Unobserved Wholesale Contracts*

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Abstract

A manufacturer with private information about product quality may earn higher expected profit when their wholesale pricing contract with a retailer is unobserved by consumers. Secret wholesale contracts may prevent distortionary signaling by the manufacturer and double marginalization by the retailer. Instead, reasonable pooling outcomes exist where wholesale pricing is independent of quality, leaving the retailer and consumers in the dark about true quality. These outcomes may increase expected consumer and total surplus. The strategic interaction is different from standard signaling games. The pooling outcomes satisfy a new equilibrium refinement that we develop in the spirit of the Intuitive Criterion.

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1 Introduction

There are many markets where producers are much better informed than end users of the quality of the product. One important set of examples relate to ethical dimensions of the production process such as the environmental footprint, the working conditions of employees (including the use of child labor), employment discrimination, the use of genetically modified organisms, and prices paid to suppliers (fair trade). Producers in these markets sell their products through intermediaries (retailers), where, importantly, the (upstream) contracts are almost always unobserved by end users: the latter only know the terms at which they can buy the product from the intermediary. In this paper, we argue that by selling through an intermediary on the basis of a contract that is unobserved by end users, (*i*) producers may hide their private information and that (*ii*) this may yield higher industry profit and more consumer surplus than when upstream contracts between producers and intermediaries are observed by end users.

The key idea behind our result is simple. When wholesale contracts are observed by end users, manufacturers can signal their private information directly to end users and the resulting equilibrium involves potentially large inefficiencies due to signaling distortions. Instead, when a manufacturer sells through a retailer and the contract between the manufacturer and the retailer is not observed by consumers, the manufacturer can *hide* information about quality by selling to the retailer at a price that is independent of quality. As a result, the retail price does not convey any information about quality to the buyers. Signaling does not work as the final buyer (Receiver) does not observe the actions of the manufacturer (Sender). The resulting pooling equilibria not only eliminate the signaling distortions that arise when the manufacturer sells directly to buyers, but also avoid double marginalization by the retailer.[1]

[1] We do allow for two-part tariffs, but as we will show the fixed fee of the two-part tariff does not play an important role in our setting with unobservable wholesale contracts.

As the pooling equilibria associated with unobservable wholesale contracts may have relatively low prices on average, consumers are better off being in the dark about product quality and *not being able to infer* quality from prices. We also that a regulatory policy that requires all communication about product quality, for example through advertising, to be truthful may reduce expected consumer surplus as in the resulting full information outcome the manufacturer maximally exploits its market power, while he does not do so in any pooling equilibrium under unobservable wholesale contracts.

To make these points, we modify a well-known model of price signaling product quality due to Bagwell and Riordan (1991) where the manufacturer sells directly to consumers and the price charged reveals the manufacturer's private information about product quality. The resulting equilibrium may, however, be quite inefficient as in order to deter imitation by a low quality type, the high quality price should be sufficiently distorted upward.[2] In our framework, the manufacturer sells through an intermediary (a retailer) instead of selling directly to end users. Our results show that signaling distortions continue to play an important role when consumers observe wholesale contracts, but that they may disappear when consumers do not observe these contracts.

Equilibrium refinements are a crucial part of the signaling literature. In particular, the strategic interaction in Bagwell and Riordan (1991) falls within the class of standard signaling games where a Sender with private information sends a message that is observed by a Receiver who then chooses an action; a significant element in their analysis involves showing that pooling equilibria do not satisfy the Intuitive Criterion (Cho and Kreps, 1987). When the privately informed manufacturer sells

[2] Ellingsen (1997) analyzes price signaling of product quality by a seller when demand for every quality is perfectly inelastic. Janssen and Roy (2010) show that price signaling may also work in an oligopoly context where sellers are uninformed about the quality sold by their competitor.

to consumers via a retailer, the game is no longer a standard signaling game and equilibrium refinements such as the Intuitive Criterion developed for standard signaling games cannot be applied. If consumers observe the wholesale contract, then the pay-off to the manufacturer depends on the beliefs of the consumer and the action taken by the retailer, which in turn depends on their belief about consumers' beliefs. If consumers do not observe the wholesale contract and only observe actions by the retailer, then they must consider whether an out-of-equilibrium retail price has to be attributed to a unilateral deviation by the manufacturer or the retailer. A methodological contribution of this paper is to develop refinements similar to the Intuitive Criterion for these two particular interaction structures and show that our result satisfies these refinements. Although this paper is about quality uncertainty, our methodological contribution applies more broadly to vertical industry structures where the manufacturer has some private information.

