Information Uncertainty*

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Abstract

In a market where buyers and sellers are uncertain about whether others are informed about the quality of an asset, inefficiency in trading arises due to incomplete learning. An uninformed seller will want to learn the asset's quality from the buyers' bids and may be willing to sell at low, but not at intermediate bids. Buyers may have incentives to pool their bids to prevent this type of learning. We outline conditions under which potential gains from trade cannot be realized in states where all traders are symmetrically informed or symmetrically uninformed.

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1 Introduction

Market participants often do not know whether other market participants are informed about the quality of goods or assets. Such cases attract newspaper attention when, for example, an owner finds a painting in their old family mansion, not knowing whether or not it was painted by an old master. Potential buyers may not know either and often also do not know whether the seller knows. Uncertainty about whether market participants know the quality of an asset is prevalent in many other markets as well. The value of a piece of land depends on whether or not it is polluted or whether it is rich in minerals or precious metal and it is unclear whether the seller and/or buyers know this information. Similarly, the value of a house depends on whether or not the foundations are properly built, and the seller and potential buyers may well be uninformed about this. In all these markets, not only is quality uncertain, it is also uncertain whether market participants know whether others, including the seller, know the true quality. We refer to this latter uncertainty as "information uncertainty".[1]

This paper investigates the ways in which information uncertainty affects outcomes of decentralized trading where multiple buyers compete to buy an asset that is owned by a seller. The seller and the buyers' valuations depend positively on asset quality.[2] Further, the seller is not pre-committed to selling. We consider the simplest setting where quality is either high or low and market participants either perfectly know the true quality or are uninformed (i.e., only know the prior distribution of quality).[3] Whether or not an agent is informed is independent of whether any other agent is informed. If quality is commonly (un)known, there are gains from trade, trade occurs and the market is efficient.

We show that information uncertainty may lead to market failure in real-

[1]Information uncertainty arises naturally in many markets where participants need to consult experts to assess asset quality and this consultation process is not publicly observable. See, e.g., Pavan and Tirole (2023) and Thereze (2023).

[2]In particular, the seller's opportunity cost of trading may reflect his expected return from retaining the asset for own future use or some other outside option.

[3]This can be seen as a stylized way of modeling more general environments where traders have private information about the informativeness of their private signals.

izations where all participants are symmetrically (un)informed about quality. In particular, trade may not take place even when (i) all market participants know quality is high, or (ii) all market participants are uninformed about quality.[4] At the core of our argument is the interaction between the incentives of the uninformed seller to learn quality from buyers' bids and the incentives of informed high quality buyers as well as uninformed buyers to strategically obfuscate their information to prevent this type of learning.

Incomplete learning arises in two types of pooling equilibria. One is a full pooling equilibrium where all buyers make a bid equal to the buyers' valuation of low quality, while only the uninformed and the low quality seller, but *not* the high quality seller, sell at this price. Thus, the asset is not traded in the realization where all market participants know quality is high. Buyers do not marginally increase their bid as the uninformed seller infers quality is high if he sees such a bid and rationally decides not to sell. Thus, an uninformed seller may rationally decide to sell at low prices, but not at somewhat higher prices. The other is a partial pooling equilibrium where informed low quality buyers and uninformed buyers pool on a bid equal to the seller's valuation of low quality and this bid is only accepted by the low quality seller and the asset is not traded when the seller and the buyers are uninformed about the true quality. Note that in this latter outcome the low quality asset may also not be traded, namely when the seller does not know quality is low.

Clearly, these pooling outcomes depend critically on what the uninformed seller believes after observing a deviation from the equilibrium bids. We characterize parametric conditions under which these inefficient pooling outcomes with partial or no learning by the uninformed seller are the *only* possible (perfect Bayesian) equilibrium outcomes. Thus, these equilibria should *not* be dismissed on the ground that they are just few among many equilibria. In addition, in the Appendix we show the beliefs supporting these equilibria are reasonable. As the bids of both buyers may signal information about the

[4] Our results are quite distinct from some other no-trade results, such as Myerson and Satterthwaite (1983), as buyer's and seller's values are independent in their model and there is a positive probability that trade is inefficient.

quality of one and the same product, standard refinements, such as the D1 criterion (Cho and Kreps 1987), do not apply. We adapt the D1 criterion to our game and show that the equilibria we construct satisfy this modified D1 criterion.

There are several parts of our arguments where subtle implications of the winner's curse come into play. First, the reason why fully revealing equilibria may not exist is that in a candidate revealing outcome uninformed buyers are not willing to bid high as they will only buy in case the other market participants know that quality is low or if they are uninformed. The low bids by uninformed buyers create, however, an incentive for buyers who know quality is high to pretend to be uninformed. Second, the winner's curse plays an important role in the beliefs sustaining pooling equilibria. In particular, the winner's curse dampens the incentive of the uninformed buyer to deviate to higher out-of-equilibrium bids. Third, the classical lemons market (Akerlof, 1970) is a special case of our model. We show that uncertainty about other market participants' knowledge creates a larger set of parameter conditions where the lemons outcome obtains. This is because due to the winner's curse an uninformed buyer knows that conditional on winning, the probability that quality is low is larger than the ex ante probability that quality is low.

Even though there is some early literature on buyers not knowing whether other buyers know quality (see, e.g., Baye et al. (1992) and Piccione and Tan (1996)), to the best of our knowledge this is the first paper that considers that buyers *and the seller* may not know of each other whether they are informed. This is important when the seller cannot commit to selling and may want to retain the asset if he learns through the bids that quality is high.[5]

The classic lemons problem has been extended in subsequent literature to include analyses of dynamic trading in settings where an informed seller faces buyers that may be informed or uninformed (see, e.g., Kaya and Kim 2018)

[5] In this sense, the paper also connects to a literature in economics and finance (see, e.g., Atakan and Ekmekci (2014)) showing that markets may not properly aggregate information, because of actions affecting the asset's value. In our paper, learning is incomplete and markets are inefficient even though the seller's action of retaining the item does not affect the asset's value.

and where an uninformed seller passively acquires information about quality over time (see, Hwang 2018). The possibility of an uninformed seller learning quality from buyers' bids and the incentive of buyers to shade their bids to hide information is not explored in this literature, however.

Our analysis also builds on the notion of the winner's curse (Wilson 1967, 1969) in that in a fully revealing equilibrium an uninformed buyer takes into account that winning means that the other buyer does not know that quality is high (as a high quality buyer will choose a higher price). Inferences drawn by an uninformed seller from the bids of buyers are crucial to our results, but they do not play any role in the auction-based winner's curse literature.

A more recent literature (see, e.g., Bergemann et al. 2015, Roesler and Szentes 2017, Bar Isaac et al. 2021 and Kartik and Zhong 2023) studies how the equilibrium payoffs of buyers and seller depend on variations in the information structure.[6] Information uncertainty as we consider it in this paper is not studied in this literature.

