i,t} = r_{i,t} \sum_{g=1}^{G} \frac{K_{i,t}}{L_{i,t}}(1 - \tau^L_{i,t})\ell_g \mu_{i,g,t} N_{i,t},$$

13

leaving the capital-labor ratio to be determined below.

Gross national income also consists of interest on bonds. Since the interest rate on bonds is equal across countries, net interest receipts from (payments to) foreigners is

$$q_{i,t}\mathcal{A}_{i,t} = q_{i,t}\sum_{g=1}^{G} a_{i,g,t}\mu_{i,g,t}N_{i,t}.$$

By assumption, aggregate excess returns—transfers—are $\mathcal{E}_{i,t} = N_{i,t}\varepsilon_{i,t}$.

On the expenditure side, aggregate consumption spending is given by

$$P_{i,t}^c C_{i,t} = \sum_{g=1}^{G}\left(\frac{\beta^{g-1}\left(1 - \mathcal{T}_{i,t}^{\mathcal{A}}\right)}{\sum_{g'=1}^{G}\beta^{g'-1}\left(1 - \mathcal{T}_{i,t+g'-g}^{\mathcal{A}}\right)}\right)\left(\frac{\mathcal{W}_i[t-g+1]}{1 + \mathcal{Q}_{t-g+1}}\right)(1 + \mathcal{Q}_t)\,\mu_{i,g,t}N_{i,t}, \quad (8)$$

which makes use of equation (5). Equation (8) implies that

$$\frac{C_{i,t+1}/N_{i,t+1}}{C_{i,t}/N_{i,t}} = \left(\frac{\psi_{i,t+1}}{\psi_{i,t}}\right)\left(\frac{1 + q_{t+1}}{P_{i,t+1}^c/P_{i,t}^c}\right)\left(1 - \tau_{i,t+1}^{\mathcal{A}}\right) \quad (9)$$

$$\psi_{i,t} = \sum_{g=1}^{G}\left(\frac{\beta^{g-1}}{\sum_{g'=1}^{G}\beta^{g'-1}\left(1 - \mathcal{T}_{i,t+g'-g}^{\mathcal{A}}\right)}\right)\left(\frac{\mathcal{W}_i[t-g+1]}{1 + \mathcal{Q}_{t-g+1}}\right)\mu_{i,g,t}. \quad (10)$$

which characterizes aggregate saving and defines the *endogenous preference shifter*, $\psi_{i,t}$.

The no arbitrage equation (6) dictates how saving is split between bonds and investment:

$$\frac{C_{i,t+1}/N_{i,t+1}}{C_{i,t}/N_{i,t}} = \left(\frac{\psi_{i,t+1}}{\psi_{i,t}}\right)\left(\frac{r_{i,t+1}}{P_{i,t+1}^x} + 1 - \delta\right)\left(\frac{P_{i,t+1}^x/P_{i,t+1}^c}{P_{i,t}^x/P_{i,t}^c}\right)\left(\frac{1 - \tau_{i,t+1}^K}{1 - \tau_{i,t}^K}\right) \quad (11)$$

This equation places a restriction on the marginal product of capital and, hence, the aggregate capital-labor ratio, thereby pinning down aggregate investment, $X_{i,t}$.

Tax rebates balance with revenue at the household level, and therefore in aggregate:

$$T_{i,t} = \sum_{g=1}^{G} t_{i,g,t}\mu_{i,g,t}N_{i,t} = (1 + q_{i,t})\mathcal{A}_{i,t}\tau_{i,t}^{\mathcal{A}} + \left(r_{i,t}K_{i,t} - P_{i,t}^x X_{i,t}\right)\tau_{i,t}^K.$$

Any difference between the aggregate income and aggregate expenditures described above is reconciled by net borrowing or lending in one-period bonds. Resultantly, the net-foreign asset position carried over to the next period describes an aggregate budget constraint:

$$A_{i,t+1} = \left(r_{i,t}K_{i,t} - P_{i,t}^x X_{i,t}\right)\left(1 - \tau_{i,t}^K\right) + w_{i,t}L_{i,t} + (1 + q_t)\mathcal{A}_{i,t}\left(1 - \tau_{i,t}^{\mathcal{A}}\right) + \mathcal{E}_{i,t} + T_{i,t} - P_{i,t}^c C_{i,t}.$$

14

Feasibility To close the model, three conditions must be satisfied: (i) gross absoprtion must equal final plus intermediate demand, (ii) gross production must be absorbed globally, and (iii) the current account equals net exports plus net-foreign income.

$$P_{i,t}Q_{i,t} = P_{i,t}^c C_{i,t} + P_{i,t}^x X_{i,t} + P_{i,t}M_{i,t},$$

$$P_{i,t}Y_{i,t} = \sum_{j=1}^{I} P_{j,t}Q_{j,t}\pi_{j,i,t},$$

$$\underbrace{\mathcal{A}_{i,t+1} - \mathcal{A}_{i,t}}_{\text{Current account}} = \underbrace{P_{i,t}Y_{i,t} - P_{i,t}Q_{i,t}}_{\text{Net exports}} + \underbrace{q_t\mathcal{A}_{i,t} + \mathcal{E}_{i,t}}_{\text{Net-foreign income}}.$$

2.3 An equivalent economy with representative households

In this section I describe a representative household framework that can replicate the aggregate outcomes arising from the OLG framework.

Firms, production, and trade are identical to that in the OLG model in Section 2.2. Each country is populated by a representative household with population in each period given by $N_{i,t}$. The representative household has lifetime utility

$$\sum_{t=1}^{\infty} \psi_{i,t} N_{i,t} \ln\left(\frac{C_{i,t}}{N_{i,t}}\right)$$

with *endogenous preference shifter* $\psi_{i,t}$ defined in equation (10).

Aggregate labor supply, $L_{i,t}$, is governed by a distortionary component and a demographic component as defined in equation (7). The age distribution, $\mu_{i,g,t}$ serves as an exogenous force that shapes aggregate labor supply, $L_{i,t}$, and the preference shifter, $\psi_{i,t}$.

The representative household enters period t with net-foreign asset position $\mathcal{A}_{i,t}$, with $\mathcal{A}_{i,1}$ given. The assets yield a common world interest rate, q_t. In each period the assets, along with interest income, are taxed at the rate $\tau_{i,t}^{\mathcal{A}}$. During the period new bonds can be traded and expire after one period. Countries engage in exogenous transfers of assets, $\mathcal{E}_{i,t}$.

The household enters period t with $K_{i,t}$ units of capital, with $K_{i,1}$ given. Unlike in the OLG model, there is no investment intermediary. The household directly rents out capital to firms. Investment, $X_{i,t}$, is purchased absent any restrictions other than a budget constraint. These outlays add to the capital stock subject to depreciation: $K_{i,t+1} = (1-\delta)K_{i,t} + X_{i,t}$.

Capital and labor are compensated $r_{i,t}$ and $w_{i,t}$, respectively. Capital income is taxed at

the rate $\tau_{i,t}^K$, while investment expenditures are deductible from capital gains taxes.[13]

Tax revenue is rebated in lump sum, $T_{i,t}$. The period budget constraint is

$$P_{i,t}^c C_{i,t} + \mathcal{A}_{i,t+1} = \left(r_{i,t} K_{i,t} - P_{i,t}^x X_{i,t}\right)\left(1 - \tau_{i,t}^K\right) + w_{i,t} L_i + (1 + q_t)\mathcal{A}_{i,t}\left(1 - \tau_{i,t}^{\mathcal{A}}\right) + \mathcal{E}_{i,t} + T_{i,t}.$$

The government runs a balanced budget in each period:

$$T_{i,t} = (1 + q_t)\mathcal{A}_{i,t}\tau_{i,t}^{\mathcal{A}} + \left(r_{i,t} K_{i,t} - P_{i,t+g-1}^x X_{i,t}\right)\tau_{i,t}^K$$

Proposition 1. *Given a set of initial conditions and exogenous forces across countries and over time,* $\left(K_{i,1}, \mathcal{A}_{i,1}, \{N_{i,t}, \mu_{i,g,t}, \chi_{i,t}^c, \chi_{i,t}^x, \nu_{i,t}, \tau_{i,t}^K, \tau_{i,t}^L, \tau_{i,t}^{\mathcal{A}}, \mathcal{E}_{i,t}, d_{i,j,t}, A_{i,t}\}_{t=1}^T\right),$ *the equilibrium prices and aggregate allocations in the representative household economy described in section 2.3 coincide with those in the OLG economy described in section 2.2.*

Proof. See Appendix A. □

3 Calibration

The model is calibrated to 28 countries (27 individual countries plus a rest-of-world aggregate) using annual data from 1970-2060. I target national accounts and bilateral trade data. Data from 1970-2014 are realized, while data from 2015-2060 are based on projections. Incorporating projections imposes the terminal conditions as of 2060, far enough beyond the period of interest (1970-2014), while providing discipline to households' expectations prior to 2014. Appendix B describes the data in detail.

The calibration involves two parts. The first part assigns values to the common parameters $(\beta, \alpha, \delta, \lambda, \theta, \eta)$, which are reported in Table 1. I assume that each household from the OLG version of the model lives for $G = 85$ periods, and works at ages 15-64 so that $\ell_g = 1$ if $g \in \{15, 16, \ldots, 64\}$ and $\ell_g = 0$ otherwise. The remaining common parameters take on standard values from the macro and trade literature.

The second part assigns values to the country-specific and time-varying parameters: $\left(K_{i,1}, \mathcal{A}_{i,1}, \{N_{i,t}, \mu_{i,g,t}, \chi_{i,t}^c, \chi_{i,t}^x, \nu_{i,t}, \tau_{i,t}^K, \tau_{i,t}^L, \tau_{i,t}^{\mathcal{A}}, \mathcal{E}_{i,t}, d_{i,j,t}, A_{i,t}\}_{t=1}^T\right)$ for all (i,j). Some of the country-specific parameters are observable. Those that are unobservable are inferred to rationalize both the observed and projected data as a solution to a perfect foresight equilibrium (see Appendix D). The data targets, roughly in order of how they map into model

---

[13]This assumption brings tractability by making the distortion separable from the gross return on capital.

Table 1: Common parameters

G	Lifespan for households	85
ℓ_g	Labor supply indicator over the life cycle	1 if $g \in \{15, \ldots, 64\}$ and 0 otherwise
β	Annual discount factor	0.96
α	Capital's share in value added	0.33
δ	Annual depreciation rate for capital stock	0.06
θ	Trade elasticity	4
η	Elasticity of substitution between varieties	2

Notes: The subscript g denotes a household's age, pertaining to the OLG version of the model, which runs from $1, \ldots, 85$. The discount factor also pertains to such households.
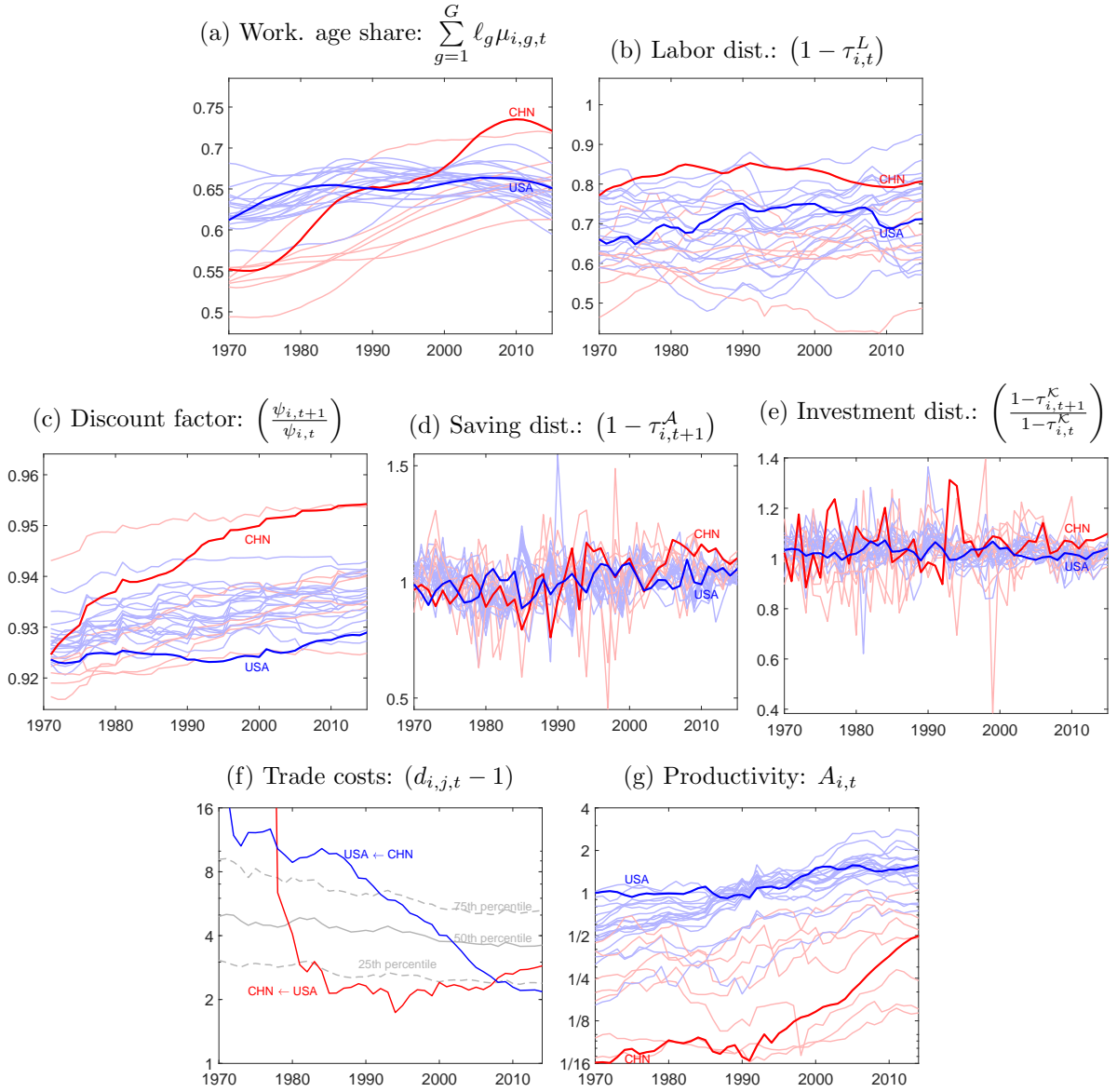
parameters, are (i) initial capital stock, (ii) initial net-foreign asset position, (iii) population, (iv) age distribution, (v) price level of consumption using PPP exchange rates, (vi) price level of investment using PPP exchange rates, (vi) ratio of value added to gross output, (vii) real investment, (viii) ratio of employment to population, (x) real consumption growth, (xi) current account, (xii) bilateral trade flows, and (xiii) price level of tradables using PPP exchange rates. The parameters are exactly identified in every country-year so the model fits these targets perfectly. Next I discuss a few important aspects of the calibration.