In the game where the manufacturer delegates setting the retail price to a retailer and upstream vertical pricing is not observed by consumers, there is a continuum of pooling equilibria that satisfy the new refinement. A key part in the argument is that even though buyers may rule out that certain high retail prices stem from a unilateral deviation by the low quality manufacturer, they may not rule out the real possibility that the retailer has unilaterally deviated to these prices allowing them to have the same beliefs about quality as the retailer. Importantly, as the retailer does not know the quality of the product his action cannot reveal quality.[3]

Our paper contributes to the literature on the role of intermediaries in markets with asymmetric information about quality that has largely focused on information or certification intermediaries that use their own information, skill or reputation to

[3]As in a pooling equilibrium his cost is independent of product quality, the retailer cannot signal quality even if he knows it, as his incentives to choose actions are independent of whether quality is low or high.

provide information to buyers (Biglaiser 1993, Lizzeri 1999, Albano and Lizzeri 2001 and Glode and Opp 2016). In our framework, the intermediary retailer has no skill or market reputation and in fact, may have no more information about product quality than the uninformed consumer. In contrast to this literature, our key result is based on the beneficiary role of using a retailer to *hide* information from final consumers, and the result does not depend on the information the retailer has.

A number of papers have analyzed the role of leasing of new durable goods in reducing the extent of the lemons problem in the used goods markets. The leasing firm's opportunity cost of selling the used good (at the end of the lease) is determined prior to the realization of actual quality or performance of the used good and therefore independent of it; see, among others, Lizzeri and Hendel (2002) and Johnson and Waldman (2003). One may view leasing as selling the used good via an intermediary (the leasing firm). Further, the timing of actions rules out the possibility of signaling. Unlike this literature, our paper focuses on private information about producer's quality. The manufacturer is informed about quality before he sets the terms under which the retailer acquires the good and he chooses whether or not the retailer's cost of acquiring the good varies with quality. Signaling by the manufacturer (to the retailer) is potentially possible when wholesale contracts are unobserved, but the manufacturer abstains from doing so. Observability of the terms of the vertical contract by final consumers affects the market outcome significantly in our setting, whereas this does not play a role in the leasing literature.

Our result on regulatory policies penalizing false advertising not being in consumers' interest contributes to a recently revitalized literature analyzing the role of false or deceptive advertising (see, e.g., Rhodes and Wilson (2018), Piccolo et al. (2015), Janssen and Roy (2022) and an earlier paper by Daughety and Reinganum (1997)). These papers do not analyze vertical industries, however, and the mechanism we uncover is therefore very different from theirs.

There is also a large literature addressing the policy debate on whether financial kickbacks for financial intermediaries or advisors in other sectors should be made public so that consumers can understand the incentives involved. Our setting, the result and the mechanism we uncover are very different from that literature, however, as it is the manufacturer in our setting that has private information and they do not set the retail price the consumer is paying.

Finally, we contribute to the literature initiated by Bonanno and Vickers (1988), Katz (1989) and Hart and Tirole (1990) on the strategic use of vertical contracts. This literature generally shows that observable contracts with downstream firms may create a strategic advantage in the presence of market competition. Pagnozzi and Piccolo (2012) demonstrate the strategic gains when competing firms sell through independent retailers and vertical contracts are unobservable to competing firms. In contrast to this literature, our paper highlights the strategic advantage of selling through a retailer and of keeping the vertical contract secret from *consumers* even when there are no competing firms.[4]

The rest of the paper is organized as follows. Section 2 outlines the basic model of a privately informed manufacturer selling through a retailer. Section 3 considers the benchmark model where the wholesale contract is unobserved by consumers, whereas Section 4 covers our main result for markets where the wholesale contract is unobserved. Section 5 concludes with a discussion. Proofs and our methodological contribution related to the refinement for the model with unobservable contracts are contained in the Appendix.

[4]Fershtman and Kalai (1997) and Ok and Kockesen (2004), among others, study the effect of strategic delegation with unobservable contracts in games of perfect information.