Somewhat similarly to our paper, Feinberg and Skrzypacz (2005) analyze one-sided bargaining focusing on the implications of the seller's private information about *his beliefs* about the buyer's valuation for the delay in trade. There is, however, no uninformed seller in their framework who may want to learn the quality of the product from the prices offered.

Finally, Andreyanov and Caoui (2022) have an empirical paper about French timber auctions where bidders are better informed about a common component of quality across lots and where the seller revises an endogenous secret reserve price using submitted bids. They do not, however, analyze how the incentives of buyers and sellers interact.

The rest of the paper is structured as follows. The next section presents the model. Section 3 analyzes conditions under which the full pooling equilibrium arises and when it is the unique equilibrium outcome, while Section 4 performs a similar analysis for the partial pooling equilibrium. Section 5 concludes. The Appendix discusses belief refinement.

[6]See, also Levin (2001) for an early paper in this direction.

2 The Model

A seller owns a unit of an asset that he may trade with one of two symmetric buyers, $i = 1, 2$.[7] The buyers make simultaneous take-it-or leave it price offers to the seller who must then decide which of the offers (if any) to accept. The valuations of the buyers as well as the reservation value of the seller, i.e., his payoff if he does not trade, depend on the quality of the asset, where quality can be either high (H) or low (L). In particular, for asset quality $s \in \{H, L\}$, each buyer's valuation is denoted by V_s, while the seller's reservation value is denoted by c_s, where

$$V_H > V_L, c_H > c_L \geq 0 \text{ and } V_s > c_s, s = L, H.$$

All agents are risk neutral. We also assume:

$$c_H > V_L \tag{1}$$

so that a buyer who knows quality to be high will not trade with a seller who knows quality to be low. The *ex ante* probability that the asset is of high quality is $\alpha \in (0, 1)$. When convenient, we will use $\bar{c} = \alpha c_H + (1 - \alpha)c_L$ and $\overline{V} = \alpha V_H + (1 - \alpha)V_L$ to denote the seller's ex ante expected (reservation) value, respectively, the buyers' ex ante expected value.

Each of the three agents (the seller and the two buyers) receive private signals of asset quality; the signal realization is either fully informative (i.e., reveals the true asset quality perfectly) or fully uninformative (i.e., signal realization is independent of true quality). The event that an agent's private signal is informative is independent of whether or not the private signal of any other agent in the market is informative. The probability that the seller's signal is informative i.e., the ex ante probability that the seller knows true product quality, is given by $\sigma \in [0, 1]$. The probability that a buyer's signal is fully informative i.e., the ex ante probability that buyer i knows true product

[7]The qualitative results do not depend on there being two buyers and can be extended to $N > 1$ buyers.

quality is identical for both buyers and given by $\beta \in [0, 1]$.

Formally, the game proceeds in three stages. First, nature draws true quality s from a binary distribution that assigns probability α to H and $1 - \alpha$ to L; this draw of nature is not observed by any agent. Second, conditional on s drawn in the first stage, nature *independently* draws the type of each market participant which can be one of three: uninformed (U), informed that quality is low (L) and informed that quality is high (H). If true quality is high (low), each buyer is of type H (L) with probability β, U with probability $1 - \beta$ and L (H) with probability zero while the seller is of type H (L) with probability σ, U with probability $1 - \sigma$ and L (H) with probability zero. The realization of each market participant's type is private information. Third, both buyers (having observed their own types) simultaneously choose their bids or price offers p_i,and the seller decides which of these bids to accept (if at all). The payoff of each market participant is her expected net surplus.

The solution concept used is symmetric perfect Bayesian equilibrium. In the rest of this paper we refer to this as simply an equilibrium. The Appendix outlines a refinement of out-of-equilibrium beliefs based on the D1 criterion and shows that the equilibria we construct meet this refinement.

3 Incomplete Learning: H type buyers hide information

In this section, we consider pooling outcomes where informed buyers that know quality is high hide their private information by bidding low and pool with uninformed types. The uninformed seller does not fully learn from the buyers' bids and further, the informed seller who knows quality is high does not sell. Thus, trade may not occur even when the seller and all buyers are fully informed that quality is high. The analysis is in two steps. We first construct an equilibrium with these features and subsequently characterize parameter values for which every equilibrium has these features.

We begin by constructing a symmetric pooling equilibrium where both buyers bid V_L regardless of their private information or type. As $c_H > V_L$, a seller who knows quality is high, would never sell at this bid. So, for a H type buyer to bid V_L, the uninformed seller must be willing to sell at that bid. As the uninformed seller acquires no additional information about asset quality from the buyers' equilibrium bids, his updated reservation value along the equilibrium path is $\overline{c}$ so that an equilibrium of this sort requires

$$\overline{c} \leq V_L. \tag{2}$$

In the candidate equilibrium each buyer buys the object with probability 1/2 when the seller is either uninformed or knows that he has a low quality asset. Thus, the payoff of a buyer who knows quality is high is $\frac{1-\sigma}{2}(V_H - V_L)$. She has no incentive to bid c_H and buy the good for sure if

$$V_H - c_H \leq \frac{1-\sigma}{2}(V_H - V_L). \tag{3}$$

This inequality also ensures there is no incentive of any other type of buyer to bid c_H or higher. The uninformed seller believes that any bid $p \in (V_L, c_H)$ comes from a H type buyer for sure (and therefore does not sell at such bids). As a result, no buyer has a unilateral incentive to increase her bid above V_L to outbid her rival buyer. Consequently, in this equilibrium, the informed seller never sells a high quality asset regardless of whether or not buyers know true asset quality.

What if (2) does not hold i.e., $\overline{c} > V_L$? Obviously, an outcome where all buyer types pool at V_L cannot be an equilibrium as a H type buyer would not be able to buy with such a bid even if the seller is uninformed. However, we may still have a partial pooling equilibrium where the uninformed and the H type buyers pool at a price $p^* \in (V_L, c_H)$ while the L type buyer bids V_L. As in the previous pooling outcome, the informed seller with high quality asset does not sell at all (even if buyers are informed). However, the uninformed seller as well as the informed seller with low quality asset sell for sure. Here, there is partial learning about product quality by the uninformed seller from the pooled bid p^*; he will sell at p^* only if p^* is somewhat larger than $\overline{c}$. So, in addition to an incentive condition that ensures the uninformed buyer has no incentive to increase her bid to c_H or above in order to buy for sure, we need an additional condition to ensure that such a buyer can at least break even by choosing p^* given the loss she is exposed to in the state where the other buyer is of type L (the winner's curse). Similar to the previous construction, neither of the pooling types has an incentive to deviate to a bid in (p^*, c_H) as the uninformed seller believes that such a unilateral deviation comes with probability one from a H type buyer.

In the Appendix, we show that the restrictions on the out-of-equilibrium beliefs of the uninformed seller outlined above are reasonable in that they satisfy a refinement criterion that adapts the D1 criterion to our setting.

The following proposition summarizes these results.