Labor supply Since the working age share and the employment-population ratio are observable, the fraction of workers that are employed (distortionary component) is recovered as a residual. Figure 1a shows the demographic component of labor supply. Working age shares in emerging economies were lower generally than those of advanced economies in 1970, and caught up by the 2000s. Specifically, the working age share in China rose more rapidly than that in any other country in the sample. The distortionary component in Figure 1b partly captures changes in female labor force participation and business cycles.

Saving The age distribution affects intertemporal marginal utility through changes in the endogenous preference shifter (i.e., the endogenous discount factor) $\psi_{i,t+1}/\psi_{i,t}$. The discount factor is itself also a function of saving distortions and endogenous prices. In order to disentangle the different components, I first measure an intertemporal wedge, $1 + \omega_{i,t+1}^{\mathcal{A}}$:

$$\frac{C_{t+1}/N_{t+1}}{C_t/N_t} = \underbrace{\left(\frac{\psi_{i,t+1}}{\psi_{i,t}}\right)\left(1 - \tau_{i,t+1}^{\mathcal{A}}\right)}_{1+\omega_{i,t+1}^{\mathcal{A}}}\left(\frac{q_{t+1}}{P_{i,t+1}^c/P_{i,t}^c}\right).$$

Figure 1: Calibrated dynamic forces

(a) Work. age share: $\sum_{g=1}^{G} \ell_g \mu_{i,g,t}$

(b) Labor dist.: $\left(1 - \tau_{i,t}^L\right)$

(c) Discount factor: $\left(\frac{\psi_{i,t+1}}{\psi_{i,t}}\right)$

(d) Saving dist.: $\left(1 - \tau_{i,t+1}^{\mathcal{A}}\right)$

(e) Investment dist.: $\left(\frac{1 - \tau_{i,t+1}^{\mathcal{K}}}{1 - \tau_{i,t}^{\mathcal{K}}}\right)$

(f) Trade costs: $(d_{i,j,t} - 1)$

(g) Productivity: $A_{i,t}$

Notes: The discount factor in panel (c) is endogenous to the model. Trade cost percentiles are computed from the entire matrix of bilateral trade costs in each year, excluding the diagonal. Productivity is shown relative to that of the United States in 1970. Emerging economies in red include Brazil, China, India, Indonesia, South Korea, Mexico, Turkey, and the rest-of-world aggregate. Advanced economies in blue include the remaining 20 countries.

This wedge is similar to that in Gourinchas and Jeanne (2013) in that it is a residual that rationalizes per-capita consumption growth with the real interest rate. Their model treats

this wedge purely as a distortion to saving. A similar wedge also appears in the models of Eaton, Kortum, Neiman, and Romalis (2016) and Reyes-Heroles (2016). These two papers treat this wedge purely as an *exogenous* discount factor. Absent additional structure, the two components—discount factor and distortion—are indistinguishable. In my model some of the wedge is determined by demographic forces, while the rest is due to factors determined outside of the model. The OLG structure provides a means to separate the two components.

Given the measured saving wedges ($\overline{\omega}^{\mathcal{A}}$) and data on prices and the age distribution ($\overline{\text{data}}$), I infer saving distortions as a solution to: $\frac{\psi_{i,t+1}\left(\{\tau_{i,t}^{\mathcal{A}}\}_{t=1}^{T}, \overline{\text{data}}\right)}{\psi_{i,t}\left(\{\tau_{i,t}^{\mathcal{A}}\}_{t=1}^{T}, \overline{\text{data}}\right)}\left(1 - \tau_{i,t+1}^{\mathcal{A}}\right) = 1 + \overline{\omega}_{i,t+1}^{\mathcal{A}}$.

Figure 1c plots the implied endogenous discount factors, $\psi_{i,t+1}/\psi_{i,t}$, evaluated at the calibrated saving distortions and observables. In general, there is a common, low-frequency, upward trend over time that reflects aging populations across countries. The discount factor starts low for most emerging economies and rises remarkably fast for China.

Figure 1d plots the distortionary component of the saving wedges. High-frequency fluctuations reflect annual saving movements that are unaccounted for by prices and demographics. The distortions exhibit little trend at lower frequencies. While the average value is about one (average saving distortion is roughly zero) there are some interesting patterns. Following China's accession to the WTO in 2001, its saving distortion falls ($1 - \tau^{\mathcal{A}}$ increases).

Given the calibrated endogenous discount factors, I recover an investment wedge, $1 + \omega_{i,t+1}^{K} = (1 - \tau_{i,t+1}^{K})/(1 - \tau_{i,t}^{K})$, as a residual in the investment Euler equation (11). Then, starting with $\tau_{i,1}^{K} = 0$, the distortions in successive periods are recovered recursively.[14] Figure 1e shows that the average wedge is about zero and that the investment wedges fluctuate at a high frequency as they account for movements in aggregate investment that cannot be explained by demographics or technology. An upward movement reflects a decreasing wedge, contributing to a higher investment share in saving.

Productivity and trade costs Fundamental productivity and bilateral trade costs are calibrated to match data on real exchange rates and bilateral trade flows.

Trade costs (Figure 1f) decline over time, capturing growth in world trade. Meanwhile, there is substantial heterogeneity across countries. For instance, there is sharp decline in bilateral trade costs between China and the United States following China's economic reforms in 1978 under Deng Xiaoping. The asymmetry in the bilateral cost between China and the United States since the 1990s captures the widening bilateral trade imbalance between the

---

[14]The wedge reflects changes in the distortion over time because investment spending is tax deductible from capital income. Therefore, a constant capital income tax would not distort intertemporal decisions.

two countries and is consistent with estimates in Alessandria, Choi, and Lu (2017).

Since 1970 most countries experienced growth in fundamental productivity (Figure 1g).[15] Both the levels and growth rates of productivity differ across countries. For instance, China's rapid economic growth beginning in early 1990s largely reflects its catch up in productivity. The United States exhibited slightly lower productivity growth than other high-income countries. All else equal, these features give rise to relatively fast growing exports in China and relatively fast growing imports in the United States.

4   Counterfactual analysis

This section quantifies the importance of demographics, and other forces, in shaping the evolution of capital flows. In each counterfactual scenario, I impose an alternative path for some exogenous processes and compute the resulting dynamic equilibrium under perfect foresight. Appendix E provides the details of the algorithm for solving for the transitional dynamics, which builds on Ravikumar, Santacreu, and Sposi (2019).

4.1   Global demographic forces and capital flows

The age distribution influences capital flows through both the endogenous discount factor and aggregate labor supply. I first consider the effects of demographics through both of these channels simultaneously, and then consider each channel separately.

The main counterfactual imposes a scenario in which the age distribution varies over time but not across countries. Specifically, each country's age distribution equals that of the whole world: $\widetilde{\mu}_{i,g,t} = \mu_{WLD,g,t} = \frac{\sum_{i=1}^{I} \mu_{i,g,t} N_{i,t}}{\sum_{i=1}^{I} N_{i,t}}$. The remaining parameters of the model, including total population, remain at the calibrated baseline values.

Figure 2a illustrates how long-run capital flows in the counterfactual compare to those in the baseline. The figure reports, for each country, the change in the net-foreign asset position from 1970-2014 as a share of cumulative GDP over that time span: $\frac{\mathcal{A}_{i,2014} - \mathcal{A}_{i,1970}}{\sum_{t=1970}^{2014} GDP_{i,t}}$ (*long-run capital flow*, henceforth). In general, long-run capital flows are significantly altered for every country. For the average country, the absolute value of the difference between the long-run capital flow in the baseline scenario and in the counterfactual scenario is 3.0% of GDP.

Relative to the baseline scenario, some countries experience relatively more inflows (less outflows), while others experience relatively less inflows (more outflows). In the counterfac-

---

[15]Fundamental productivity is not the same as measured TFP, which can also grow due to an expanding import share: $\text{TFP}_{i,t} \propto A_{i,t} \left(\pi_{i,i,t} \right)^{-\frac{1}{\theta \nu_{i,t}}}$.

Figure 2: Capital flows: Baseline and counterfactual demographic scenarios
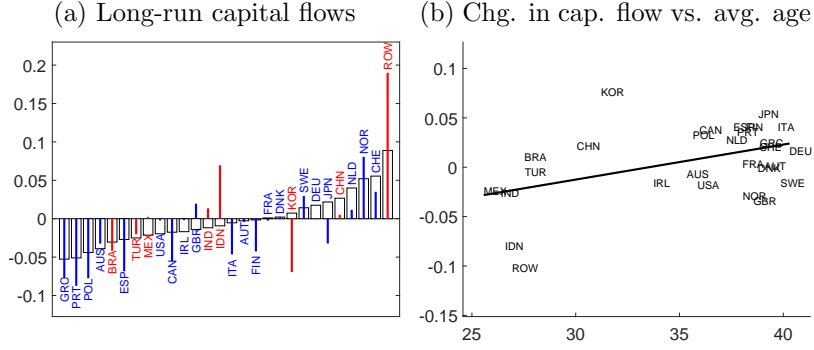
(a) Long-run capital flows

(b) Chg. in cap. flow vs. avg. age

Notes: Long-run capital flows are defined as the change in net-foreign asset position from 1970-2014, relative to cumulative GDP: $\frac{\mathcal{A}_{i,2014}-\mathcal{A}_{i,1970}}{\sum_{t=1970}^{2014} GDP_{i,t}}$. In panel (a), black-white bars refer to the baseline scenario; Colored bars refer to the counterfactual. Emerging economies are in red and advanced economies, in blue. In panel (b), change in capital flows is defined as the long-run capital flows in the baseline scenario minus that in the counterfactual. Mean age is taken over the period 1970-2014 and is the same for all countries in the counterfactual. The counterfactual scenario imposes that the age distribution in every country is equal to that of the world: $\widetilde{\mu}_{i,g,t} = \mu_{WLD,g,t}$.

tual China remains a net creditor, although its net lending declines. China's long-run capital flow is 0.5% of GDP in the counterfactual compared to 2.7% in the baseline. Similarly, the U.S. remains a net borrower, although its its net borrowing declines. The U.S. long-run capital flow is -0.2% of GDP in the counterfactual compared to -2.0% in the baseline. On average, countries whose mean age is one year higher than the world average realize a 0.4 percentage point increase in their current account as a share of GDP, relative to the scenario with all countries experiencing a common age distribution (Figure 2b).

Demographic effects through two distinct channels The following two counterfactuals isolate the individual importance of the discount factor and labor supply channels.

In one scenario the counterfactual demographic process operates only through the preference shifter: $\widetilde{\psi}_{i,t} = \sum_{g=1}^{G} \left(\frac{\beta^{g-1}}{\sum_{g'=1}^{G} \beta^{g'-1}\left(1-\mathcal{T}_{i,t+g'-g}^{\mathcal{A}}\right)} \right) \left(\frac{\widetilde{\mathcal{W}}_i[t-g+1]}{1+\widetilde{\mathcal{Q}}_{t-g+1}} \right) \widetilde{\mu}_{i,g,t}$. Note also that counterfactual prices differ from the baseline, further affecting the endogenous discount factor. Labor supply remains unchanged relative to the baseline model.