2 The Model

Our basic framework is the one used by Bagwell and Riordan (1991) to analyze price signaling of product quality by a monopoly firm. The firm, who we shall henceforth refer to as the manufacturer, produces a good whose quality can be either high (H) or low (L). The unit cost of production is constant and depends only on the quality of the good. In particular, high quality has a unit cost $c > 0$, while the cost of low quality is normalized to zero. There is a unit mass of consumers. All consumers have unit demand. They have identical valuation $v_L > 0$ for the low quality product, while their valuation of high quality is uniformly distributed on $[v_L, 1 + v_L]$. Thus, if consumers face a price p and assign common probability μ to high quality, then the quantity demanded $d(p, \mu)$ is given by:

$$\begin{aligned} d(p,\mu) &= 0, \text{ if } p \geq \mu + v_L \\ &= 1 - \frac{p - v_L}{\mu}, \text{ if } p \in [v_L, \mu + v_L] \\ &= 1, \text{ if } p \leq v_L. \end{aligned} \tag{1}$$

The prior probability that quality is high is common knowledge and denoted by $\alpha \in (0, 1)$. The realized quality of the good is observed only by the manufacturer.

The manufacturer sells his product exclusively through an intermediary retailer. The manufacturer offers a wholesale pricing contract to the retailer that takes the form of a two part tariff with a fixed fee $f \geq 0$ and and a per unit wholesale price $w \geq 0$. If the retailer accepts the contract, he determines the retail price p at which he sells to the consumers. If the retailer does not accept the contract, the manufacturer does not sell. The retailer has no specific expertise, does not know the quality of the good provided by the manufacturer, and his outside option is zero. The only cost incurred by the retailer is what he pays the manufacturer for the good. His payoff is his expected profit net of this payment. The manufacturer's payoff is his expected profit and each consumer maximizes her expected net surplus.

We maintain the following assumption on the parameters in the rest of this paper:

$$c + v_L < 1, \tag{2}$$

$$\alpha > \max\{c, v_L\}. \tag{3}$$

The first assumption guarantees that signaling involves a distortion from optimal pricing under full information, while the second assumption guarantees (as we will see) that interesting pooling equilibria exist when wholesale contracts are unobserved.

3 Benchmark: Observable Vertical Contracts

In this section, we consider the version of the basic model where in addition to the retail price, consumers are able to observe the vertical contract, i.e., the two-part tariff set by the manufacturer.

The extensive form is as follows. First, nature draws the type τ (product quality) of the manufacturer from a distribution that assigns probability α to H (high quality) and $(1-\alpha)$ to L; only the manufacturer observes this move of nature. Next, the manufacturer chooses a two-part wholesale tariff (w, F) at which he offers to sell to the retailer; here, F is the fixed fee and w is the marginal wholesale price; the two-part tariff is observed by the retailer and the consumers. Next, the retailer chooses his retail price p which is also observed by consumers. Finally, consumers update their beliefs and make their purchase decisions. The expected payoff of the manufacturer of type τ is given by $(w - c_\tau)d(p,\mu) + F$ and that of the retailer is given by $(p - w)d(p,\mu) - F$.

We focus on symmetric pure strategy PBE with reasonable restrictions on the consumers' out-of-equilibrium beliefs. The manufacturer's strategy is given by $(w_\tau, F_\tau)_{\tau=H,L}$ where w_τ is the marginal wholesale price and F_τ is the fixed fee charged by manufacturer of type τ. The retailer's strategy is described by a function $p(w,F)$ that indicates the retail price charged when (w,F) is the two-part tariff set by the manufacturer.

This game is close to a standard signaling game where criteria like the Intuitive Criterion may apply. To apply such a criterion, the main question now is how to define the notion of equilibrium domination in view of the fact that whether a message is equilibrium dominated for the Sender (the manufacturer) may now depend on the action taken by the Intermediary (the retailer) and the effect this has on the Receiver's (buyers') response.

For our price signaling game, to see which type of manufacturer may have an incentive to deviate to some out-of-equilibrium contract $(\widehat{w}, \widehat{F})$ what matters is consumer demand $d(p, \mu(\widehat{w}, \widehat{F}))$, which depends on consumer beliefs and on the price set by the retailer, which in turn depends on $(\widehat{w}, \widehat{F})$ and on the *second-order belief* of the retailer about what consumers would believe about product quality. Note that while in any PBE (both on and off-the-equilibrium path) the second-order beliefs of the retailer must necessarily coincide with consumers' first-order beliefs as specified in the equilibrium, a potential issue arises when we want to determine the reasonableness of *out-of-equilibrium* beliefs by looking at the relative incentives of different types of the manufacturer to choose an out-of-equilibrium wholesale price.