Proposition 1 *Assume that*

$$\frac{V_H - c_H}{V_H - \max\{V_L, \bar{c}\}} < \frac{1-\sigma}{2} \tag{4}$$

(a) If $\bar{c} \leq V_L$,then there exists a pooling equilibrium where regardless of their private information buyers offer V_L for sure. In this equilibrium, the uninformed seller cannot learn from buyers' bids.

(b) If $\bar{c} > V_L$ and further,

$$\frac{(1-\alpha)V_L + \alpha(1-\sigma)V_H}{(1-\alpha) + \alpha(1-\sigma)} > \bar{c}, \tag{5}$$

then there exists $\beta_0 > 0$ such that for every $\beta \in (0, \beta_0)$ there is a partial pooling equilibrium where the informed buyer who knows quality is high and the uninformed buyer pool to offer $p^* \in (V_L, c_H)$ for sure.

In both equilibrium outcomes (a) and (b), the informed seller with a high quality asset never trades and, in particular, the high quality asset is not traded even if the seller as well as one or both buyers are informed.

Proof. (a) Here, (4) reduces to

$$\frac{V_H - c_H}{V_H - V_L} < \frac{1-\sigma}{2}. \tag{6}$$

Consider a candidate equilibrium where buyers bid $p^* = V_L$ regardless of type, the uninformed and low quality types of the seller sell for sure and the informed high quality seller does not sell at all. In such an equilibrium, the equilibrium payoffs of buyer types $\tau = L, H, U$ are:

$$S_L = 0, S_H = \frac{1-\sigma}{2}(V_H - V_L), S_U = \frac{1}{2}(1-\sigma)\alpha(V_H - V_L).$$

We specify the following restriction on the out-of-equilibrium belief of the uninformed seller about asset quality after observing a *unilateral* deviation by a buyer to a price offer p :

$$\mu(p^*, p) = 1 \text{ for } p \in (V_L, c_H). \tag{7}$$

Under (7), only a L-type seller would sell at price $p < c_H$ and no buyer can gain from unilaterally deviating to $p \in (V_L, c_H)$. Obviously, a L type buyer has no incentive to raise his bid above $p^* = V_L$. As (6) implies (3) a H type buyer has no incentive to deviate to a bid p larger than c_H even if she buys for sure at that bid. An uninformed buyer also does not gain by deviating to set p larger than c_H if $\alpha V_H + (1-\alpha)V_L - c_H \leq S_U$, i.e.,

$$\frac{\alpha V_H + (1-\alpha)V_L - c_H}{\alpha V_H + (1-\alpha)V_L - V_L} \leq \frac{1-\sigma}{2},$$

which is implied by (6).

(b) Here, (4) reduces to

$$V_H - \frac{2}{1-\sigma}(V_H - c_H) > \bar{c} = \alpha c_H + (1-\alpha)c_L. \tag{8}$$

Note that (5) and (8) imply that for $\beta = 0$ the following holds:

$$\begin{aligned} & \min\left\{V_H - \frac{2}{1-\sigma}(V_H - c_H), \frac{(1+\sigma)(1-\alpha)V_L + \alpha(1-\sigma)V_H}{(1+\sigma)(1-\alpha)+\alpha(1-\sigma)}\right\} \quad (9) \\ > & \frac{\alpha c_H + (1-\beta)(1-\alpha)c_L}{\alpha + (1-\beta)(1-\alpha)}. \end{aligned}$$

As the right (left) hand side of the inequality in (9) is increasing (decreasing) in β, there exists $\beta_0 > 0$ such that (9) holds for all $\beta \in (0, \beta_0)$. Choose $\beta \in (0, \beta_0)$ and choose

$$p^* = \frac{\alpha c_H + (1-\beta)(1-\alpha)c_L}{\alpha + (1-\beta)(1-\alpha)}. \quad (10)$$

Note that $c_H > p^* \geq \bar{c} > V_L$. An uninformed seller who receives two price offers equal to p^* will sell as:

$$\begin{aligned} p^* & \geq \frac{\beta\alpha}{\beta\alpha + 1 - \beta} c_H + \frac{1-\beta}{\beta\alpha + 1 - \beta}(\alpha c_H + (1-\alpha)c_L) \\ & = \frac{\alpha c_H + (1-\beta)(1-\alpha)c_L}{\beta\alpha + 1 - \beta}, \end{aligned}$$

which follows from (10). If one price offer is p^* and the other offer is V_L the uninformed seller will always sell. Thus, the equilibrium payoff of a type τ buyer, $\tau = L, H, U$, is as follows:

$$\begin{aligned} S_L & = 0, S_H = \frac{1-\sigma}{2}(V_H - p^*), \\ S_U & = \frac{1}{2}\left[(1+\beta)(1-\alpha)(V_L - p^*) + \alpha(1-\sigma)(V_H - p^*)\right]. \end{aligned}$$

Note that $S_U \geq 0$ iff

$$p^* \leq \frac{(1+\beta)(1-\alpha)V_L + \alpha(1-\sigma)V_H}{(1+\beta)(1-\alpha)+\alpha(1-\sigma)} \quad (11)$$

and this is satisfied using (9) and (10). The out-of-equilibrium belief of the uninformed seller following a unilateral deviation by a buyer to a price offer

$p \in (p^*, c_H)$ is specified as follows:

$$\mu(p, V_L) = 0, \mu(p, p^*) = 1. \tag{12}$$

Given (12), it is optimal for the uninformed seller to not sell when facing bids p^* and $p \in (p^*, c_H)$. A buyer that deviates unilaterally to $p \in (p^*, c_H)$ can buy only if quality is low and either the seller or the rival buyer (who has not deviated) is informed. Thus, as $p^* > V_L$, unilateral deviation to $p \in (p^*, c_H)$ cannot be gainful regardless of buyer type. We also need to ensure that neither the uninformed nor the H type buyer has an incentive to deviate upwards to an offer $p \geq c_H$ where it would buy with probability one. If the deviating buyer is of H type, the other (non-deviating) buyer must be uninformed or of type H and therefore offers p^*. There is no incentive to unilaterally deviate to an offer $p \geq c_H$ by an H type buyer if $(V_H - c_H) \leq \frac{1-\sigma}{2}(V_H - p^*)$ which reduces to:

$$p^* \leq V_H - \frac{2}{1-\sigma}(V_H - c_H) \tag{13}$$

and using (9) and (10), we can see that this is satisfied. If the deviating buyer is uninformed, the other (non-deviating) buyer may be offering p^* or V_L. The uninformed buyer has no incentive to deviate from p^* to $p \geq c_H$ if

$$\alpha V_H + (1-\alpha)V_L - c_H \leq \frac{1}{2}\left[(1+\beta)(1-\alpha)(V_L - p^*) + \alpha(1-\sigma)(V_H - p^*)\right], \tag{14}$$

which is always satisfied if $\alpha V_H + (1-\alpha)V_L \leq c_H$ and (11) holds. If $\alpha V_H + (1-\alpha)V_L > c_H$, (14) follows from (13). This concludes the proof. ■

The conditions in Proposition 1 are easier to hold if the seller is uninformed with higher probability (σ is smaller) as it increases the incentive of H type buyers to hide their information. The conditions are also more likely to hold if $(V_H - c_H)$ is small as this limits the surplus an H type buyer can obtain by deviating to a bid above c_H. The partial pooling equilibrium outcome described in part (b) of Proposition 1 is more likely to exist if β is smaller as this lowers the probability of the uninformed buyer winning the bidding competition in a state where the rival buyer knows quality is low.