In the other scenario the counterfactual demographic process operates only through labor supply: $\widetilde{L}_{i,t} = (1 - \tau_{i,t}^L) \sum_{g=1}^{G} \ell_g \widetilde{\mu}_{i,g,t} N_{i,t}$. The preference shifters are computed using the ob-

served age distribution, μ, but may differ from the baseline model as a result of counterfactual prices.

There are notable differences in each counterfactual. By and large, the qualitative effects of counterfactual demographics operating through the endogenous discount factor resemble the total effects operating through both channels. The main difference is in the magnitude. With only the counterfactual discount factor channel, the average absolute difference in long-run capital flows between the counterfactual and baseline is 3.4% of GDP, a slightly greater effect than the 3.0% found with both channels operative. In this scenario China's net lending declines to the point that it becomes a net borrower: its long-run capital flow is -3.4% of GDP in the counterfactual compared 2.7% in the baseline. U.S. net borrowing declines to the point that it becomes a net lender: its long run capital flow is 0.7% of GDP in the counterfactual compared to -2.0% in the baseline.

Figure 3a illustrates how counterfactual demographic dynamics alter the path for the endogenous discount factor. China's discount factor is uniformly lower in the counterfactual compared to the baseline, which drives its reduced appetite for saving. In the United states the observed age distribution is more stable over time, so its discount factor in the counterfactual hovers around its value from the baseline model. As a result of changes in foreign demand for saving, the U.S. current account deficit is smaller in the counterfactual than in the baseline since there is less foreign saving to absorb. This result provides demographic-based support for the global saving glut hypothesis of Bernanke (2005).

Figure 3: Endogenous discount factor and labor supply
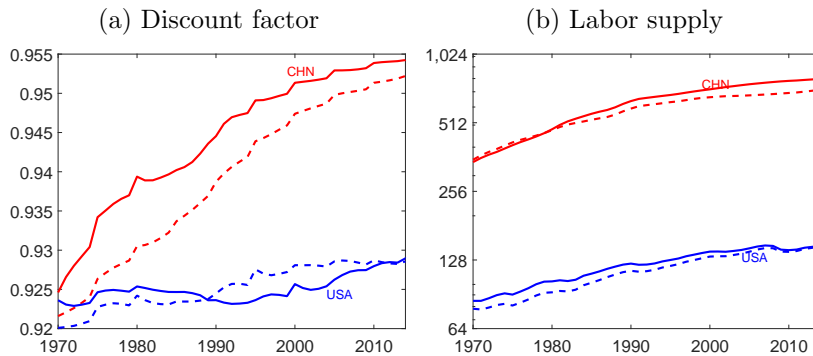
Notes: Solid lines refer to the baseline model scenario (data). Dashed lines refer to the counterfactual. The discount factor at time t is defined as $\psi_{i,t+1}/\psi_{i,t}$. Labor supply refers to millions of people employed. The counterfactual scenario imposes that the age distribution in every country is equal to that of the world: $\widetilde{\mu}_{i,g,t} = \mu_{WLD,g,t}$.

The effects through the labor supply channel tend to go in the opposite direction with respect to the discount factor channel. With counterfactual labor supply, both the U.S. current account deficit and China's surplus are magnified. China's long-run capital flow is 5.9% of GDP in the counterfactual compared to 2.7% in the baseline. The U.S. long-run capital flow is −2.7% of GDP in the counterfactual compared to −2.0% in the baseline.

Figure 3b shows that the counterfactual yields slower growth in labor supply in China and higher growth in labor supply in the United States, compared to the baseline. Since capital takes time to accumulate, in the counterfactual capital becomes relatively more scarce in the United States and more abundant in China, relative to labor. This gives rise to an increasing rate of return to capital in the United States, which, all else equal, widens both the U.S. current account deficit and China's current account surplus.

In sum, the two channels tend to have opposing forces on capital flows in response to demographic forces. The direction of both channels squares with the implications highlighted by Brooks (2003). On net, I find the discount factor channel to be more prominent.

4.2   Unilateral change in demographics in key countries

The previous counterfactual scenarios alter every country's age distribution simultaneously. Next I consider changing the age distribution in only one country at a time. I do this for four countries: the U.S., Japan, Germany, and China. These countries account for the largest shares of global imbalances. In each case I counterfactually set the age distribution in one *domestic* country equal to the world age distribution, and keep all other *foreign* countries' age distributions at their baseline values. I then examine how the incidence of changes in the domestic country's capital flows is distributed across all foreign countries.

Counterfactually changing the U.S. age distribution makes the United States relatively younger compared to the baseline. The U.S., in turn, borrows more and its liabilities increase by 0.8% of GDP (long-run capital flows change from −2.0% to −2.8%). The increase in U.S. liabilities is offset by an increase in assets (a decrease in liabilities) in other countries. The cross-country incidence correlates positively with the U.S. share in foreign country's trade. For instance, the United States accounts for 71% of Canada's trade and only 3% of Poland's trade. Canada's assets increase by 0.9% of GDP (its long-run capital flow changes from −1.8% in the baseline to −0.9% in the counterfactual), while Poland's assets increase by only 0.3% of GDP (from −4.4% to −4.1%). In general, the correlation between the change in foreign long-run capital flows and the U.S. share in each foreign country's trade is 0.55.

I also unilaterally change the age distribution in each of Japan, Germany, and China. In

each scenario, the domestic country realizes a decline in its long-run capital flows compared to the baseline: from 2.2% in the baseline to -4.1% in the counterfactual for Japan, from 1.8% to -1.5% for Germany, and from 2.7% to -4.0% for China. Meanwhile, the respective correlations between foreign capital flows and trade shares is 0.79, 0.74 and 0.34.

To summarize, the incidence of demographic-induced changes in a country's saving is distributed across all other countries in a manner that correlates with observed trade patterns. This is because the current account is ultimately connected to net exports. As an aging country accumulates more debt for life-cycle reasons, its net imports tend to rise, increasing the net exports and current accounts of countries that trade more with it.

4.3   Gravity, the real exchange rate, and capital flows

To highlight the importance of gravity I consider an alternative specification where gravity is removed. The *no-gravity* framework is characterized by frictionless trade ($d_{i,j,t} = 1$) and a high trade elasticity ($\theta = 12$). With a high trade elasticity, the model behaves more like one with a single homogeneous good. With frictionless trade, composite good prices equalize across countries so that the law of one price holds: $P_{i,t} = P_{j,t}$ for all country pairs (i, j) and the real exchange rate (RER) is constant at one. All parameters, aside from trade costs, are re-calibrated to match the same targets as in the baseline model.

Capital flows in the baseline demographic scenario are practically identical in both the gravity and no-gravity specifications—by design—as are all other objects except for real exchange rates and trade flows. The ratio of world trade to GDP in 2014 is 1.83 in the no-gravity model and is 0.18 in the gravity model.

The absence of gravity makes it "easy" to trade and, ultimately, easy to borrow and lend. A high trade elasticity makes it is easy to adjust trade patterns so capital flows become more sensitive to changes in demographics. Under counterfactual demographics, the magnitude of capital flows is greater in the no-gravity model than in the model with gravity. The average country's absolute change in long-run capital flows, relative to the baseline scenario, is 5.8% in the no-gravity model; In the model with gravity, the average change is just 3.0%. In addition, for some countries the direction of capital flows reverses compared to the model with gravity. In the no-gravity model, under counterfactual demographics China experiences a significant increase in capital outflows, relative to the baseline scenario. In contrast, in the model with gravity, China's capital outflows decline relative to the baseline demographic scenario. This capital flow "reversal" has to do with the pattern of the RER.

Country i's RER is defined as a trade-weighted average of all of its bilateral RERs, $\frac{P_i}{P_j}$. In

the model with gravity, under the baseline demographic scenario China's RER is high in 1970, declines gradually until the mid-90s, then slowly rises thereafter (Figure 4a) – mimicking the dynamics of its import costs. However, its current account does not grow substantially until after 2000 because its export costs are high until 2000. In the no-gravity model, the RER is invariant to demographic forces and China's counterfactual current account peaks at a high level in the mid-90s, then sharply declines after 2000 (Figure 4b).

Figure 4: Real exchange rate and current account dynamics
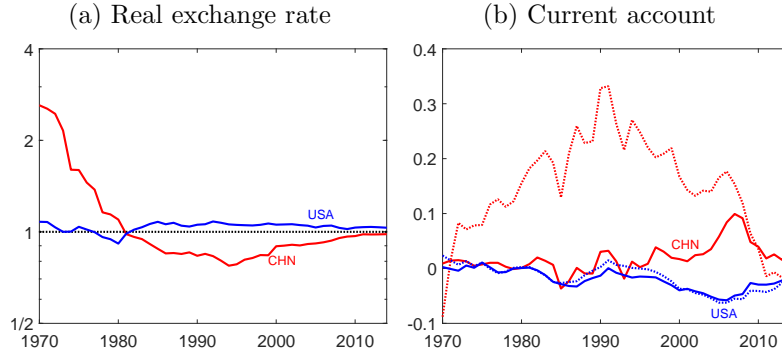
Notes: Solid lines refer to the baseline scenario in the model with gravity. Dotted lines refer to the counterfactual scenario in the no-gravity model. The counterfactual scenario imposes that the age distribution in every country is equal to that of the world: $\widetilde{\mu}_{i,g,t} = \mu_{WLD,g,t}$.

Unilateral demographic shifts in a no-gravity environment In the no-gravity model, the bilateral incidence of changes in capital flows stemming from a unilateral change in one country's demographics is uncorrelated with bilateral trade shares. For instance, by unilaterally setting the U.S. age distribution equal to that of the world, U.S. liabilities increase substantially: long-run capital flows change from -2.0% of GDP to -6.0%. This direction is the same as in the specification with gravity, but the magnitude of the decline is considerably larger. More interestingly, the bilateral incidence of the change in capital flows is uncorrelated with observed trade patterns. In the model with gravity, the incidence is born more by countries where the United States accounts for a large share of trade (e.g., Canada and Mexico). In the no-gravity model, Poland absorbs a greater share of the incidence than Canada, yet the United States accounts for only a very small share of Poland's trade.

I also consider unilaterally changing the age distribution in each of Japan, Germany, and China. When doing so, the incidence of the change in capital flows across foreign countries is uncorrelated with observed trade patterns, different than in the model with gravity.

5   Conclusion

This paper builds a multicountry trade model featuring overlapping generations where dynamics are driven by capital accumulation and cross-country borrowing and lending. Differential demographic structures across countries give rise to capital flows as a result of differences in aggregate propensities to save and labor supply. Integrating capital accumulation and gravity provides a novel structure to sort out whether changes in demographic-induced saving behavior lands in net exports versus investment, and is crucial for studying current account dynamics. In addition, the bilateral trade structure sheds new light on the cross-country dispersion in capital flows in response to changes in any one country's demographics.

I demonstrate how to replicate the equilibrium with an economy where each country is inhabited by a representative household. The representative household apparatus facilitates computation, thereby providing a window for quantitative analysis.

Capital flows arise endogenously as the result of shifts in technologies, trade costs, factor market distortions, and most importantly, demographics. All of the exogenous forces are calibrated using a wedge accounting procedure so that the model rationalizes past and projected age distributions, national accounts data, and bilateral trade flows.

I study capital flows between 28 countries from 1970-2014. On average, a one-year increase in a country's mean population boosts its current account by almost one-half of a percent of GDP. The cross-country incidence of changes in one country's current account correlates positively with observed bilateral trade shares.

One interesting direction for future research is to incorporate productivity changes over the life-cycle, as in Bárány, Coeurdacier, and Guibaud (2019). This aspect, combined with gravity will allow for novel dynamics between demographics and comparative advantage.

References

Aguiar, Mark and Manuel Amador. 2011. "Growth in the Shadow of Expropriation." *Quarterly Journal of Economics* 126 (2):651–697.

Alessandria, George and Horag Choi. 2019. "The Dynamics of the Trade Balance and the Real Exchange Rate: The J Curve and Trade Costs?" Working Paper 25563, National Bureau of Economic Research.

Alessandria, George, Horag Choi, and Dan Lu. 2017. "Trade Integration and the Trade Balance in China." *IMF Economic Review* 65 (3):633–674.

Alfaro, Laura, Sebnem Kalemli-Ozcan, and Vadym Volosovych. 2008. "Why Doesn't Capital Flow From Rich to Poor Countries? An Empirical Investigation." *Review of Economics and Statistics* 90 (2):347–368.

Alvarez, Fernando. 2017. "Capital Accumulation and International Trade." *Journal of Monetary Economics* 91 (C):1–18.

Alvarez, Fernando and Robert E. Lucas. 2007. "General Equilibrium Analysis of the Eaton-Kortum Model of International Trade." *Journal of Monetary Economics* 54 (6):1726–1768.

Anderson, James E., Mario Larch, and Yoto Yotov. 2020. "Growth and Trade with Frictions: A Structural Estimation Framework." *Economic Journal* 130 (630):1583–1607.

Backus, David, Thomas Cooley, and Espen Henriksen. 2014. "Demography and Low-Frequency Capital Flows." *Journal of International Economics* 92:S94–S102.

Backus, David, Patrick J. Kehoe, and Finn E. Kydland. 1992. "International Real Business Cycles." *Journal of Political Economy* 100 (4):745–775.