Whether or not a deviation is profitable depends on the relation between consumer beliefs and the retailer's *second-order belief* about consumer beliefs. Obviously, the more optimistic consumers are about product quality and the less optimistic the retailer believes the consumer is, the more incentives the manufacturer has to deviate. To give the Intuitive Criterion some bite, it is natural to impose that first- and second-order beliefs are coordinated, i.e., that the retailer holds correct beliefs about the beliefs of consumers: if after observing a deviation by the manufacturer, consumers believe with probability μ that the manufacturer sells high quality, then the retailer also believes that consumers have belief μ. Coordinated beliefs are implied by, but weaker than, the retailer and the consumer having a "common prior" about the quality the manufacturer sells in the continuation game following the manufacturer's action (w, F). Coordinated beliefs seem natural as the manufacturer cannot control these beliefs and there does not seem to be any reason why the manufacturer should entertain the possibility that consumers' beliefs about quality should be different from the retailer's second-order beliefs of consumers' beliefs. Note that the retailer's own belief about quality does not play any role in determining his response to a deviation.

To define the Intuitive Criterion while requiring that first- and second-order beliefs are coordinated, suppose the manufacturer deviates from the equilibrium contract and chooses some out-of-equilibrium contract $(\widehat{w}, \widehat{F})$ and that the retailer sets his retail price assuming that demand is $d(p, \mu)$ at any retail price p, where for notational simplicity we suppress that μ may depend on $(\widehat{w}, \widehat{F})$. The optimal response of the retailer, denoted by $p((\widehat{w}, \widehat{F}), \mu)$, is then given by:

$$p((\widehat{w}, \widehat{F}), \mu) = \arg\max_{p \geq \widehat{w}} \left[(p - \widehat{w})d(p, \mu)\right].$$

Using this price reaction of the retailer to an out-of-equilibrium contract $(\widehat{w}, \widehat{F})$, and defining the equilibrium pay-off for type τ manufacturer as $\pi_\tau^* = (w_\tau^* - c_\tau)d(p^*(w_\tau^*)) + F^*, \tau = L, H$, we can directly apply the logic of the Intuitive Criterion and require that if for some $\tau \in \{H, L\}$ the out-of-equilibrium contract $(\widehat{w}, \widehat{F})$ is equilibrium dominated, i.e.,

$$\pi^*_\tau > (\widehat{w} - c_\tau)d(p(\widehat{w}, \mu), \mu) + \widehat{F} \text{ for all } \mu \in [0, 1],$$

while for $\tau' \in \{H, L\}, \tau' \neq \tau$ the out-of-equilibrium contract $(\widehat{w}, \widehat{F})$ is *not* equilibrium dominated, i.e.,

$$\pi^*_{\tau'} < (\widehat{w} - c_{\tau'})d(p(\widehat{w}, \mu), \mu) + \widehat{F} \text{ for some } \mu \in [0, 1],$$

then the out-of-equilibrium belief $\mu(\widehat{w}, \widehat{F})$ should be such that consumers believe that the manufacturer of type τ' has deviated with probability one.

We now characterize all equilibria that satisfy the Intuitive Criterion with coordinated beliefs as defined above. We begin by showing that there is no such equilibrium where the low and high quality types of the manufacturer pool on the same two-part tariff contract with positive probability.

Lemma 1 *When the manufacturer's upstream (two-part tariff) pricing is observable by consumers, there is no pooling or partially pooling equilibrium satisfying the Intuitive Criterion with coordinated beliefs.*

The proof of this result argues that when $w^* > v_L$, one can find deviations $(\widehat{w}, \widehat{F})$ such that, using the Intuitive Criterion with coordinated beliefs, consumers have to believe that they come from a high quality manufacturer, making these deviations profitable.[5] If $\widehat{w}$ is sufficiently large, low quality manufacturers would never have an incentive to deviate, while due to higher production cost one can still find wholesale prices in this range (and appropriately chosen fixed fees) that may be profitable for

[5]This part of the argument is similar to the argument used by Bagwell and Riordan (1991) to eliminate pooling equilibria under direct selling, but in our setting the manufacturer also has to take double marginalization by the retailer into account.

the high quality manufacturer. For $0 \leq w^* \leq \upsilon_L$, a low type manufacturer gains by reducing the marginal wholesale price to zero and charging a fixed fee close to its full information monopoly profit regardless of the beliefs of buyers.