The equilibrium outcomes in Proposition 1 should not to be dismissed as just few of the many possible outcomes of the game. We will now show that for a subset of the parameter space considered in Proposition 1, the only equilibrium outcomes that exist are such that with positive probability the asset is not traded even if all agents know quality is high. We begin with a lemma that specifies a natural condition under which *uninformed* buyers never bid above c_H and therefore, do not trade when the seller knows quality to be high.

Lemma 1 *Suppose that*

$$\frac{V_H - c_H}{V_H - V_L} < \frac{1-\alpha}{1-\alpha\beta}. \tag{15}$$

Then the uninformed buyer bids below c_H with probability one and the high quality informed seller never trades when both buyers are uninformed.

Proof. Let $F(p)$ denote the (possibly) mixed strategy of an uninformed buyer such that $F(c_H) < 1$; let the support of F be $[\underline{p}_U, \overline{p}_U]$ (possibly with mass points). Note that the uninformed buyer cannot pool with H type buyer at any bid $p \geq c_H$; if they would pool there would have to be a mass point at $p \leq \alpha V_H + (1-\alpha)V_L < V_H$ and therefore, the H type buyer would be strictly better off at a slightly higher bid. We first show that there is no bid $\widetilde{p} < \overline{p}_U$ in the support of the H type buyer's equilibrium strategy. To see this, let $q(p)$ be the probability a buyer buys when bidding price p. Given the strategy of the other bidder it must be that $q(p)$ is (weakly) increasing. As $\widetilde{p}$ and $\overline{p}_U$ are optimal prices for the two types of bidders we should have

$$q(\overline{p}_U)(\alpha V_H + (1-\alpha)V_L - \overline{p}_U) \geq q(\widetilde{p})(\alpha V_H + (1-\alpha)V_L - \widetilde{p}) \tag{16}$$

$$q(\widetilde{p})(V_H - \widetilde{p}) \geq q(\overline{p}_U)(V_H - \overline{p}_U) \tag{17}$$

(17) implies that $q(\widetilde{p})(V_H - \widetilde{p}) + q(\widetilde{p})(1-\alpha)(V_L - V_H) \geq q(\overline{p}_U)(V_H - \overline{p}_U) + q(\overline{p}_U)(1-\alpha)(V_L - V_H) = q(\overline{p}_U)(\alpha V_H + (1-\alpha)V_L - \overline{p}_U)$ so that (16) must hold with equality. Similarly, (16) implies that (17) should hold with equality.

However, both (16) and (17) cannot simultaneously hold with equality. A similar argument shows that it cannot be that L type buyer bids a price $\widetilde{p} > \underline{p}_U$. Therefore, in a revealing equilibrium where $\overline{p}_U$ is set with probability x the pay-off of an uninformed buyer at bid $\overline{p}_U \geq c_H$ equals

$$
\begin{aligned}
& (1-\alpha)\beta(V_L - \overline{p}_U) + (1-\beta)(1 - x + \frac{x}{2})(\alpha V_H + (1-\alpha)V_L - \overline{p}_U) \\
\leq \; & (1-\alpha)\beta(V_L - c_H) + (1-\beta)(\alpha V_H + (1-\alpha)V_L - c_H) \\
< \; & 0 \text{ (using (15))},
\end{aligned}
$$

a contradiction. ■

Condition (15) in the previous lemma can be rewritten as

$$\frac{\alpha(1-\beta)V_H + (1-\alpha)V_L}{1-\alpha\beta} < c_H$$

which can be read as a generalized "lemons" condition. In the classical market for lemons where uninformed buyers interact with a fully informed seller, buyers do not buy high quality if $\alpha V_H + (1-\alpha)V_L < c_H$. In our model where there is uncertainty about whether buyers are uninformed, this "lemons" outcome can obtain even if the ex ante expected valuation of an uninformed buyer exceeds the informed high quality seller's cost. This is because the uninformed buyer knows that the only chance to buy high quality is when the competing buyer is also uninformed and takes this into account when assessing her willingness to buy. In fact, regardless of the ex-ante distribution of quality, an uninformed buyer is not willing to bid enough to buy high quality from an informed seller if the probability β that the competing buyer is informed is large enough.

In Proposition 2 below, we outline sufficient conditions under which in *any* equilibrium, there is a strictly positive probability that the high quality good is not traded *even if the seller as well as one or both buyers are informed about true quality* and despite the positive gains from trade in such a state. In particular, an informed buyer who knows quality is high bids strictly below c_H with strictly positive probability.

Proposition 2 *If $\overline{c} < V_L$ and*

$$\frac{V_H - c_H}{V_H - V_L} < \frac{(1-\alpha)(1-\sigma)}{2\left(\alpha(1-\beta)(1-\sigma) + (1-\alpha)\right)} \tag{18}$$

then in every equilibrium with strictly positive probability the high quality good is not traded when the seller as well as one or both buyers are informed about true quality.

Proof. Consider an equilibrium where H type buyers bid above c_H with probability one. From Lemma 1 and the fact that (18) implies (15), we know there is no equilibrium where uninformed buyers bid above c_H with positive probability. Therefore, the uninformed buyer's equilibrium bid must be below c_H with probability one. Further, the H type buyer must randomize over an interval $[c_H, \overline{p}_H]$ with no mass point and her equilibrium payoff is $(1-\beta)(V_H - c_H)$. Let $\overline{p}_U$ be the upper bound of the support of the uninformed type's bids. There are two possibilities: (a) $\overline{p}_U > V_L$ and (b) $\overline{p}_U = V_L$. Consider first case (a). Here, the uninformed type reveals her type at bid $\overline{p}_U$ and as $\overline{c} < V_L$, the uninformed seller sells with probability one when he receives this bid. Further, $\overline{p}_U \leq \widetilde{p}_U$ where $\widetilde{p}_U$ is the bid of the uninformed type that generates zero surplus in a hypothetical scenario where the uninformed type of the rival buyer bids less than $\widetilde{p}_U$ for sure :

$$\widetilde{p}_U = \frac{\alpha(1-\beta)(1-\sigma)V_H + (1-\alpha)V_L}{\alpha(1-\beta)(1-\sigma) + (1-\alpha)}.$$

If the high type buyer deviates to bid $\overline{p}_U$, the lowest probability with which it sells is $(\frac{1-\sigma}{2})(1-\beta)$ and as $\overline{p}_U \leq \widetilde{p}_U$ his deviation surplus is at least as large as $(\frac{1-\sigma}{2})(1-\beta)(V_H - \widetilde{p}_U)$ which exceeds his equilibrium payoff of $(1-\beta)(V_H - c_H)$ leading to a contradiction if $V_H - \frac{2}{1-\sigma}(V_H - c_H) > \widetilde{p}_U$ i.e.,

$$\frac{\alpha(1-\beta)(1-\sigma)V_H + (1-\alpha)V_L}{\alpha(1-\beta)(1-\sigma) + (1-\alpha)} < V_H - \frac{2}{1-\sigma}(V_H - c_H)$$

and this reduces to (18).