Bai, Yan and Jing Zhang. 2010. "Solving the Feldstein–Horioka Puzzle With Financial Frictions." *Econometrica* 78 (2):603–632.

Bárány, Zsófia, Nicolas Coeurdacier, and Stéphane Guibaud. 2019. "Capital Flows in an Aging World." Working paper.

Bernanke, Ben S. 2005. "The Global Saving Glut and the U.S. Current Account Deficit." Tech. rep.

Blanchard, Olivier J. 1985. "Debt, Deficits, and Finite Horizons." *Journal of Political Economy* 93 (2):223–247.

Brooks, Robin. 2003. "Population Aging and Global Capital Flows in a Parallel Universe." *IMF Economic Review* 50 (2):200–221.

Buera, Francisco J. and Yongseok Shin. 2017. "Productivity Growth and Capital Flows: The Dynamics of Reforms." *American Economic Journal: Macroeconomics* 9 (3):147–85.

Caliendo, Lorenzo, Fernando Parro, Esteban Rossi-Hansberg, and Pierre-Daniel Sarte. 2017. "The Impact of Regional and Sectoral Productivity Changes on the U.S. Economy." *The Review of Economic Studies* 85 (4):2042–2096.

Carroll, Christopher D., Jody Overland, and David N. Weil. 2000. "Saving and Growth with Habit Formation." *American Economic Review* 90 (3):341–355.

Caselli, Francesco and James Feyrer. 2007. "The Marginal Product of Capital." *Quarterly Journal of Economics* 122 (2):535–568.

Chang, Yongsung, Sun-Bin Kim, and Jaewoo Lee. 2013. "Accounting for Global Dispersion of Current Accounts." *Review of Economic Dynamics* 16 (3):477–496.

Chari, V.V., Patrick J. Kehoe, and Ellen R. McGrattan. 2007. "Business Cycle Accounting." *Econometrica* 75 (3):781–836.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum. 2007. "Unbalanced Trade." *American Economic Review* 97 (2):351–355.

Domeij, David and Matin Flodén. 2006. "Population Aging and International Capital Flows." *International Economic Review* 47 (3):1013–1032.

Eaton, Jonathan and Samuel Kortum. 2002. "Technology, Geography, and Trade." *Econometrica* 70 (5):1741–1779.

Eaton, Jonathan, Samuel Kortum, and Brent Neiman. 2016. "Obstfeld and Rogoff's International Macro Puzzles: A Quantitative Assessment." *Journal of Economic Dynamics and Control* 72:5–23.

Eaton, Jonathan, Samuel Kortum, Brent Neiman, and John Romalis. 2016. "Trade and the Global Recession." *American Economic Review* 106 (11):3401–3438.

Feenstra, Robert C., Robert Inklaar, and Marcel P. Timmer. 2015. "The Next Generation of the Penn World Table." *American Economic Review* 105 (10):3150–3182.

Ferrero, Andrea. 2010. "A Structural Decomposition of the U.S. Trade Balance: Productivity, Demographics and Fiscal Policy." *Journal of Monetary Economics* 57 (4):478–490.

Gourinchas, Pierre-Olivier and Olivier Jeanne. 2013. "Capital Flows to Developing Countries: The Allocation Puzzle." *The Review of Economic Studies* 80 (4):1422–1458.

28

İmorohoğlu, Ayşe and Kai Zhao. 2018. "The Chinese Saving Rate: Long-Term Care Risks, Family Insurance, and Demographics." *Journal of Monetary Economics* 96:33–52.

———. 2020. "Household Saving, Financial Constraints, and the Current Account in China." *International Economic Review* 61 (1):71–103.

Kehoe, Timothy J., Kim J. Ruhl, and Joseph P. Steinberg. 2018. "Global Imbalances and Structural Change in the United States." *Journal of Political Economy* 126 (1):761–796.

Krueger, Dirk and Alexander Ludwig. 2007. "On the Consequences of Demographic Change for Rates of Returns to Capital, and the Distribution of Wealth and Welfare." *Journal of Monetary Economics* 54 (1):49–87.

Lane, Philip R. and Gian Maria Milesi-Ferretti. 2018. "The External Wealth of Nations Revisited: International Financial Integration in the Aftermath of the Global Financial Crisis." *IMF Economic Review* 66:189–222.

Lucas, Robert E. 1990. "Why Doesn't Capital Flow from Rich to Poor Countries?" *American Economic Review* 80 (2):92–96.

Maliar, Lilia, Serguei Maliar, John Taylor, and Inna Tsener. 2015. "A Tractable Framework for Analyzing a Class of Nonstationary Markov Models." Working Paper 21155, National Bureau of Economic Research.

Obstfeld, Maurice and Kenneth Rogoff. 2001. *The Six Major Puzzles in International Macroeconomics: Is there a Common Cause?* Cambridge, MA: MIT Press, 339–390.

Ohanian, Lee E., Paulina Restrepo-Echavarria, and Mark L. J. Wright. 2018. "Bad Investments and Missed Opportunities? Postwar Capital Flows to Asia and Latin America." *American Economic Review* 108 (12):3541–3582.

Organization for Economic Cooperation and Development. 2014. "Long-Term Baseline Projections, No. 95 (Edition 2014)." Tech. rep.

Papetti, Andrea. 2020. "Population Aging, Relative Prices and Capital Flows across the Globe." Working Paper 21155, Bank of Italy.

Prasad, Eswar S., Raghuram G. Rajan, and Arvind Subramanian. 2007. "Foreign Capital and Economic Growth." *Brookings Papers on Economic Activity* 1:153–230.

Ravikumar, B., Ana Maria Santacreu, and Michael Sposi. 2019. "Capital Accumulation and Dynamic Gains from Trade." *Journal of International Economics* 119:93–110.

Reyes-Heroles, Ricardo. 2016. "The Role of Trade Costs in the Surge of Trade Imbalances." Working paper.

Rodriguez, Maria Jose. 2019. "Investment for the Demographic Window in Latin America." Working Papers 2019-15, Banco de México.

Song, Zheng, Kjetil Storesletten, and Fabrizio Zilibotti. 2011. "Growing Like China." *American Economic Review* 101 (1):196–233.

Sposi, Michael. 2012. "Evolving Comparative Advantage, Structural Change, and the Composition of Trade." Working paper.

Steinberg, Joseph. 2019. "On the Source of U.S. Trade Deficits: Global Saving Glut or Domestic Saving Draught?" *Review of Economic Dynamics* 31:200–223.

Timmer, Marcel P., Erik Dietzenbacher, Bart Los, Robert Stehrer, and Gaaitzen J. de Vries. 2015. "An Illustrated User Guide to the World Input-Output Database: the Case of Global Automobile Production." *Review of International Economics* 23 (3):575–605.

United Nations. 2015. "World Population Prospects: The 2015 Revision, Key Findings and Advance Tables." Working Paper ESA/P/WP.241, United Nations, Department of Economic and Social Affairs, Population Division.

Wei, Shang-Jin and Xiaobo Zhang. 2011. "The Competitive Saving Motive: Evidence from Rising Sex Ratios and Savings Rates in China." *Journal of Political Economy* 119 (3):511–564.

Yaari, Menahem E. 1965. "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer." *The Review of Econmic Studies* 32 (2):137–150.

Yang, Dennis Tao, Junsen Zhang, and Shaojie Zhou. 2012. *Why Are Saving Rates So High in China?* University of Chicago Press, 249–278.

Zylkin, Thomas. 2016. "Feeding China's Rise: The Growth Effects of Trading with China, 1993-2011." Working paper.

A   Equilibrium

The set of equilibrium conditions for the aggregate variables in both the OLG and representative household models are described in Table A.1.

Table A.1: Equilibrium conditions for aggregates

F1	$r_{i,t}K_{i,t} = \alpha\nu_{i,t}P_{i,t}Y_{i,t}$	$\forall(i,t)$
F2	$w_{i,t}L_{i,t} = (1-\alpha)\nu_{i,t}P_{i,t}Y_{i,t}$	$\forall(i,t)$
F3	$P_{i,t}M_{i,t} = (1-\nu_{i,t})P_{i,t}Y_{i,t}$	$\forall(i,t)$
F4	$P_{i,t} = \gamma\left(\sum_{j=1}^{I}\left((A_{j,t})^{-\nu_{j,t}}\,u_{j,t}d_{i,j,t}\right)^{-\theta}\right)^{-\frac{1}{\theta}}$	$\forall(i,t)$
F5	$P_{i,t}^{c} = \frac{P_{i,t}}{\chi_{i,t}^{c}}$	$\forall(i,t)$
F6	$P_{i,t}^{x} = \frac{P_{i,t}}{\chi_{i,t}^{x}}$	$\forall(i,t)$
F7	$\pi_{i,j,t} = \left((A_{j,t})^{-\nu_{j,t}}\,u_{j,t}d_{i,j,t}\right)^{-\theta} \Big/ \sum_{j'=1}^{I}\left((A_{j',t})^{-\nu_{j',t}}\,u_{j',t}d_{i,j',t}\right)^{-\theta}$	$\forall(i,j,t)$
H1	$L_{i,t} = (1-\tau_{i,t}^{L})\sum_{g=1}^{G}\ell_{g}\mu_{i,g,t}N_{i,t}$	$\forall(i,t)$
H2	$P_{i,t}^{c}C_{i,t} + \mathcal{A}_{i,t+1} = \left(r_{i,t}K_{i,t} - P_{i,t}^{x}X_{i,t}\right)\left(1-\tau_{i,t}^{K}\right) + w_{i,t}L_{i,t}$	
	$\qquad\qquad +(1+q_{t})\mathcal{A}_{i,t}\left(1-\tau_{i,t}^{A}\right) + \mathcal{E}_{i,t} + T_{i,t}$	$\forall(i,t)$
H3	$K_{i,t+1} = (1-\delta)K_{i,t} + X_{i,t}$	$\forall(i,t)$
H4	$\frac{C_{i,t+1}/N_{i,t+1}}{C_{i,t}/N_{i,t}} = \left(\frac{\psi_{i,t+1}}{\psi_{i,t}}\right)\left(\frac{1+q_{t+1}}{P_{i,t+1}^{c}/P_{i,t}^{c}}\right)\left(1-\tau_{i,t+1}^{A}\right)$	$\forall(i,t)$
H5	$\frac{C_{i,t+1}/N_{i,t+1}}{C_{i,t}/N_{i,t}} = \left(\frac{\psi_{i,t+1}}{\psi_{i,t}}\right)\left(\frac{r_{i,t+1}}{P_{i,t+1}^{x}}+1-\delta\right)\left(\frac{P_{i,t+1}^{c}/P_{i,t+1}^{x}}{P_{i,t}^{x}/P_{i,t}^{c}}\right)\left(\frac{1-\tau_{i,t+1}^{K}}{1-\tau_{i,t}^{K}}\right)$	$\forall(i,t)$
H6	$\psi_{i,t} = \sum_{g=1}^{G}\left(\frac{\beta^{g-1}}{\sum_{g'=1}^{G}\beta^{g'-1}\left(1-\mathcal{T}_{i,t+g'-g}^{A}\right)}\right)\left(\frac{\mathcal{W}_{i}[t-g+1]}{1+\mathcal{Q}_{t-g+1}}\right)\mu_{i,g,t}$	$\forall(i,t)$
G1	$T_{i,t} = \left(r_{i,t}K_{i,t} - P_{i,t}^{x}X_{i,t}\right)\tau_{i,t}^{K} + (1+q_{t})\mathcal{A}_{i,t}\tau_{i,t}^{A}$	$\forall(i,t)$
M1	$P_{i,t}Q_{i,t} = P_{i,t}^{c}C_{i,t} + P_{i,t}^{x}X_{i,t} + P_{i,t}M_{i,t}$	$\forall(i,t)$
M2	$P_{i,t}Y_{i,t} = \sum_{j=1}^{I}P_{j,t}Q_{j,t}\pi_{j,i,t}$	$\forall(i,t)$
M3	$\mathcal{A}_{i,t+1} - \mathcal{A}_{i,t} = P_{i,t}Y_{i,t} - P_{i,t}Q_{i,t} + q_{t}\mathcal{A}_{i,t} + \mathcal{E}_{i,t}$	$\forall(i,t)$

Notes: $u_{j,t} = \left(\frac{r_{j,t}}{\alpha\nu_{j,t}}\right)^{\alpha\nu_{j,t}}\left(\frac{w_{j,t}}{(1-\alpha)\nu_{j,t}}\right)^{(1-\alpha)\nu_{j,t}}\left(\frac{P_{j,t}}{1-\nu_{j,t}}\right)^{1-\nu_{j,t}}$ is the unit input cost in country j at time t and $\gamma = \Gamma(1+(1-\eta)/\theta)^{1/(1-\eta)}$ is a constant, where $\Gamma(\cdot)$ is the Gamma function. In condition H6, $\mathcal{W}_{i}[t]$ is the net present value of lifetime wealth for a household born in country i at time t as defined in equation (4), $1+\mathcal{Q}_{t} = \prod_{t'=1}^{t}(1+q_{t})$ is the compounded interest rate and $1-\mathcal{T}_{t} = \prod_{t'=1}^{t}(1-\tau_{t})$ is the compounded non-taxed portion of assets.