From Lemma 1, it follows that only fully separating outcomes are consistent with reasonable restrictions on out-of-equilibrium beliefs. Our next result outlines a separating equilibrium that satisfies our refinement criterion. Further, we show that this is the least distortionary separating equilibrium. The retail prices faced by buyers are never lower than in this equilibrium, while the manufacturer's profit and consumer surplus generated in the low and high quality states are never higher.

Lemma 2 *Suppose the manufacturer 's upstream (two-part tariff) pricing is observable by consumers.*

(i) There exists a fully separating perfect Bayesian equilibrium satisfying the Intuitive Criterion with coordinated beliefs where the low quality manufacturer sets two-part tariff $(w_L, F_L) = (0, \upsilon_L)$ *earning profit* υ_L *and the high quality manufacturer sets two-part tariff* $(w_H, F_H) = (1 - \upsilon_L, (\upsilon_L)^2)$ *earning profit* $\upsilon_L(1-c)$*. In this equilibrium, the high quality good is sold at a retail price* $p_H = 1$ *while the low quality good is sold at retail price* υ_L*; the retailer earns zero profit, the ex ante expected manufacturer profit is*

$$\overline{\pi} = \upsilon_L(1 - \alpha c) \tag{4}$$

and the ex ante expected consumer surplus is $\frac{\alpha}{2}(\upsilon_L)^2$.

(ii) In any fully separating perfect Bayesian equilibrium satisfying the Intuitive Criterion with coordinated beliefs, the ex ante expected industry profit (and therefore, the ex ante expected manufacturer's profit) is bounded above by $\overline{\pi}$ *and the ex ante expected consumer surplus is bounded above by* $\frac{\alpha}{2}(\upsilon_L)^2$.

It is obvious that in any separating equilibrium, the low quality manufacturer must earn his full information profit υ_L. In the equilibrium outlined in Lemma 1(i), the high quality manufacturer signals his type by setting a two-part tariff (w_H, F_H) such that the incentive constraint of the low quality type is binding, i.e., the latter is indifferent between imitating the high quality type's action and sticking to his equilibrium action. As the two-part tariff (w_H, F_H) reveals quality to be high for sure, the optimal retail price set by a retailer accepting this contract is $(1/2)(1+\upsilon_L+w_H) = 1$ and the quantity sold by the high quality manufacturer is υ_L. Note that under full information, the high quality manufacturer optimally sets the marginal wholesale price $w = c$ and the retail price is then $\frac{1+\upsilon_L+c}{2} < 1$ ($\upsilon_L + c < 1$ under assumption (2)). Thus, the signaling equilibrium involves an upward distortion of the marginal wholesale and retail prices with consequent loss of profit and consumer surplus in the high quality state (relative to the full information outcome). The extent of this distortion is higher when c or υ_L is lower.

The high quality manufacturer would be eager to reduce the marginal wholesale price below w_H (and set a higher fixed fee) if buyers would be optimistic about quality after the deviation. To deter such a deviation, for any observed deviation to a two-part tariff (w, F) where $w \in (0, 1 - \upsilon_L)$, we choose out-of-equilibrium beliefs of buyers that assign probability one to the manufacturer being of low quality. These pessimistic beliefs are reasonable: we show that if the high quality manufacturer gains from such a deviation for some belief of buyers, then the low quality manufacturer can also gain from the same deviation. In particular, these beliefs meet the Intuitive Criterion with coordinated beliefs. The out-of-equilibrium beliefs assign probability one to high quality when the manufacturer deviates to a contract where the marginal wholesale price is higher than w_H. As the high quality manufacturer has higher marginal cost, this is a reasonable belief and is consistent with the Intuitive Criterion with coordinated beliefs.

It is worth noting that the equilibrium outlined in Lemma 2(i) yields the same outcome in terms of prices paid by buyers (and distortion in industry profits and consumer surplus) as in the unique equilibrium satisfying the Intuitive Criterion when the manufacturer sells directly to buyers. The latter is fully characterized in Bagwell and Riordan (1991).

Part (ii) of Lemma 2 asserts that any separating equilibrium satisfying our refinement must generate at least as much distortion in profits and consumer surplus as the equilibrium outlined in part (i). In other words, the equilibrium described in part (i) is the least distortionary separating equilibrium which is not surprising as the low quality manufacturer's incentive constraint is binding in this equilibrium.