Consider next case (b) where $\overline{p}_U \leq V_L$. The pay-off to the H type buyer of imitating this bid is at least $(\frac{1-\sigma}{2})(1-\beta)(V_H - V_L) > (1-\beta)(V_H - c_H)$, the H type's equilibrium payoff (the inequality follows from (18)), a contradiction. Thus, in any equilibrium, H type buyers bid strictly below c_H with strictly positive probability and therefore does not trade if the seller is informed quality is high. The proof is complete. ■

It is easy to verify that condition (18) implies (4) when $\bar{c} < V_L$ i.e., the conditions of Proposition 2 are stronger than the conditions in Proposition 1(a). In particular, a full pooling equilibrium as described in Proposition 1(a) exists under the conditions of Proposition 2.

It is important to qualitatively distinguish the outcome highlighted by Proposition 2 from the "lemons" outcome in Lemma 1. Whereas the latter describes a market where an informed seller does not trade high quality with *uninformed* buyers, Proposition 2 highlights the inability to trade a high quality good in a state of the world where *both* sides of the market know that quality is high. This market failure arises from the uncertainty that market participants continue to have about how well others are informed (even after observing their actions).

It is intuitive that a buyer who is informed that quality is high would not bid less than an uninformed buyer. It follows that the conditions under which the informed buyers bid less than c_H for a high quality good, ought to be stronger than condition (15) in Lemma 1 which ensures uninformed buyers bid below c_H. Indeed, condition (18) is stronger than condition (15) in Lemma 1.

The conditions in Proposition 2 are more likely to hold with smaller σ, smaller α and/or larger β. As the probability of the seller being uninformed increases, it becomes more attractive for the H type buyer to hide his information. If the other buyer is informed or quality is low with higher probability, the uninformed buyer is likely to lower his bid as he faces a higher risk of buying in the state where rival buyer is informed and quality is low. The latter makes it more attractive for the H type buyer to pool with the uninformed type.

4 Incomplete Learning II: Uninformed buyers hide

In the pooling outcomes considered in the previous section, the H type buyer pools with the uninformed type to hide her private information from the uninformed seller (at least partially) and the high quality asset is not traded when the seller is informed. However, ex ante gainful trade always occurs when the buyers and the seller are symmetrically uninformed. Further, the low quality asset is always traded. In this section, we consider a very different kind of inefficient outcome where no trade occurs when the seller as well as buyers are (symmetrically) uninformed and even the *low* quality asset is not traded when the seller is uninformed.

In particular, if $\bar{c} > V_L$ we can construct an equilibrium where an H type buyer always bids above c_H but the uninformed buyer pools with an L type buyer and bids V_L. We impose a condition under which only the informed seller who knows quality is low sells at the pooled bid V_L. The uninformed seller's updated valuation at bid V_L is higher than the bid and so he does not sell. The uninformed seller believes that when the other buyer's bid is equal to V_L, an out-of-equilibrium bid $p \in (V_L, c_H)$ can come only from an uninformed buyer; an informed seller will sell at that bid only if quality is low while the uninformed seller will sell only if the bid exceeds $\bar{c}$. Under certain conditions, the uninformed buyer is not willing to raise her bid to $\bar{c}$ to induce the uninformed seller to sell as she is deterred by the negative surplus she makes in the state where the rival buyer is of L type (winner's curse). Further, she is unwilling to raise her bid to c_H even though she can buy for sure at that bid. All of these conditions are formally stated in the next proposition.

In the Appendix, we show that restrictions on the out-of-equilibrium beliefs of the uninformed seller mentioned above are reasonable in that they satisfy our adaptation of the D1 refinement.

Proposition 3 *Suppose that $\overline{c} > V_L$ and further,*

$$\frac{(1-\beta)\,\alpha}{1-\alpha\beta} \geq \frac{V_L - c_L}{c_H - c_L}, \tag{19}$$

$$\max\left\{\frac{V_H - c_H}{c_H - V_L}, (1-\sigma)\frac{V_H - \overline{c}}{\overline{c} - V_L}\right\} \leq \frac{1-\alpha}{\alpha(1-\beta)}, \tag{20}$$

$$\frac{V_H - c_H}{V_H - \overline{c}} \geq (1-\sigma). \tag{21}$$

Then, there exists a symmetric equilibrium where (a) the uninformed and informed buyers that know quality is low pool to offer V_L with probability one, (b) informed buyers that know quality is high randomize over bids on an interval $[c_H, \overline{p}_H]$ and (c) the uninformed seller does not sell when both buyers offer V_L and thus never trades if buyers are either informed that quality is low or are uninformed.

Proof. Consider the candidate equilibrium. It is optimal for the uninformed seller to not sell when both buyers offer V_L if, and only if, $c_L + \frac{(1-\beta)\alpha(c_H - c_L)}{1-\alpha\beta} \geq V_L$ i.e., (19) holds. The equilibrium payoffs of the different types of buyers are as follows: $S_L = S_U = 0, S_H = (1-\beta)(V_H - c_H)$.The L type buyer has no incentive to offer any price above V_L.The uninformed buyer does not want to imitate the H type buyer if $\alpha(1-\beta)(V_H - c_H) + (1-\alpha)(V_L - c_H) \leq 0$ i.e., (20) holds. The out-of-equilibrium belief of the uninformed seller are as follows: if one buyer offers an out-of-equilibrium price $p \in (V_L, c_H)$ and the non-deviating buyer offers p^*, then

$$\begin{aligned} \mu(p, p^*) &= 1, \text{ if } p^* \in [c_H, \overline{p}_H] \\ &= \alpha, \text{ if } p^* = V_L \end{aligned} \tag{22}$$

Given these beliefs, the uninformed type buyer has no incentive to deviate to a price offer $p \in (V_L, c_H)$ if

$$\frac{V_H - \overline{c}}{\overline{c} - V_L} \leq \frac{1-\alpha}{\alpha(1-\beta)(1-\sigma)}. \tag{23}$$