Proof of Proposition 1. It suffices to show that the representative household economy satis-

fies the conditions in Table A.1.

First consider the firm optimality conditions. Conditions F1-F7 are unchanged since they characterize firm's optimal behavior.

Second, consider the representative household's optimality conditions. Aggregate labor supply is described by H1, by construction. The aggregate budget constraint in condition H2 is clearly satisfied. The law of motion for capital is technological, so H3 holds by construction. H6 also holds by definition, while H4 and H5 follow by solving the representative household's dynamic program.

Third, consider the government. The government's budget must balance in the aggregate, equivalent to condition G1.

Finally, consider market clearing. Conditions M1-M3 hold due to feasibility, independent of how the aggregates are constructed. □

B   Data

This section of the Appendix describes the sources of data as well as adjustments made to the data. Sources include the 2016 release of the World Input-Output Database (Timmer, Dietzenbacher, Los, Stehrer, and de Vries, 2015, (WIOD)), version 9.0 of the Penn World Table (Feenstra, Inklaar, and Timmer, 2015, (PWT)), Organization for Economic Cooperation and Development (2014) Long-Term Projections Database (OECD), 2015 revision of the United Nations (2015) World Population Prospects (UN), the International Monetary Fund Direction of Trade Statistics (IMFDOTS), Federal Reserve Economic Data (FRED), and data on external assets and liabilities from Lane and Milesi-Ferretti (2018) (LM-F). Table B.1 summarizes the data raw data.

Selection of countries is based on constructing a panel with data spanning 1970-2060. The countries (3-digit isocodes) are: Australia (AUS), Austria (AUT), Brazil (BRA), Canada (CAN), China (CHN), Denmark (DNK), Finland (FIN), France (FRA), Germany (DEU), Greece (GRC), India (IND), Indonesia (IDN), Ireland (IRL), Italy (ITA), Japan (JPN), South Korea (KOR), Mexico (MEX), Netherlands (NLD), Norway (NOR), Poland (POL), Portugal (PRT), Spain (ESP), Sweden (SWE), Switzerland (CHE), Turkey (TUR), United Kingdom (GBR), United States (USA), and Rest-of-world (ROW). Below I provide descriptions of how the data from 1970-2014 are constructed, and separately for 2015-2060.

Constructing realized data from 1970-2014

Table B.1: Model variables and corresponding data sources

Variable description	Model counterpart	Data source 1970-2014	Data source 2015-2060
Age distribution	$\mu_{i,g,t}$	UN	UN
Population	$N_{i,t}$	PWT	UN
Employment	$L_{i,t}$	PWT	OECD
Value added*	$w_{i,t}L_{i,t} + r_{i,t}K_{i,t}$	PWT & WIOD	OECD
Price of consumption**	$P_{i,t}^c$	PWT	Imputed
Price of investment**	$P_{i,t}^x$	PWT	Imputed
Price of composite intermediate**	$P_{i,t}$	PWT	Imputed
Risk free nominal interest rate	q_t	FRED	Imputed
Consumption***	$C_{i,t}$	PWT	Imputed
Investment***	$X_{i,t}$	PWT	OECD
Initial capital stock***	K_{i1}	PWT	N/A
Gross output*	$P_{i,t}Y_{i,t}$	Imputed & WIOD	Imputed
Bilateral trade flow*	$P_{i,t}Q_{i,t}\pi_{i,j,t}$	IMFDOTS & WIOD	Imputed
Absorption*	$P_{i,t}Q_{i,t}$	Imputed & WIOD	Imputed
Current account*	$\mathcal{A}_{i,t+1} - \mathcal{A}_{i,t}$	LM-F	OECD

Notes: The age distribution tracks the working age share and the retired share of the population: $\mathbf{s}_{i,t} = \left(s_{i,t}^{15-64}, s_{i,t}^{65+} \right)$. *Values are measured in current prices using market exchange rates. **Prices are measured using PPP exchange rates. ***Quantities are measured as values deflated by prices. LM-F refers to Lane and Milesi-Ferretti (2018).

- Age distribution data from 1970-2014 come from the UN. For the ROW aggregate I take the age distribution data for the "world" aggregate that the UN reports, and subtract the sum of the data for the countries in my sample.

- Population data from 1970-2014 come directly from PWT. For the ROW aggregate, the population is computed as the sum of the entire population across all countries in PWT, minus the sum of the population across countries in my sample.

- Employment data from 1970-2014 come directly from PWT. For the ROW aggregate, employment is computed as the sum of across all countries in PWT, minus the sum across countries in my sample.

- Value added in current U.S. dollars is taken from various sources. From 2000-2014, these data are obtained from WIOD and are computed as the sum of all value added across every industry in each country-year. From 1970-2000 these data are obtained

from PWT and computed as output-side real GDP at current PPP times the price level of GDP at current PPP exchange rate (relative to the U.S.). The data from PWT are multiplicatively spliced to WIOD as of the year 2000.

- Price of consumption from 1970-2014 is computed directly from PWT. For ROW, it is computed as the ratio of consumption in current prices relative to consumption in PPP prices. Consumption in ROW is computed as the sum across all countries in PWT minus the sum across countries in my sample.

- Price of investment is computed analogously to the price of consumption.

- Price of the composite intermediate good from 1970-2014 is constructed using various data in PWT. I take a weighted average of the price level of imports and of exports, each of which come directly from PWT. The weight applied to imports is the country's import share in total absorption, and the weight applied to exports is the country's home trade share in total absorption. Trade and absorption data are described below. For ROW, I compute the price of imports as the ratio of ROW imports in current prices divided by ROW imports in PPP prices. ROW imports is computed as the sum of imports across all countries in PWT minus the sum across countries in my sample.

- The risk free nominal rate of interest is defined as the annual yield on the 10-year U.S. Treasury note, taken from Federal Reserve Economic Data (FRED).

- Consumption quantities are calculated in two steps. The first step computes total consumption expenditures as total value added minus investment expenditures, minus net exports. The second step deflates the consumption expenditure by the price of consumption. This way, consumption includes that of both households and government, and necessarily ensures that the national income identity holds in both the data and in the model.

- Investment quantities from 1970-2014 are computed from various data in the PWT. I begin by computing the nominal investment rate (ratio of expenditures on investment as a share of GDP in current prices). I then multiply the nominal investment rate by GDP in current U.S. dollars to arrive at total investment spending in current U.S. dollars. Finally, I deflate the current investment expenditures by the price of investment.

- Initial capital stock is taken directly from PWT. Capital stock in ROW is computed as the sum across all countries in PWT minus the sum across countries in my sample.

- Gross output in current U.S. dollars from 2000-2014 is obtained from WIOD and is computed as the sum of all gross output across every industry in each country-year. Prior to 2000, I impute these data using the ratio of value added to gross output in 2000, and applying that ratio to scale value added in each year prior to 2000.

- Bilateral trade in current U.S dollars from 2000-2014 is computed directly from WIOD as the sum of all trade flows (intermediate usage and final usage) across all industries (goods and services). Prior to 2000, bilateral trade flows are obtained from the IMFDOTS, which includes only trade in merchandise. These data are multiplicatively spliced to the WIOD data as of the year 2000. Bilateral trade flows with ROW are computed as imports (exports) to (from) the world, minus the sum of imports (exports) to (from) the countries in my sample.

- Absorption in current U.S. dollars from 2000-2014 is computed using WIOD data as gross output minus net exports, summed across all industries.

- Current account in U.S. dollars from 1970-2014 comes from LM-F. I use the variable "Current account balance" when available, and using the variable "CA balance (WEO)" when the former is missing.

Constructing projected data from 2015-2060

- Age distribution data from 2015-2060 come from the UN. For the ROW aggregate I take the age distribution data for the "world" aggregate that the UN reports, and subtract the sum of the data for the countries in my sample.

- Population data from 2015-2060 are taken directly from UN and spliced to the PWT levels as of the year 2014.

- Employment data from 2015-2060 are taken directly from OECD and spliced to the PWT levels as of the year 2014.

- From 2015-2060, data on value added in current U.S. dollars are obtained from OECD projections and computed as real GDP per capita (in constant, local currency units) times the population, times the price level (in local currency units), times the PPP

level (relative to the U.S.), times the nominal exchange rate (local currency per U.S. dollar at current market prices). The data from OECD are multiplicatively spliced to WIOD as of the year 2014.

- Price of consumption from 2015-2060 is imputed by assuming equal growth rates to the price deflator for aggregate GDP, where the price deflator for aggregate GDP is directly computed using data in the OECD projections.

- Price of investment from 2015-2060 is imputed using information on its co-movement with the price of consumption. In particular, I estimate the relationship between growth in the relative price against a constant and a one-year lag in the relative price growth, for the years 1972-2014.

$$\ln\left(\frac{P_{i,t}^c/P_{i,t}^x}{P_{i,t-1}^c/P_{i,t-1}^x}\right) = \beta_0 + \beta_1 \ln\left(\frac{P_{i,t-1}^c/P_{i,t-1}^x}{P_{i,t-2}^c/P_{i,t-2}^x}\right) + \epsilon_{i,t}. \tag{B.1}$$

I use the estimates from equation (B.1) to impute the sequence of prices for investment from 2015-2060, given the already imputed data for the price of consumption during these years.

- Price of the composite intermediate good from 2015-2060 is imputed by first constructing data for the price of imports and exports. Prices of imports and exports are each computed analogously to the price of investment by estimating equation (B.1) for each series. I then take a weighted average of the price levels of imports and of exports to determine the price of the composite good. The weight applied to imports is the country's import share in total absorption, and the weight applied to exports is the country's home trade share in total absorption. Trade and absorption data are described below.

- The risk free nominal rate of interest is constant from 2015-2060 at a value of 2.14 percent.

- Consumption quantities from 2015-2060 are calculated in the same exact way as done for 1970-2014.

- Investment quantities from 2015-2060 are computed from various variables in the OECD projections. I begin by imputing the nominal investment rate (ratio of expenditures on investment to GDP in current prices) using information on its co-movement with the relative prices. I estimate the relationship between the investment rate against

a country-fixed effect, the lagged investment rate, the contemporaneous and lagged relative price of investment, and the contemporaneous and lagged real GDP per capita for the years 1971-2014. Letting $\rho_{i,t} = \frac{P^x_{i,t} X_{i,t}}{GDP_{i,t}}$ denote the investment rate,

$$
\begin{aligned}
\ln\left(\frac{\rho_{i,t}}{1-\rho_{i,t}}\right) &= \alpha_i + \beta_1 \ln\left(\frac{\rho_{i,t-1}}{1-\rho_{i,t-1}}\right) \\
&+ \beta_2 \ln\left(\frac{P^x_{i,t}}{P^c_{i,t}}\right) + \beta_3 \ln\left(\frac{P^x_{i,t-1}}{P^c_{i,t-1}}\right) + \beta_4 \ln(y_{i,t}) + \beta_5 \ln(y_{i,t-1}) + \epsilon_{i,t} \quad (B.2)
\end{aligned}
$$

Using $\ln(\rho/(1-\rho))$ to ensures that the imputed values of ρ are bounded between 0 and 1. I use the estimated coefficients from equation (B.2) together with projections on the relative price and income per capita to construct projections for the investment rate. I then multiply the nominal investment rate by GDP in current U.S. dollars (available in the OECD projections) to arrive at total investment spending in current U.S. dollars. Finally, I deflate the investment expenditures by the price of investment.

- Gross output in current U.S. dollars from 2015-2060 is imputed using the ratio of value added to gross output in 2014, and applying that ratio to scale value added in each year after 2014. The value added data (GDP in current U.S. dollars) after 2014 is obtained directly from OECD projections.

- Bilateral trade in current U.S dollars from 2015-2060 are constructed in multiple steps. First, let $x_{i,j,t} = \frac{X_{i,j,t}}{GO_{j,t}-EXP_{j,t}}$ be the ratio of country j's exports to country i, relative to country j's gross output net of its total exports. I then estimate how changes in this trade share co-moves with changes in the importer's aggregate import price index, changes in the exporter's aggregate export price index, and changes in both the importer's and exporter's levels of GDP:

$$
\begin{aligned}
\ln\left(\frac{x_{i,j,t}}{x_{i,j,t-1}}\right) &= \beta_1 \ln\left(\frac{P^m_{i,t}}{P^m_{i,t-1}}\right) + \beta_2 \ln\left(\frac{P^x_{j,t}}{P^x_{j,t-1}}\right) \\
&+ \beta_3 \ln\left(\frac{GDP_{i,t}}{GDP_{i,t-1}}\right) + \beta_4 \ln\left(\frac{GDP_{j,t}}{GDP_{j,t-1}}\right) + \epsilon_{i,t}. \quad (B.3)
\end{aligned}
$$

I use the estimated coefficients from equation (B.3) together with projections of prices of imports and of exports and levels of GDP to construct trade shares from 2015-2060.