Combining Lemmas 1 and 2, we have the main result of this section:

Proposition 1 *When the manufacturer"s upstream (two-part tariff) pricing is observable by consumers, in any perfect Bayesian equilibrium satisfying the Intuitive Criterion with coordinated beliefs, the ex ante expected industry profit (sum of manufacturer and retailer's profit) is bounded above by $\overline{\pi} = \upsilon_L(1 - \alpha c)$ and the ex ante expected consumer's surplus is bounded above by $\frac{\alpha}{2}(\upsilon_L)^2$.*

4 Unobservable Wholesale Contracts

In this Section, we consider the situation where consumers do not observe the wholesale contracts set by the manufacturer. We show that this can lead to pooling equilibrium outcomes that yield higher profits for the manufacturer and may also lead to higher consumer surplus.

The extensive form game is as follows. As in the previous section, nature draws the type τ (product quality) of the manufacturer from a distribution that assigns probability α to H (high quality) and $(1-\alpha)$ to L. Only the manufacturer observes this move of nature. Following this, the manufacturer chooses a wholesale contract (w, F) which he offers to the retailer. The key difference with the previous section is that this two-part wholesale tariff is observed by the retailer but not by the consumers. Next, the retailer chooses his retail price p which is observed by consumers. Consumers update their beliefs on the basis of the retail price and make their purchase decisions.

Note that this information structure is very different from a standard signaling game as the "Receivers" (consumers) do not observe the actions (w and F) of the privately informed "Sender" (the manufacturer), but only the action p of the retailer (an intermediary) who is himself not informed. Deviations from the equilibrium path are observed by consumers ("Receivers") only if the retail price is different from the equilibrium retail price. In forming their beliefs in such situations, consumers consider whether the observed retail price is due to a deviation by the manufacturer or the retailer or both. It is easy to see that different notions of equilibrium refinements for standard signaling games cannot be readily applied here. In Appendix B, we outline a notion of equilibrium refinement for a class of games of this sort (which we call *Intermediated Signaling Games*), where a Sender with private information chooses an action that is only observed by an intermediary who himself chooses another action that is observed by the receiver and where the Sender's pay-offs depend on the actions taken by the Intermediary and the Receiver. The refinement, the *Intuiitive Criterion for Intermediated Signaling Games (IC-I)* is based on considerations similar to the Intuitive Criterion for standard signaling games. In outlining the pooling equilibria in this section, we describe informally how IC-I works and how the beliefs underlying these equilibria satisfy this refinement.

Consider pooling equilibria where the manufacturer sets a wholesale price w^* and a fixed fee $F^* = 0$ regardless of product quality. On the equilibrium path, the retailer follows up by selling at a retail price $p^* = w^*$. After observing the retail price p^*, a buyer's updated belief is identical to her prior belief, i.e., $\mu(p^*) = \alpha$, while the manufacturer and the retailer sell quantity $d(p^*, \alpha)$. Under assumption (3), $\alpha > \max\{\upsilon_L, c\}$. We focus on outcomes where

$$\alpha > p^* = w^* \geq \max\{\upsilon_L, c\}. \tag{5}$$

The second inequality above must be satisfied as otherwise the retailer wants to deviate (to υ_L) if p^* were smaller than υ_L, or the high quality manufacturer would want to deviate (if $w^* < c$). The first inequality will be used later to ensure the low quality manufacturer has no incentive to reduce his wholesale price. Note that (5) implies $p^* = w^* < \alpha + \upsilon_L$ so that the manufacturer sells a strictly positive quantity.

We begin by specifying the symmetric out-of-equilibrium belief $\mu(p)$ of buyers that we use to sustain the above set of outcomes in an equilibrium of the game:

$$\begin{aligned} \mu(p) &= 0 \text{ if } p \in (\upsilon_L, p^*) \text{ or if } p \in (p^*, \alpha + \upsilon_L) \\ &= \alpha \text{ if } p \in [\alpha + \upsilon_L, 1 + \alpha). \end{aligned} \tag{6}$$

Below, we argue when these beliefs are reasonable (and satisfy our equilibrium refinement). We allow for any belief $\mu(p) \in [0, 1]$ for $p \geq 1 + \alpha$ as no buyer buys at such retail price regardless of belief. Similarly, we allow for any belief $\mu(p) \in [0, 1]$ for retail price $p \leq \upsilon_L$ as it is optimal for all buyers to buy regardless of their belief.

Given these beliefs, it is easy to see that the buyers' optimal strategy is summarized by the quantity $q(p)$ purchased where $q(p) = d(p, \mu(p)), p \neq p^*$, and $q(p^*) = d(p^*, \alpha)$.