To see this, only a seller of type L would sell at $p \in (V_L, \overline{c})$ which would yield negative surplus and so the most gainful deviation in (V_L, c_H) would be $p = \overline{c}$, namely if the uninformed seller sells for sure at this price. This would yield the uninformed buyer a pay-off of $\alpha(1-\beta)(1-\sigma)(V_H - \overline{c}) + (1-\alpha)(V_L - \overline{c})$ and this is smaller than 0 if, and only if, (23) holds. Similarly, a H type buyer's best deviation in the range (V_L, c_H) is to set $p = \overline{c}$ if the uninformed seller sells for sure at this price and this is not gainful if $(V_H - c_H) \geq (1-\sigma)(V_H - \overline{c})$ as he will only buy if the rival buyer and the seller are uninformed, which follows from (21). ■

Unlike the pooling equilibria described in Proposition 1, the pooling equilibrium in Proposition 3 is more likely when the seller is more likely to be informed (σ is higher) as it reduces the incentive of the uninformed buyer to increase her bid to sell to the uninformed seller. In fact, for σ sufficiently close to 1, all of the conditions in Proposition 3 are satisfied as long as:

$$\frac{(1-\beta)\,\alpha}{1-\alpha\beta} \geq \frac{V_L - c_L}{c_H - c_L} \text{ and } \frac{V_H - c_H}{c_H - V_L} < \frac{1-\alpha}{\alpha(1-\beta)}$$

which hold as long as the gains from trade $V_L - c_L$ and $V_H - c_H$ are somewhat small relative to the difference in the opportunity cost of selling low and high quality assets.

The key inefficiency in the equilibrium outlined in Proposition 3 is that the uninformed seller does not sell to uninformed buyers i.e., despite the potential gains from trade in the state where all agents are symmetrically uninformed, trade does not occur at all. Next, we provide a sufficient condition under which all equilibria exhibit this specific feature of the equilibrium outcome in Proposition 3.

Proposition 4 *If:*

$$\frac{V_L - c_L}{c_H - c_L} < \alpha \min\left\{\frac{1-\beta}{1-\alpha\beta}, \frac{1}{2}\left(\beta - \left(\frac{2-\beta}{1-\alpha}\right)\left(\frac{V_H - c_H}{c_H - c_L}\right)\right)\right\} \qquad (24)$$

then the asset is not traded when the seller and the buyers are uninformed.

Proof. We begin by noting that under condition (24), in any equilibrium where the uninformed type buyer pools entirely with the L type buyer, the uninformed seller never sells at the pooling bid. To see this, note that it must be that the pooling price $p^* \leq V_L$. Further, p^* must be the upper bound of the equilibrium bid distribution of the L type buyer. As (24) implies $c_L + \frac{(1-\beta)\alpha(c_H-c_L)}{1-\alpha\beta} > V_L$ i.e.,

$$\frac{(1-\alpha)c_L + \alpha(1-\beta)c_H}{(1-\alpha)+\alpha(1-\beta)} > V_L \geq p^*,$$

the uninformed seller would reject a bid of p^* independent of the probability mass placed by H and L types of the buyer at p^*. Thus, in such an equilibrium outcome, trade does not occur when the seller and the two buyers are uninformed.

We now argue that in every equilibrium, the uninformed type buyer must pool entirely with the L type buyer. Suppose to the contrary there is an equilibrium where the uninformed buyer does not pool entirely with the L type buyer. Let $\overline{p}_U$ be the upper bound of the support of the uninformed buyer's bids. Then, the L type buyer bids strictly below $\overline{p}_U$ for sure. If $\overline{p}_U < \overline{c}$, then the uninformed seller will not sell at bid $\overline{p}_U$ (regardless of the probability with which the H type buyer also bids $\overline{p}_U$) and only the informed seller who knows quality is low will sell at such a bid so that the uninformed seller will earn negative expected surplus by bidding $\overline{p}_U$. Therefore, $\overline{p}_U \geq \overline{c}$. At bid $\overline{p}_U$, the uninformed buyer buys for sure when the rival buyer is of L type. An upper bound on the equilibrium payoff of the uninformed buyer at bid $\overline{p}_U$ can be obtained if we suppose that with this bid: (i) she buys with probability one in the state where rival buyer is uninformed (regardless of the private information of the seller), and (ii) she buys with probability $\frac{1}{2}$ in the state where the rival buyer is of H type (the equilibrium bid of the rival H type buyer cannot be strictly less than $\overline{p}_U$ and so in equilibrium, she cannot buy with a higher probability in this state). As $\overline{p}_U \geq \overline{c}$ this upper bound on the expected surplus is given by

$$(1-\alpha)\beta(V_L-\overline{c})+(1-\beta)(\alpha V_H+(1-\alpha)V_L-\overline{c})+\frac{1}{2}\alpha\beta(V_H-\overline{c}),$$

which is negative if

$$(1-\alpha)V_L+\alpha\left(1-\frac{1}{2}\beta\right)V_H<\left(1-\frac{1}{2}\alpha\beta\right)(\alpha c_H+(1-\alpha)c_L).$$

This reduces to

$$\frac{V_L-c_L}{c_H-c_L}<\frac{\alpha}{2}\left[\beta-\frac{2-\beta}{1-\alpha}\left(\frac{V_H-c_H}{c_H-c_L}\right)\right],$$

which is implied by (24). Thus, the uninformed buyer would earn negative expected surplus in any such equilibrium, a contradiction. ■

As one would expect, condition (24) in Proposition 4 imposes fairly strong restrictions on the parameter space. In particular, (24) imposes upper bounds on how large the gains from trade can be relative to the cost disadvantage of high quality over low quality. Note that condition (24) is more likely to hold when the gains from trade (V_H-c_H and V_L-c_L) are relatively small, while the cost difference c_H-c_L is relatively large.

Proposition 4 ensures that all equilibria satisfy the key feature of the equilibrium constructed in Proposition 3: the uninformed buyer pools entirely with L type buyer to make a bid at which the uninformed seller does not sell. Based on this, one may expect the condition in Proposition 4 to be stronger than that in Proposition 3. Indeed, the first two conditions ((19) and (20)) of Proposition 3 are implied by condition (24) of Proposition 4. However, condition (24) of Proposition 4 does not impose any restriction on σ, the probability that the seller is informed. In contrast, condition (21) of Proposition 3 requires σ to be somewhat large, which is not required in Proposition 4. To understand why note that Proposition 4 does not ensure that equilibria satisfy *all* of the features of the specific equilibrium constructed in Proposition 3.

5 Conclusion

In markets where buyers and sellers are uncertain about how well other market participants are informed, failure to trade can occur even when all buyers and sellers are symmetrically informed or symmetrically uninformed. Incomplete learning is at the heart of this inefficiency: as the seller's opportunity cost of selling depends on true quality, the uninformed seller would like to learn from the buyers' bids about information the latter may have about quality in order to determine whether or not to sell. This learning can lead to non-monotonic behavior in the willingness to sell of such a seller: at certain low prices, an uninformed seller may sell if they infer that quality is likely low, while at somewhat higher prices they may prefer not to sell if the updated belief tells them it is likely that quality is high. For reasonable beliefs, this learning may, in principle, happen both on and off the equilibrium path.