In the second step, I use the fact that country i's domestic sales is determined by

$$X_{i,i,t} = \frac{GO_{i,t}}{1 + \sum_{j \neq i} x_{j,i,t}},$$

where gross output and trade shares from 2015-2060 have already been constructed. Finally, given projected domestic sales and trade shares, the bilateral trade flow is

$$X_{i,j,t} = X_{jj,t} x_{i,j,t}.$$

- Absorption in current U.S. dollars from 2015-2060 is gross output minus net exports.

- Current account in U.S. dollars from 2015-2016 are taken directly from OECD and spliced to the data from LM-F.

C   Equilibrium conditions

This section describes the solution to a perfect foresight equilibrium.

Remark   The world interest rate is strictly nominal. As such, the value plays essentially no role other than pinning down a numéraire. Since my choice of numéraire is world GDP in each period, the world interest rate reflects the relative valuation of world GDP at two points in time. This interpretation is useful in guiding the solution procedure and also makes for straightforward mapping between model and data. That is, in the model the prices map into current units, as opposed to constant units. In other words, the model can be rewritten so that all prices are quoted in time-1 units (like an Arrow-Debreu world) with the world interest rate of zero and the equilibrium would yield identical quantities.

D   Calibration procedure

This section describes the block recursive procedure for calibrating the country-specific and time-varying parameters of the model.

Initial conditions and demographics   For each country the initial stock of capital, K_{i1}, initial net-foreign asset position, $\mathcal{A}_{i1}$, are taken directly from the data in 1970. In terms of demographics, both the population, $N_{i,t}$, and the age distribution, $\mu_{i,g,t}$, are observable.

Value added shares and relative prices The parameter $\nu_{i,t}$ is defined as the ratio of aggregate value added to gross production, and is constructed as such (condition F3 in Table A.1). The (inverse) relative price of consumption, $\chi^c_{i,t}$, is computed as the price of intermediates relative to consumption (condition F5). Similarly, $\chi^x_{i,t}$ is the price of intermediates relative to investment (condition F6). Prices of intermediates, consumption, and investment are observable.

Labor distortions Given labor supply over the life cycle, ℓ_g, along with the demographic structure of each country $(N_{i,t}, \mu_{i,g,t})$, labor distortions $(\tau^L_{i,t})$ are computed directly from aggregate employment data using condition H1 in Table A.1. Recall that the aggregate employment-population ratio is decomposed into a demographic component (working age share) and a discretionary component.

Saving distortions The saving distortions can be recovered using the intertemporal Euler equation (9). First, I recover a "wedge", $\omega^{\mathcal{A}}_{i,t+1} = \left(\frac{\psi_{i,t+1}}{\psi_{i,t}}\right)\left(1 - \tau^{\mathcal{A}}_{i,t+1}\right)$, which is comprised on the age distribution, saving distortions, and various endogenous variables including the interest rate and income. The saving wedge can be computed given a couple of common parameters and data on a risk-free interest rate, consumer price growth, and per-capita consumption growth. The interest rate is the annual, nominal yield on the 10-year U.S. Treasury note. Consumption—sum of both private and public—is measured as GDP minus investment expenditures minus net exports, all divided by the consumer price level. The consumer price level is measured in 2011 U.S. dollars at current purchasing power parities (PPP).

Given the fact that the preference shifters are a function of the distortions, along with various observables:

$$\psi_{i,t}\left(\{\tau^{\mathcal{A}}_{i,t}\}; \text{data}\right) = \sum_{g=1}^{G}\left(\frac{\beta^{g-1}}{\sum_{g'=1}^{G}\beta^{g'-1}\left(1 - \mathcal{T}^{\mathcal{A}}_{i,t+g'-g}\right)}\right)\left(\frac{\mathcal{W}_i[t-g+1]}{1+\mathcal{Q}_{t-g+1}}\right)\mu_{i,g,t},$$

the distortions can be isolated using an iterative procedure.[16] The net-present value of lifetime wealth is $\mathcal{W}_i[t] = \sum_{g=1}^{G}\frac{(1-\rho_{i,t})(1-\tau^L_{i,t})w_{i,t+g-1}\ell_g/(1-\alpha)}{1+\mathcal{Q}_t}$, where $\rho_{i,t} = \frac{P^x_{i,t}X_{i,t}}{r_{i,t}K_{i,t}+w_{i,t}L_{i,t}}$ is the

---

[16]First make an initial guess for the entire sequence of distortions, $\{\tau^{\mathcal{A}}_{i,t}\}$, with $\tau^{\mathcal{A}}_{i,1} = 0$. Then compute the preference shifters as a function of those distortions and observed data: $\psi_{i,t}\left(\{\tau^{\mathcal{A}}\}; \text{data}\right)$. Then update the guess using $1 - \tau^{\mathcal{A},\text{new}}_{i,t} = \frac{\psi_{i,t-1}\left(\{\tau^{\mathcal{A}}_{i,t}\}; \text{data}\right)}{\psi_{i,t}\left(\{\tau^{\mathcal{A}}_{i,t}\}; \text{data}\right)}\omega^{\mathcal{A}}_{i,t}$, where $\omega^{\mathcal{A}}_{i,t}$ is the "fixed" wedge backed out from the Euler equation. These recursions converge to a fixed point for the entire sequence of distortions rather quickly.

aggregate investment rate and $(1 - \tau_{i,t}^L)w_{i,t+g-1}\ell_{i,t+g-1}/(1-\alpha) = r_{i,t}K_{i,t} + w_{i,t}L_{i,t}$ due to firm optimality conditions F1 and F2. Data on the wage rate and aggregate investment rate are both observable.[17]

The preference shifter in any one period involves the intricate inetraction between the history of the age distribution along with histories of saving distortions and income. Even then, there are clear patterns that emerge. The counterfactuals in the next section will isolate the contribution from the age distribution.

Investment distortions Without loss of generality, I initialize $\tau_{i,1}^K = 0$. The remaining investment distortions require measurements of the capital stock in every period. Given capital stocks at time 1, K_{i1}, and data on investment in physical capital, $X_{i,t}$, I construct the sequence of capital stocks iteratively using $K_{i,t+1} = (1-\delta)K_{i,t} + X_{i,t}$.

Given the constructed sequence of capital stocks, I recover $\tau_{i,t+1}^K$ iteratively using the Euler condition H5 for investment in physical capital. Prices of consumption and investment come from the national accounts data, as does per-capita consumption. The rental rate for capital, defined according to the theory, is constructed as: $r_{i,t} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_{i,t}L_{i,t}}{K_{i,t}}\right)$.

Excess returns (transfers) The excess return on assets (transfers) for country i at time t, $\mathcal{E}_{i,t}$, absorbs the difference between the observed current account and net exports after imposing the restriction of a common world interest rate in the model:

$$\mathcal{E}_{i,t} = q_t\mathcal{A}_{i,t} + NX_{i,t} - CA_{i,t}$$

where NX and CA are observed net exports and current account, respectively. The interest rate q_t is the annual yield on the 10-year U.S. treasury bond. The net foreign asset position begins at zero in 1970. In each successive period it is computed as $\mathcal{A}_{i,t+1} = \mathcal{A}_{i,t} + CA_{i,t}$. SInce data on current accounts and net exports both sum to zero across countries in every period, the excess returns (transfers) do as well.

---

[17]This requires information about the distortions and prices faced by the "initial old" prior to 1970. That is, in the initial year, 1970, one needs information about the net present value of lifetime income for 85 year-olds, which requires making assumptions about certain variables in the year 1886. For such calculations I assume the data are constant over time prior to 1970 and that distortions are zero.

Trade costs   The trade cost for any given country pair is computed using data on prices and bilateral trade shares using the following structural equation:

$$\frac{\pi_{i,j,t}}{\pi_{j,j,t}} = \left(\frac{P_{j,t}}{P_{i,t}}\right)^{-\theta} d_{i,j,t}^{-\theta},$$

where $\pi_{i,j,t}$ is the share of country i's absorption that is sourced from country j and $P_{i,t}$ is the price of tradables in country i. I set $d_{i,j,t} = 10^8$ for observations in which $\pi_{i,j,t} = 0$ and set $d_{i,j,t} = 1$ if the inferred value is less than 1. As a normalization, $d_{i,i,t} = 1$.

Productivity   I back out productivity, $A_{i,t}$, using price data and home trade shares,

$$P_{i,t} = \left(\frac{\gamma\,(\pi_{i,i,t})^{\frac{1}{\theta}}}{(A_{i,t})^{\nu_{i,t}}}\right) \left(\frac{r_{i,t}}{\alpha\nu_{i,t}}\right)^{\alpha\nu_{i,t}} \left(\frac{w_{i,t}}{(1-\alpha)\nu_{i,t}}\right)^{(1-\alpha)\nu_{i,t}} \left(\frac{P_{i,t}}{1-\nu_{i,t}}\right)^{1-\nu_{i,t}}.$$

More simply, $P_{i,t} = \frac{u_{i,t}}{Z_{i,t}}$, where $u_{i,t}$ is the unit cost of an input bundle and $Z_{i,t} = \left(\frac{(A_{i,t})^{\nu_{i,t}}}{\gamma(\pi_{i,i,t})^{\frac{1}{\theta}}}\right)$ is measured productivity of gross output.[18]

E   Solution algorithm

In this section of the Appendix I describe the algorithm for computing the equilibrium transition path. Before going further into the algorithm, I introduce some notation. I denote the cross-country vector of a given variable at a point in time using vector notation, i.e., $\vec{K}_t = \{K_{i,t}\}_{i=1}^{I}$ is the vector of capital stocks across countries at time t.

E.1   Computing the equilibrium transition path

Given the initial conditions—$(\vec{K}_1, \vec{\mathcal{A}}_1)$—the equilibrium transition path consists of 18 objects: $\{\vec{w}_t, \vec{r}_t, q_t, \vec{\psi}_t \vec{P}_t, \vec{P}_t^c, \vec{P}_t^x, \vec{Y}_t, \vec{Q}_t, \vec{C}_t, \vec{X}_t, \vec{K}_{t+1}, \vec{L}_t, \vec{M}_t, \vec{B}_t, \vec{\mathcal{A}}_{t+1}, \vec{T}_t, \vec{\overset{\rightrightarrows}{\pi}}_t\}_{t=1}^{T}$ (the double-arrow notation on $\vec{\overset{\rightrightarrows}{\pi}}_t$ indicates that this is an $I \times I$ matrix in each period t). Table A.1 provides a list of 17 equilibrium conditions that these objects must satisfy.

The solution procedure is boils down to two loops, similar to the algorithm in Ravikumar, Santacreu, and Sposi (2019). The outer loop iterates on the rate of investment in physical

---

[18]The parameter $\gamma = \Gamma(1 + (1-\eta)/\theta)^{1/(1-\eta)}$ is a constant, where $\Gamma(\cdot)$ is the Gamma function.

capital. The inner loop iterates on a subset of prices as in Sposi (2012), given the guess for investment rates.

First, for the outer loop, guess at the sequence of investment rates for every country in every period. Given the investment rate, within the inner loop start with an initial guess for the entire sequences of wage vectors and the world interest rate on bonds. Form these two objects, recover all remaining prices and quantities, across countries and throughout time, using optimality conditions and market clearing conditions, excluding the balance of payments condition. Then use departures from the the balance of payments condition to update the wages, and use deviations from intertemporal price relationships to update the world interest rate. Iterate on this until the wages and world interest rate satisfy the balance of payments condition and the intertemporal condition for prices. Then back up to the outer loop and check if the investment rate decisions satisfy optimality. If not, update the guess of investment rates and solve the inner loop again. The details to this procedure follow, making reference to equilibrium conditions in Table A.1.

1. Solve for aggregate labor supply in every country, $\{\vec{L}_t\}_{t=1}^T$ as a function of the age distribution and labor distortions:

$$L_{i,t} = (1 - \tau_{i,t}^L) \sum_{g=1}^G \ell_g \mu_{i,g,t} N_{i,t}.$$

Guess a sequence of nominal investment rates, $\{\vec{\rho}_t\}_{t=1}^T$ with

$$0 < \rho_{i,t} = \frac{P_{i,t}^x X_{i,t}}{r_{i,t} K_{i,t} + w_{i,t} L_{i,t}} < 1,$$

and guess at a terminal net-foreign asset position in each country, $\mathcal{A}_{i,T+1}$, with $\sum_{i=1}^I \mathcal{A}_{i,T+1} = 0$. Take these as given for the next sequence of steps.

   (a) Guess the entire paths for wages, $\{\vec{w}_t\}_{t=1}^T$, across countries, and the world interest rate, $\{q_t\}_{t=2}^T$, such that $\sum_i \frac{w_{i,t} L_{i,t}}{1-\alpha} = 1$, for all t (world GDP is the numéraire in each period.

   (b) In period 1 use conditions F1 and F2 and set $\vec{r}_1 = \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\vec{w}_1 \vec{H}_1}{\vec{K}_1}\right)$, where the initial stock of capital is predetermined. Compute prices $\vec{P}_1, \vec{P}_1^c, \vec{P}_1^x$ using conditions F4, F5, and F6.

   (c) Solve for the endogenous preference shifters, $\frac{\vec{\psi}_2}{\vec{\psi}_1}$, using condition H6.