Thus,

$$
\begin{aligned}
q(p) &= 1 \text{ if } p \le \upsilon_L && (7)\\
&= 0 \text{ if } p \in (\upsilon_L, p^*) \text{ or if } p > p^*\\
&= 1 - \frac{p - \upsilon_L}{\alpha} \text{ if } p = p^*
\end{aligned}
$$

Note that $q(p) = d(p, \alpha) = 0$ for $p \ge \alpha + \upsilon_L$ as given the belief $\mu(p) = \alpha$ if $p \in [\alpha + \upsilon_L, 1 + \alpha)$ buyers find the price too high. Also, buyers do not buy at retail price $p \in (p^*, \alpha + \upsilon_L)$ or $p \in (\upsilon_L, p^*)$ as they believe the product is of low quality for sure.

We stipulate that the retailer's equilibrium pricing strategy if he accepts a wholesale contract with two part tariff (w, F) is as follows:

$$
\begin{aligned}
p(w, F) &= w, \quad \text{if } w \ge p^* && (8)\\
&= p^*, \text{ if } w \le p^*
\end{aligned}
$$

and further, the retailer accepts this contract if, and only if,

$$F \le [p(w, F) - w]q(p(w, F)),$$

i.e., the retailer makes non-negative profit.

We have already noted that the buyers' strategy is optimal given the beliefs. Consider the retailer's strategy (8). Note that the retailer's equilibrium profit is zero. If the per unit wholesale price $w \ge p^*$, the retailer can never sell a positive quantity by deviating to a retail price above w (as $q(p) = 0$ for $p > p^*$). Hence, it optimal for the retailer to set the retail price equal to w. In particular, the out-of-equilibrium beliefs eliminate double marginalization by the retailer. If the per unit wholesale price $w \in (\upsilon_L,\ p^*)$, the only retail price above w at which the retailer can sell a positive quantity is p^* and so it is optimal to charge p^*. If $w \in [0, \upsilon_L]$, the retailer can choose between charging υ_L (and selling to all buyers) and charging p^* (selling $q(p^*) = d(p^*, \alpha)$). Using the fact that $p^* < \alpha$ one can check the latter is optimal for the retailer.[6]

We now argue that regardless of his type, the manufacturer has no incentive to deviate from the pooling two-part tariff ($w^* = p^*, F^* = 0$). The equilibrium payoffs of the low, respectively high, quality manufacturer are given by

$$\pi_L^* \equiv \left(1 - \frac{p^* - \upsilon_L}{\alpha}\right) p^* \text{ and } \pi_H^* \equiv \left(1 - \frac{p^* - \upsilon_L}{\alpha}\right)(p^* - c),$$

If the manufacturer deviates to a contract $(\widehat{w}, \widehat{F})$ where $\widehat{w} \in [0, p^*)$ and if such a contract is accepted, then (as per the retailer's equilibrium strategy) it leads to the same retail price as at $w = w^* = p^*$ and therefore cannot be more profitable. On the other hand, the manufacturer can never sell by deviating to a contract $(\widehat{w}, \widehat{F})$ where $\widehat{w} > w^* = p^*$ as the retailer, if he accepts such a contract, will set a retail price equal to $\widehat{w}$ and buyers are too pessimistic about product quality to buy at such a retail price. Thus, the strategies and beliefs outlined above constitute a perfect Bayesian equilibrium (PBE).

The out-of-equilibrium beliefs (6) play a very important role in sustaining the pooling PBE outcome outlined above. The IC-I refinement we formally develop in Appendix B applies to actions of the intermediary that can arise due to a *unilateral* deviation by the sender and/or the intermediary. For all such actions, the IC-I uses the notion of equilibrium domination that is also at the heart of the Intuitive Criterion to impose restrictions on beliefs. We informally describe the main idea behind the

[6] For $w \in [0, \upsilon_L]$, $(p^* - w)[1 - \frac{p^* - \upsilon_L}{\alpha}] \geq \upsilon_L - w$ if $w \geq p^* - \alpha$ and the latter holds as $p^* \leq \alpha$.

refinement here and discuss how it applies to the beliefs (6) that sustain the pooling PBE described above.

In the pooling equilibrium described above, any observed off-equilibrium retail price $p > p^*$ can result from a unilateral deviation by the retailer (given the pooling equilibrium strategy (w^*, F^*) of the manufacturer), but it can also be the consequence of a unilateral deviation by the manufacturer (given the equilibrium strategy (8) of the retailer). Our refinement IC-I gives any account that attributes a retail price $p > p^*$ to a unilateral deviation of one of the players priority over attributing the observed retail price to multiple deviations.