An informed buyer who knows quality is high (and to a lesser extent, an uninformed buyer), has a strategic incentive to prevent the uninformed seller from learning her type in order to lower the latter's belief about quality and their valuation of the asset. In particular, such buyers may lower their bid to imitate the bid of a buyer who knows quality is low or is uninformed to enjoy the advantage of increasing the probability the uninformed seller will sell. We identify parameter conditions that guarantee buyers' incentives to shade bids are strong enough to outweigh the risk of not buying in case the seller is symmetrically informed or uninformed.

It is plausible that the possibility of learning from the bids creates an incentive for sellers to not commit (even if they could) to selling mechanisms like auctions where they always sell to the highest bidder. With a commitment to sell, the equilibrium bids will be fully revealing, but the high quality bidders' bid may be significantly smaller than the seller's reservation value for high quality, which may make it optimal to not commit to sell.

Our paper makes a first step towards understanding markets with information uncertainty, where buyers and sellers with interdependent valuations have private information about whether they know the quality of the asset to be traded. In our paper this private information is modelled as a signal that is either fully informative or fully uninformative and independent across market participants. Allowing these private signals to be partially informative and/or correlated across various market participants, allowing for dynamic trading or considering other market structures are interesting directions for future research.

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APPENDIX

In this appendix we formally define a belief refinement for Perfect Bayesian Equilibria (PBE) of our signaling game where two senders potentially have information about the same state of the world; the refinement is a natural extension of D1 refinement (Cho and Kreps, 1987) for the standard signaling game with single sender and single receiver. We then show that the beliefs underlying the PBE constructed in the proof of Propositions 1 and 3 satisfy this refinement.

Belief Refinement.

We have a signaling game with two senders (the two buyers) and one receiver (the seller) and both buyers may signal some information about the true asset quality to the uninformed seller. We outline two principles that must be satisfied by out-of-equilibrium beliefs.

For any PBE and realized price offer p_i^*, $i = 1, 2$, made by buyer i that is on the equilibrium path, one can use buyer $i's$ equilibrium strategy to derive the conditional probability $\mu_i(p_i^*)$ that asset quality is high. This conditional probability is based only on the price offered by buyer i. Now, consider a *unilateral* deviation by buyer j to an out of equilibrium price p, while the non-deviating buyer $i \neq j$ offers a price p_i^*. The first principle is that the *uninformed* seller should rely entirely on the information revealed by the non-deviating buyer's bid if her bid fully reveals quality for sure.

(P1) If $\mu_i(p_i^*) \in \{0, 1\}$ then regardless of the bid of the other buyer, the updated belief of the uninformed seller should be equal to $\mu_i(p_i^*)$.

Principle (P1) reflects the idea behind some other belief refinements in the literature such as Unprejudiced Beliefs due to Bagwell and Ramey (1991) that a single deviation is more likely than multiple deviations. The next principle specifies restrictions on beliefs when uncertainty about product quality remains after observing the non-deviating player's action.

Let $M_j(p)$ be the set of updated beliefs (updated probability of high quality) based on the out-of-equilibrium price p offered by player j that are consistent with an adaptation of the D1 criterion outlined below.

(P2) If $0 < \mu_i(p_i^*) < 1$ i.e., the non-deviating buyer $i's$ price does not reveal asset quality perfectly, then the out-of-equilibrium belief of the uninformed seller after observing price offers (p_i^*, p) can only assign probability $\mu(p_i^*, p)$ to high quality if

$$\mu(p_i^*, p) = \lambda\mu_i(p_i^*) + (1-\lambda)\mu_j \text{ for some } \mu_j \in M_j(p) \text{ and some } \lambda \in [0,1].$$

The refinement permits any out-of-equilibrium belief that is some weighted average of the beliefs derived separately from the non-deviating and the deviating player's actions. For instance, suppose that p_i^* is a pooling price that is offered in equilibrium with probability one by uninformed and H types; then $\mu_i(p_i^*) = \frac{\alpha}{\alpha\beta+(1-\beta)}$. If, further, the adapted D1 criterion assigns probability one to the deviating player being of type H, implying $M_j(p) = \{1\}$, then only out-of-equilibrium beliefs $\mu(p_i^*, p) \in \left[\frac{\alpha}{\alpha\beta+(1-\beta)}, 1\right]$ are consistent with principle (P2) of our refinement. On the other hand, if the adapted D1 criterion does not yield any restriction based on the deviating player's action i.e., $M_j(p)$ is the $[0,1]$ interval, then our refinement imposes no restriction on beliefs.

We now outline how we apply the D1 criterion to our setting under principle (P2). Consider a unilateral deviation to an out of equilibrium bid p by buyer j. To apply the D1 criterion, we need to determine the undominated responses of the seller for which this deviation is gainful (for each type of buyer j). As buyer j deviates before observing the bid of the non-deviating buyer $i \neq j$, we need to consider all possible equilibrium bids that may simultaneously come from buyer i and the seller's undominated response to each pair of bids that he might then observe. We have two reasonable restrictions here. When the equilibrium bid of buyer i is strictly higher than p, the seller sells to the deviating buyer j with probability zero. Further, if the equilibrium bid of buyer i reveals product quality fully (for sure), the uninformed seller's response is simply her optimal action based on the (deterministic) revealed quality. We can then determine the profiles of undominated responses by the uninformed seller (one for each possible equilibrium bid of the non-deviating buyer) for which each type of deviating buyer j earns an expected pay-off that is higher than her equilibrium payoff and compare the sets of such profiles for the various types of buyer j. If the set of such profiles of responses for one type of buyer j is a strict subset of that for another type, then the adapted D1 criterion assigns probability zero to the first type.

The equilibria constructed in the proof of Proposition 1 satisfy this refinement.

Consider the full pooling equilibrium constructed in the proof of part (a) of Proposition 1. As this is a symmetric full pooling equilibrium, the equilibrium action of the non-deviating buyer is uninformative and so our refinement suggests the uninformed seller use the D1 criterion to decide which buyer type of the deviating player has the strongest incentive to deviate to such a bid. We show that the H type buyer can gain from such an upward deviation for a larger set of undominated responses from the uninformed seller and therefore, the D1 criterion supports the specified belief. In particular, we check that the out-of-equilibrium belief (7) in the proof of Proposition 1 satisfies our belief refinement. Consider a unilateral deviation by buyer j to $p \in (V_L, c_H)$. As there is full pooling at price V_L, $\mu_i(V_L) = \alpha \in (0,1)$ so that only Principle (P2) can be applied. As $\mu_i(V_L) \in (0,1)$, every probability of selling is an undominated action of the uninformed seller after observing bids (p, V_L). Let $q_H(p), q_U(p)$ be the probabilities of selling by the uninformed seller that make the H and U type buyers indifferent between offering $p^* = V_L$ and deviating to p, i.e.,

$$q_H(p) = \frac{1}{1-\sigma}\left(\frac{S_H}{V_H - p}\right) = \frac{1}{2}\frac{V_H - V_L}{V_H - p}$$

$$q_U(p) = \frac{1}{1-\sigma}\left(\frac{S_U - \sigma(1-\alpha)(V_L - p)}{\alpha V_H + (1-\alpha)V_L - p}\right).$$

We show that $q_H(p) < q_U(p)$ for $p \in (V_L, c_H)$ so that using the D1 criterion, $M_j(p) = \{1\}$ and therefore, the specified belief $\mu(p^*, p) = 1$ meets principle (P2) of the refinement criterion R if we choose $\lambda = 0$. Note that $q_H(p) < q_U(p)$ if, and only if,

$$\frac{\alpha V_H + (1-\alpha)V_L - p}{V_H - p} < \alpha - \frac{2\sigma(1-\alpha)}{(1-\sigma)}\left[\frac{V_L - p}{V_H - V_L}\right].$$

As the left-hand side is strictly decreasing and the right-hand side is strictly increasing in p, the inequality holds for all $p \in (V_L, c_H)$ if, and only if, it holds weakly at $p = V_L$, which is true.