(d) Solve for physical investment, $\vec{X}_1$, using the guess for the nominal investment rate together with prices: $\vec{X}_1 = \frac{\vec{\rho}_1 \vec{w}_1 \vec{L}_1}{(1-\alpha)\vec{P}_1^x}$. Given the investment, solve for the next-period capital stock, $\vec{K}_2$, using condition H3. Repeat this set of calculations for period 2, then for period 3, and all the way through period T to construct the entire sequence of investment and capital.

(e) Given prices, compute the bilateral trade shares $\{\vec{\pi}_t\}_{t=1}^T$ using condition F7.

(f) This step is slightly more involved. I show how to compute the path for consumption and bond purchases by solving the intertemporal problem of the household. This is done in three parts. First I derive the lifetime budget constraint, second I derive the fraction of lifetime wealth allocated to consumption at each period t, and third I recover the sequences for bond purchases and the stock of net-foreign assets.

Deriving the lifetime budget constraint   To begin, compute the lifetime budget constraint for the representative household (omitting country subscripts for now). Begin with the period budget constraint from condition H2 and combine it with the the balanced tax revenue in condition G1:

$$\mathcal{A}_{t+1} = r_t K_t - P_t^x X_t + w_t L_t - P_t^c C_t + (1+q_t)\mathcal{A}_t + \mathcal{E}_t.$$

Iterate the period budget constraint forward through time and derive a lifetime budget constraint. Given $A_{i1} > 0$, compute the net-foreign asset position at time $t = 2$:

$$\mathcal{A}_2 \;=\; r_1 K_1 - P_1^x X_1 + w_1 L_1 - P_1^c C_1 + (1+q_t)\mathcal{A}_1 + \mathcal{E}_1.$$

Similarly, compute the net-foreign asset position at time $t = 3$:

$$
\begin{aligned}
\mathcal{A}_3 \;&=\; r_2 K_2 - P_2^x X_2 + w_2 L_2 - P_2^c C_2 + (1+q_2)\mathcal{A}_2 + \mathcal{E}_2 \\
\Rightarrow \mathcal{A}_3 \;&=\; r_2 K_2 - P_2^x X_2 + w_2 L_2 + (1+q_2)\left(r_1 K_1 - P_1^x X_1 + w_1 L_1\right) \qquad \text{(E.1)} \\
&\;-\; P_2^c C_2 - (1+q_2)P_1^c C_1 + (1+q_2)(1+q_1)\mathcal{A}_{i1} + (1+q_2)\mathcal{E}_1 + \mathcal{E}_2.
\end{aligned}
$$

It is useful to define $(1 + Q_t) = \prod_{n=1}^{t}(1 + q_n)$. By induction, for any time t,

$$
\mathcal{A}_{t+1} = \sum_{t'=1}^{t} \frac{(1 + \mathcal{Q}_t)\left(r_{t'}K_{t'} - P_{t'}^x X_{t'} + w_{t'}L_{t'} + \mathcal{E}_{t'}\right)}{1 + \mathcal{Q}_{t'}} \tag{E.2}
$$

$$
- \sum_{t'=1}^{t} \frac{(1 + \mathcal{Q}_t)P_{t'}^c C_{t'}}{1 + \mathcal{Q}_{t'}} + (1 + \mathcal{Q}_t)\mathcal{A}_1
$$

$$
\Rightarrow \mathcal{A}_{t+1} = (1 + \mathcal{Q}_t)\left(\sum_{t'=1}^{t} \frac{r_{t'}K_{t'} - P_{t'}^x X_{t'} + w_{t'}L_{t'} + \mathcal{E}_{t'}}{1 + \mathcal{Q}_{t'}} - \sum_{t'=1}^{t} \frac{P_{t'}^c C_{t'}}{1 + \mathcal{Q}_{t'}} + \mathcal{A}_1\right).
$$

Observing the previous expression as of $t = T$ yields the lifetime budget constraint:

$$
\sum_{t'=1}^{T} \frac{P_{t'}^c C_{t'}}{1 + \mathcal{Q}_{t'}} = \underbrace{\sum_{t'=1}^{T} \frac{r_{t'}K_{t'} - P_{t'}^x X_{t'} + w_{t'}L_{t'} + \mathcal{E}_{t'}}{1 + \mathcal{Q}_{t'}} + \mathcal{A}_1 - \frac{\mathcal{A}_{T+1}}{1 + \mathcal{Q}_T}}_{\mathcal{W}}. \tag{E.3}
$$

In the lifetime budget constraint (E.3), $\mathcal{W}$ denotes the net present value of lifetime wealth, taking both the initial and terminal net-foreign asset positions as given.

Solving for the path of consumption Next, compute how the net-present value of lifetime wealth is optimally allocated throughout time. For this it is useful to define $(1 - \mathcal{T}_t^A) = \prod_{t'=1}^{t}(1 - \tau_{t'}^A)$. The Euler equation for bonds (condition H4) implies the following relationship between any two periods t and t':

$$
C_{t'} = \left(\frac{N_{t'}}{N_t}\right)\left(\frac{\psi_{t'}}{\psi_t}\right)\left(\frac{1 - \mathcal{T}_{t'}^A}{1 - \mathcal{T}_t^A}\right)\left(\frac{1 + \mathcal{Q}_{t'}}{1 + \mathcal{Q}_t}\right)\left(\frac{P_t^c}{P_{t'}^c}\right)C_t
$$

$$
\Rightarrow \frac{P_{t'}^c C_{t'}}{1 + \mathcal{Q}_{t'}} = \left(\frac{N_{t'}}{N_t}\right)\left(\frac{\psi_{t'}}{\psi_t}\right)^{\sigma}\left(\frac{1 - \mathcal{T}_{t'}^A}{1 - \mathcal{T}_t^A}\right)\left(\frac{P_t^c C_t}{1 + \mathcal{Q}_t}\right).
$$

Since equation (E.3) implies that $\sum_{t'=1}^{T} \frac{P_{i,t'}^c C_{i,t'}}{1+\mathcal{Q}_{t'}} = \mathcal{W}_i$, rearrange the previous expression (putting country subscripts back in) to obtain

$$
\frac{P_{i,t}^c C_{i,t}}{1 + \mathcal{Q}_{i,t}} = \underbrace{\left(\frac{N_{i,t}\psi_{i,t}\left(1 - \mathcal{T}_{i,t}^A\right)}{\sum_{t'=1}^{T} N_{i,t'}\psi_{i,t'}\left(1 - \mathcal{T}_{i,t'}^A\right)}\right)}_{\xi_{i,t}} \mathcal{W}_i. \tag{E.4}
$$

That is, in period t, country i's net-present value of consumption spending is a fraction, $\xi_{i,t}$, of its lifetime wealth, with $\sum_{t=1}^{T} \xi_{i,t} = 1$ for all i.

Computing the net-foreign asset positions In period 1 take as given consumption spending, investment spending, capital income, labor income, and net income from the initial net-foreign asset position; each of which have already been computed in previous steps. Then solve for next period's net-foreign asset position, $\vec{\mathcal{A}}_2$, using the period budget constraint in condition H2. Repeat this set of calculations iteratively for periods $2, \ldots, T$.

(g) Given final demand $\{\vec{C}_t, \vec{X}_t\}_{t=1}^{T}$ and prices, solve for gross absorption and gross production, $\{\vec{Q}_t, \vec{Y}_t\}_{t=1}^{T}$, and intermediate-input demand, $\{\vec{M}_t\}_{t=1}^{T}$ using conditions F3, M1, and M2.

(h) Given gross national income, calculate the tax rebates, $\vec{T}_{i,t}$, using condition G1.

(i) Update the guesses for wages, the interest rate, and tax rebates.

Balance of payments condition I generalize Alvarez and Lucas (2007) and compute an excess demand equation by imposing that net exports equal the current account less net-foreign income from assets.

$$Z_{i,t}^{w}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right) = \frac{P_{i,t}Y_{i,t} - P_{i,t}Q_{i,t} + q_t\mathcal{A}_{i,t} + \mathcal{E}_{i,t} - \mathcal{A}_{i,t+1} + \mathcal{A}_{i,t}}{w_{i,t}}.$$

Condition M3 requires that $Z_{i,t}^{w}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right) = 0$ for all (i,t). If this is different from zero in at some country at some point in time, update the wages:

$$1 + w_{i,t}^{new} = \Lambda_{i,t}^{w}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right) = \left(1 + \kappa\frac{Z_{i,t}^{w}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right)}{L_{i,t}}\right)w_{i,t}$$

where κ is chosen to be sufficiently small to ensure that $\Lambda^w > 0$.

Normalizing model units The last part of this step updates the world interest rate. Recall that the numéraire is defined to be world GDP at each point in time: $\sum_{i=1}^{I}(r_{i,t}K_{i,t} + w_{i,t}L_{i,t}) = 1$. For an arbitrary sequence of $\{q_{t+1}\}_{t=1}^{T}$, this condition

need not hold. Update the the world interest rate:

$$q_t^{new} = \Lambda_{i,t}^q \left(\{\vec{w}_t, q_t\}_{t=1}^T \right) = \frac{\sum_{i=1}^I (r_{i,t-1} K_{i,t-1} + \Lambda_{i,t-1}^w L_{i,t-1})}{\sum_{i=1}^I (r_{i,t} K_{i,t} + \Lambda_{i,t}^w L_{i,t})} (1 + q_t) - 1 \; (t \geq 2)$$

and $q_1^{new} = q_1$. The values for capital stock and the rental rate of capital are computed in step 2, while the values for wages are the updated values Λ^w above. I set $q_1 = \frac{1-\beta}{\beta}$ (the interest rate that prevails in a steady state) and chose $A_{i,1}$ so that $q_1 A_{i,1}$ matches the desired initial net-foreign asset position in current prices.

Having updated the wages, the world interest rate, and tax rebates, repeat steps 1b-1g. Iterate through this procedure until the excess demand is sufficiently close to zero. In the computations I find that my preferred convergence metric:

$$\max_{t=1}^T \left\{ \max_{i=1}^I \left\{ |Z_{i,t}^w \left(\{\vec{w}_t, q_t\}_{t=1}^T \right)| \right\} \right\}$$

converges roughly monotonically towards zero.

2. The last step of the algorithm is to update the investment rate and terminal net-foreign asset position. Until now, the optimality condition H5 for the investment in physical capital have not been used. To this end, compute a "residual" from each of these first-order conditions as

$$
\begin{aligned}
Z_{i,t}^\rho \left(\{\vec{\rho}_t\}_{t=1}^T \right) &= \left(\frac{\psi_{i,t+1}}{\psi_{i,t}} \right) \left(\frac{r_{i,t+1}}{P_{i,t+1}^x} + (1 - \delta) \right) \left(\frac{P_{i,t+1}^x / P_{i,t+1}^c}{P_{i,t}^x / P_{i,t}^c} \right) \left(\frac{1 - \tau_{i,t+1}^K}{1 - \tau_{i,t}^K} \right) \\
&- \left(\frac{C_{i,t+1}/N_{i,t+1}}{C_{i,t}/N_{i,t}} \right).
\end{aligned}
$$

Condition H5 requires that $Z_{i,t}^\rho \left(\{\vec{\rho}_t\}_{t=1}^T \right) = 0$. Update the investment rate as

$$\rho_{i,t}^{new} = \Lambda_{i,t}^\rho \left(\{\vec{\rho}_t\}_{t=1}^T \right) = \left(1 + \kappa Z_{i,t}^\rho \left(\{\vec{\rho}_t\}_{t=1}^T \right) \right) \rho_{i,t},$$

where κ is a constant value small enough to ensure that the updated guesses remain positive. Given the updated sequence of labor supply and investment rate, return to step 1 using the updated labor supply and investment rate as the "guess" for the next iteration. Iterate until condition H5 holds.

With T chosen to be sufficiently large, the turnpike theorem implies that the terminal

net-foreign asset position has minimal bearing on the transition path up to some time $t^\star < T$ (see Maliar, Maliar, Taylor, and Tsener, 2015). Therefore I update $\mathcal{A}_{i,T+1}$ to be the value of $\mathcal{A}_{i,T}$ from the previous iteration to smooth out the transition in the final periods.

F Extensions

F.1 The allocation puzzle

Standard neoclassical theory predicts that capital should flow from countries with low productivity growth to countries with high productivity growth. That is, countries with relatively higher productivity growth should run a current account deficit. However, as has been documented by Prasad, Rajan, and Subramanian (2007) and Gourinchas and Jeanne (2013), such a correlation is not present in the data. Figure F.1 depicts the lack of a negative correlation between the ratio of current account to GDP and TFP growth – also known as the *allocation puzzle*. If anything, the correlation is positive; The elasticity of the ratio of current account to GDP with respect to TFP growth (slope of the best fit line) is 29.2%.