Next, consider the partial pooling equilibrium outlined in the proof of part (b) of Proposition 1. We can verify that the out-of-equilibrium belief restriction (12) satisfies our refinement criterion. Suppose the uninformed seller observes a unilateral deviation by buyer j to a price offer $p \in (p^*, c_H)$. If the non-deviating buyer i offers V_L, then as $\mu_i(V_L) = 0$ principle (P1) of our refinement implies $\mu(p, V_L) = 0$ and the only undominated action of the uninformed seller after observing pair of prices (p, V_L) is to sell with probability one. Now, suppose the non-deviating buyer i offers p^*, then $\mu_i(p_i^*) = \frac{\alpha}{\alpha\beta+(1-\beta)} \in (0,1)$ so that only Principle (P2) applies. For any pair of prices (p, p^*), as $\mu_i(p^*) \in (0,1)$, every probability of selling is an undominated action of the uninformed seller. We will now argue that an H type deviating buyer j has the strongest incentive to deviate to p using the D1 criterion so that $M_j(p) = \{1\}$ and therefore, the specified belief $\mu(p^*, p) = 1$ meets principle (P2) of the refinement criterion (by choosing $\lambda = 0$). Note that only an H type buyer has an incentive to deviate to $p > \alpha V_H + (1-\alpha)V_L$ and so $M_j(p) = \{1\}$ for such p. So, consider $p \in (p^*, \min\{\alpha V_H + (1-\alpha)V_L, c_H\})$. Let $q_\tau(p), \tau = H, U$ be the probability with which an uninformed seller sells after observing a bidder bidding p and the other bidding p^* such that a type τ buyer j is indifferent between deviating to bid p and not (where it is understood that in case the other buyer i offers V_L, the uninformed seller will sell for sure). Check that

$$q_H(p) = \frac{1}{1-\sigma}\left(\frac{S_H}{V_H - p}\right) = \frac{1}{2}\frac{V_H - p^*}{V_H - p},$$

$$q_U(p) = \frac{1}{2}\frac{(1-\alpha)(1+\beta)(V_L - p^*) + \alpha(1-\sigma)(V_H - p^*) - 2(1-\alpha)\beta(V_L - p)}{(1-\beta)(1-\alpha)(V_L - p) + \alpha(1-\sigma)(V_H - p)}.$$

It is sufficient to show that for all $p \in (p^*, \min\{\alpha V_H + (1-\alpha)V_L, c_H\})$, $q_H(p) <$

$q_U(p)$ which reduces to

$$\frac{V_H - p^*}{V_H - p} < \frac{(V_H - p^*) + \frac{(1-\alpha)(1-\beta)(V_L - p^*) + 2(1-\alpha)\beta(p-p^*)}{\alpha(1-\sigma)}}{(V_H - p) + \frac{(1-\beta)(1-\alpha)(V_L - p)}{\alpha(1-\sigma)}}.$$

which always holds.[8]

The equilibrium constructed in the proof of Proposition 3 satisfies this refinement.

Consider the partial pooling equilibrium constructed in the proof of Proposition 3. We show that the restriction (22) on out-of-equilibrium belief of the uninformed seller in the proof of Proposition 3 satisfies our refinement. Consider a unilateral deviation to price $p \in (V_L, c_H)$ by buyer j. The first restriction $\mu(p, p^*) = 1$, if $p^* \in [c_H, \overline{p}_H]$ follows directly from principle (P1) of the refinement. If the non-deviating buyer i offers $p^* = V_L$, as $\mu_i(V_L) = \frac{\alpha(1-\sigma)}{(1-\sigma)+\sigma(1-\alpha)} \in (0,1)$ principle (P1) does not apply but principle (P2) does and we can derive restrictions on beliefs based on the deviating buyer's incentives in the spirit of the D1 criterion. Note that at price (p, V_L), as $\mu_i(V_L) \in (0,1)$, every probability of selling is an undominated action of the uninformed seller. On other hand, for $p^* \in [c_H, \overline{p}_H]$, as $p < p^*$ the uninformed seller will never sell to the deviating buyer (it is a dominated action). We will now argue that U type of the deviating buyer j has the strongest incentive to deviate to p using the D1 criterion so that $M_j(p) = \{\alpha\}$ and the specified belief $\mu(p, V_L) = \alpha$ meets principle (P2) of the refinement criterion (by choosing $\lambda = 0$). It is clear that the L type buyer never has an incentive to deviate to $p > V_L$. When a H type buyer deviates to p, he cannot buy if either the rival buyer is informed or the seller is informed (as $p < c_H$). Let $q_\tau(p)$ be the probability of buying from an uninformed seller at price p when the rival buyer offers V_L that makes the

[8] This inequality can be rewritten as $\frac{V_H - p^*}{V_H - p} < \frac{V_H - p^* + x}{V_H - p + y}$ for appropriately chosen x and y; this holds if $(V_H - p^*)y < (V_H - p)x$ which reduces to

$$-(1-\beta)(V_H - V_L)(p - p^*) < 2\beta(V_H - p)(p - p^*)$$

and this always holds as the left hand side is negative, while the right hand side is positive.

τ type buyer indifferent between deviating and not deviating to p. Then,

$$\frac{1}{1-\beta}\frac{V_H - c_H}{V_H - p} = q_H(p)$$

$$q_U(p) = \frac{(1-\alpha)\sigma(p - V_L)}{(1-\sigma)\{(1-\beta)\alpha(V_H - p) + (1-\alpha)(V_L - p)\}}.$$

It follows that $q_H(p) \geq q_U(p)$ iff

$$(1-\beta)\alpha + (1-\alpha)\frac{V_L - p}{V_H - p} \geq \frac{(1-\alpha)\sigma(p - V_L)}{V_H - c_H}$$

and as the right hand expression is increasing in p and the left hand expression is decreasing in p the inequality holds for all $p \in (V_L, c_H)$ iff it holds at $p = V_L$, which is obviously the case. This concludes the proof.