Figure F.1: Ratio of current account to GDP against TFP growth
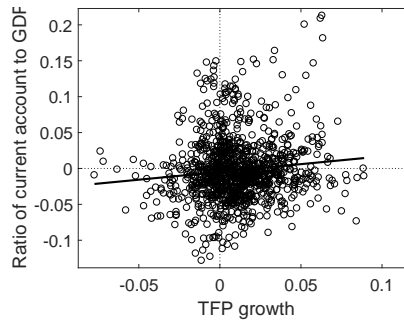
Notes: Horizontal axis is the average annual growth in labor productivity during five year windows. Vertical axis is the average ratio of net exports to GDP during five year windows. Windows run from [1970,1974]-[2010,2014]. The line corresponds to the best fit curve using OLS.

This section explores how different exogenous forces contribute to the allocation puzzle by simulating scenarios in which a given exogenous force is counterfactually changed. In each scenario I compute the resulting elasticity of the ratio of current account to GDP with respect to TFP growth and report the results in Table F.1.

Table F.1: Elasticity between ratio of current account to GDP and TFP growth

Specification	Elasticity	(S.E.)
Baseline model	29.2%	(6.0%)
Every country's age distribution set to average of world	27.4%	(9.5%)
Emr. country's age distribution set to average of adv. countries	10.1%	(10.3%)
Adv. country's age distribution set to average of emr. countries	23.0%	(11.0%)
Constant age distribution	97.8%	(19.4%)
Half saving distortions	−72.5%	(13.3%)
Half labor distortions	28.6%	(6.6%)
Half investment distortions	24.3%	(8.7%)
Half bilateral trade distortions	50.2%	(8.6%)
Constant average trade costs	36.1%	(8.1%)

Notes: The elasticity is the slope of the line corresponding to the best fit curve using OLS (see Figure F.1 for the case of the baseline model). The y-variable in the regression is the average annual growth in TFP during five year windows. The x-variable in the regression is the average ratio of net exports to GDP during five year windows. Windows run from [1970,1974]-[2010,2014].

Counterfactual age distribution   I consider four counterfactual scenarios pertaining to the age distribution. In the first scenario the age distribution of every country is set equal to that of the world, as in the in the previous section. In this scenario the relationship between capital flows and TFP growth is similar to that in the baseline scenario.

In the second scenario the age distribution of every emerging economy is set equal to the average (population weighted) age distribution of advanced economies, while the age distribution of every advanced economy is held at observed values. In this scenario the relationship between capital flows and TFP growth is moderately lower than that in the baseline scenario. The lack of capital flows from slow- to fast-growing countries is partially explained by demographic forces in emerging economies. That is, the observed demographic structure in emerging economies facilitates a demand for saving among them, drawing capital flows from advanced economies.

In the third scenario the age distribution of every advanced economy is set equal to the average (population weighted) age distribution of emerging economies, while the age distribution of every emerging economy is held at observed values. In this scenario the relationship between capital flows and TFP growth is slightly lower than, but not much different from, that in the baseline scenario. Since the world age distribution resembles that of emerging economies more-so than that of advanced economies, the end result is similar to that in the first scenario.

In the fourth scenario the age distribution of every country is held constant over time at observed 1970 values. This scenario isolates differences in "changes" over time from differences in "levels" of the age distribution across countries. In this scenario the elasticity is higher than in the baseline suggesting that the allocation puzzle is not explained by differential *changes* in demographics over time, but more-so by differences in the *levels* of the age distribution across countries.

Counterfactual saving distortions   In this counterfactual I reduce the saving distortion in each country by half: $\widetilde{\tau}_{i,t}^A = \tau_{i,t}^A/2$. The relationship between current account imbalances and TFP growth becomes strongly negative, with an elasticity of -72.5%. This finding suggests that the saving distortion encompasses important frictions that give rise to the allocation puzzle. This is no surprise, given the findings in Gourinchas and Jeanne (2013). However, there is a subtle but important difference here: The saving distortion in my model is only one part of the overall saving wedge explored by Gourinchas and Jeanne (2013). The other part is the demographic-driven, endogenous discount factor. Demographics chip away at saving wedge in this sense, but a sizable chunk remains to be explained.

There is room for more analysis to open up the saving wedge further, perhaps along the lines of public saving, as suggested by Alfaro, Kalemli-Ozcan, and Volosovych (2008). While large portions of government saving are influenced by demographics through pensions, government saving is likely affected by non-demographic forces such as un-modeled political economy features that correlate with productivity. Another possibility is that the consumption-saving trade-off depends on past levels of consumption. Carroll, Overland, and Weil (2000) posit that habits can account for the fact that a boom income is not immediately met with a boom in consumption. Such mechanisms are beyond the scope of this paper.

Counterfactual labor distortions   In this counterfactual I reduce the labor distortion in each country by half: $\widetilde{\tau}_{i,t}^L = \tau_{i,t}^L/2$. The elasticity of the ratio of current account to GDP with respect to TFP growth is 28.6%. This elasticity is similar to that in the baseline, implying that labor distortions do not systematically explain the allocation puzzle.

Ohanian, Restrepo-Echavarria, and Wright (2018) argue that the labor wedge explains the observed capital flows from Latin America and to Asia from 1950-1970. In their model the labor wedge is computed differently than in my model, and they do not disentangle the distortionary component from the demographic component. I do not rule out the possibility that the labor distortion is important for certain regional flows over certain time periods; I instead study the period 1970-2014 across a much larger set of countries.

Counterfactual investment distortions   In this countrfactual I reduce the investment wedge in each country by half. Since the effective investment wedge is $1 + \omega_{i,t+1}^K = (1 - \tau_{i,t+1}^K)/(1 - \tau_{i,t+1}^K)$, I construct a counterfactual investment distortion as $1 - \widetilde{\tau}_{i,t+1}^K = \left(1 + \omega_{i,t+1}^K/2\right)\left(1 - \widetilde{\tau}_{i,t}^K\right)$, with $\widetilde{\tau}_{i,1}^K = 0$ as in the baseline specification. The elasticity of the ratio of current account to GDP with respect to TFP growth is 24.3%. This elasticity is barely lower than that in the baseline, indicating that investment distortions may systematically explain some of the allocation puzzle, but not to a significant degree.

These distortions capture various frictions pertaining to investment that can potentially influence current account dynamics. Buera and Shin (2017) argue that financial frictions restrict the extent that investment can respond to fast GDP growth, implying that a fast growing country will tend to run a current account surplus in the short run. Aguiar and Amador (2011) put forward a theoretical justification that highly indebted developing countries have little incentive to invest due to the risk of being expropriated by their government. As a result, saving shows up in the current account rather than in domestic investment.

Counterfactual trade costs   I decompose the bilateral trade costs (the portion that melts away, $d - 1$) into a trend component and a bilateral distortionary component:

$$(d_{i,j,t} - 1) = \mathcal{D}_t \times \varepsilon_{i,j,t}^d.$$

The trend component steadily declines from 7.7 in 1970 to 3.6 in 2014, capturing reductions in shipping costs as well as large-scale tariff reductions implemented under the General Agreement on Tariffs and Trade and subsequently the World Trade Organization.[19] The distortionary component captures changes in bilateral trade costs, such as bilateral trade agreements, for instance, or other policies implemented unilaterally.

I consider two counterfactual scenarios pertaining to trade costs. In the first scenario I reduce the bilateral trade distortions between each country by half and construct counterfactual trade costs: $\widetilde{d}_{i,j,t} = 1 + \widehat{\mathcal{D}}_t \times \widehat{\varepsilon}_{i,j,t}^d/2$, where "hats" denote the estimated values. The elasticity of the ratio of current account to GDP with respect to TFP growth is 50.2%.

In the second scenario I hold the trend component constant at its 1970 level, thereby removing average declines in trade costs: $\widetilde{d}_{i,j,t} = 1 + \widehat{\mathcal{D}}_{1970} \times \widehat{\varepsilon}_{i,j,t}^d$. The elasticity of the ratio of current account to GDP with respect to TFP growth increases to 36.1%.

---

[19]The estimated trend component is then $\widehat{\mathcal{D}}_t = \exp\left(\frac{1}{I \times (I-1)} \sum_{i=1}^{I} \sum_{\substack{j=1 \\ j \neq i}}^{I} \ln(d_{i,j,t} - 1)\right)$.

In both scenarios the elasticity is higher than in the baseline specification. This result does not mean trade costs are unimportant for explaining specific bilateral trade imbalances. Indeed, in the counterfactual with trade distortions reduced by half, the U.S. bilateral trade deficit with China is 3.4% of U.S. GDP in 2014, compared to only 1.4% in the baseline. This finding is consistent with Alessandria and Choi (2019), who argue that asymmetric changes in trade costs intensified the U.S. trade deficit.

The scenario with constant average trade costs indicates that declining trade costs have not systematically altered the direction of capital flows, but instead perpetuated the magnitude of the flows. The volume of global imbalances, measured as the sum of the absolute value of long-run capital flows across countries as a share of global GDP, is 2.7% in the counterfactual compared to 3.1% in the baseline. This is due to lower trade volumes in the counterfactual: In the counterfactual trade in 2014 is 2.5% of world GDP, compared to 17.8% in the baseline.

F.2   Moving toward a model with a single good

In order to bring the model closer to one with a single homogenous good, I consider a specification with a high trade elasticity, $\theta = 12$. All remaining parameters are calibrated to match the same targets as in the specification above. One notable feature is that the high-trade-elasticity specification requires much smaller trade costs to replicate observed trade flows. In the high-trade-elasticity specification, the median trade cost in 2014 is 1.7, compared to 4.7 in the low-trade-elasticity specification. I then evaluate the effects of counterfactually altering the demographic structure. Results are reported in Table F.2.

The first dimension I study is the change in long-run capital flows under the counterfactual demographic scenario relative to the baseline demographic scenario. For each country I compute the absolute value of the difference between long-run capital flows in the counterfactual and those in the baseline, then take the average across countries. In the high-trade-elasticity specification, the average change capital flows in response to changes in underlying demographics is greater than in the low-trade-elasticity specification. In both specifications, the comparison across counterfactual demographic scenarios has similar ranking: Imposing a constant age distribution has the largest impact on long-run capital flows, while setting emerging economy age distributions equal to the average of advanced economies has the smallest impact.

Another dimension is the elasticity of the ratio of current account to GDP with respect to TFP growth in the context of the allocation puzzle. The pattern of the elasticity

(correlation) across counterfactual demographic scenarios is similar in both the high- and low-trade-elasticity specifications. Setting emerging economy age distributions equal to the average of advanced economies mitigates the allocation puzzle, while holding age distribution constant in all countries exaggerates it.

Table F.2: Capital flows with low and high trade elasticity

Specification	Avg. abs. change in L.R. cap. flow	Elasticity of C.A w.r.t. TFP
Low trade elasticity ($\theta = 4$)		
Baseline model	–	29.2%
Every age distribution set to average of world	3.0%	27.4%
Emr. age distribution set to average of adv.	2.1%	10.1%
Adv. age distribution set to average of emr.	3.6%	23.0%
Constant age distribution	6.1%	97.8%
High trade elasticity ($\theta = 12$)		
Baseline model	–	34.6%
Every age distribution set to average of world	3.6%	33.3%
Emr. age distribution set to average of adv.	2.6%	13.9%
Adv. age distribution set to average of emr.	4.5%	27.7%
Constant age distribution	6.9%	109.0%

Notes: Long-run capital flows are defined as the change in net-foreign asset position from 1970-2014, as a percent of cumulative GDP: $100 \times \frac{\mathcal{A}_{i,2014} - \mathcal{A}_{i,1970}}{\sum_{t=1970}^{2014} GDP_{i,t}}$. The average absolute change is the absolute value of the difference in this value, relative to the baseline demographic scenario, averaged across countries. The elasticity of the current account with respect to TFP is the slope of the line corresponding to the best fit curve using OLS (see Figure F.1 for the case of the baseline model). The y-variable in the regression is the average annual growth in TFP during five year windows. The x-variable in the regression is the average ratio of net exports to GDP during five year windows. Windows run from [1970,1974]-[2010,2014].

F.3   China's high saving rate

This paper also speaks to the large and growing body of work on China, which has become the poster child for studies on capital flows due to its position as a large net creditor. The literature has found evidence that both demographics and market distortions have played important roles. Wei and Zhang (2011) argue that male-biased gender ratios encourage men to save by purchasing real estate to attract scarce female partners. Yang, Zhang, and Zhou (2012) argue that successive cohorts of young Chinese workers face increasingly flat

life-cycle earnings profiles, thereby reducing household borrowing in the face of higher future aggregate productivity growth. İmorohoğlu and Zhao (2018) argue that, due to China's one-child policy, the elderly rely more on personal saving to replace lacking family support during retirement. İmorohoğlu and Zhao (2020) show that financial constraints faced by firms are equally important as the one-child policy in accounting for the rise in China's saving rate. In Song, Storesletten, and Zilibotti (2011), financial market imperfections imply that private firms in China finance the adaptation of technology through internal saving. My findings do not rule out any of these theories but, instead, offer a systematic assessment of the role of age-induced saving behavior across many countries over a long time period. Alessandria, Choi, and Lu (2017) argue that increased barriers on Chinese imports, relative to Chinese exports, have fueled China's current account surplus, therefore illustrating the importance of trade in current account dynamics. In addition to the above channels, my findings suggest that the age distribution played a quantitatively important role in contributing to China's trade and current account surplus.