Given the equilibrium strategy (8) of the retailer, an off equilibrium retail price $p \in (p^*, \alpha + \upsilon_L)$ may be attributed by the buyers entirely to a unilateral deviation by the manufacturer. A low quality manufacturer can strictly gain by deviating to a two part tariff $(w = p, F = 0)$ which will lead to a retail price $p \in (p^*, \alpha + \upsilon_L)$ when buyers assign probability 1 to high quality if

$$\pi_L^* = \left(1 - \frac{p^* - \upsilon_L}{\alpha}\right) p^* < d(p, 1)p.$$

If the inequality holds, such a deviation is not equilibrium dominated for the low quality manufacturer and it is perfectly reasonable for the buyer to think the observed retail price is caused by a unilateral deviation of the low quality manufacturer. Thus, the buyer may assign $\mu(p) = 0$ for $p \in (p^*, \alpha + \upsilon_L)$ when it observes such a retail price. For this reason, the belief $\mu(p) = 0$ for $p \in (p^*, \alpha + \upsilon_L)$ meets our refinement as long as $\pi_L^* < d(p, 1)p$ for all such p. It is easy to see the above inequality is satisfied for p slightly above p^*. Using the concavity of the profit $d(p, 1)p$ one can see that the inequality holds for all $p \in (p^*, \alpha + \upsilon_L)$ as long as

$$\left(1 - \frac{p^* - \upsilon_L}{\alpha}\right) p^* < d(\alpha + \upsilon_L, 1)(\alpha + \upsilon_L) \ = (1 - \alpha)(\alpha + \upsilon_L). \tag{9}$$

The next lemma states parameters for which this is the case.

Lemma 3 *There exists* $\underline{p} \in [\max\{\upsilon_L, c\}, \alpha)$ *such that (9) holds for all* $p^* \in [\underline{p}, \alpha]$ *if, and only if,* $\upsilon_L < 1 - \alpha$. *Further, we can choose* $\underline{p} = \max\{\upsilon_L, c\}$ *if* $\upsilon_L < \alpha(3 - 4\alpha)$.

Let us now consider $p \in [\alpha + \upsilon_L, 1 + \upsilon_L)$. It is clear that not all these prices can be attributed to a unilateral deviation of the low quality manufacturer. In particular, prices close to $1 + \upsilon_L$ will yield a profit that is close to 0 and therefore lower than the equilibrium profit. However, if we can argue that these deviations can be attributed to a unilateral deviation by the retailer, then it is reasonable to have $\mu(p) = \alpha$ for all $p \in [\alpha + \upsilon_L, 1 + \upsilon_L)$. It is clear that given the equilibrium strategy of the manufacturer, a unilateral deviation by the retailer to any $p \in [\alpha + \upsilon_L, 1 + \upsilon_L)$ can yield strictly positive profit to the retailer and is therefore strictly gainful if buyers assign sufficiently high probability to quality being high. In particular, no such retail price is equilibrium dominated for the retailer. Thus, it is reasonable for the buyers to attribute a retail price $p \in [\alpha + \upsilon_L, 1 + \upsilon_L)$ entirely to a unilateral opportunistic deviation by the retailer. As the retailer is uninformed and only observes a pooling contract from the manufacturer when he sets the retail price, his deviation does not provide any more information about product quality. Thus, as a buyer may attribute the deviation to the retailer his belief $\mu(p)$ could be identical to her prior belief α.

Finally, consider off equilibrium retail price $p \in (\upsilon_L, p^*)$. For any $p \in (\upsilon_L, p^*)$, it is clear that such a price cannot be attributed to a unilateral deviation by the retailer (as it is equilibrium dominated given the manufacturer's strategy), while given the strategy of the retailer it can also not be attributed to a unilateral deviation by the manufacturer. The refinement criterion IC-I we outline in Appendix B confines attention to incentives for unilateral deviation and does not impose any restriction on beliefs in the more complex case of joint deviations by the intermediary. Thus, a belief $\mu(p) = 0$ for all $p \in (v_L, p^*)$ meets our refinement. However, for our specific game and equilibrium, one can can intuitively argue that the low quality manufacturer has a stronger incentive (than the high quality manufacturer) to deviate to a lower per unit wholesale price that would be needed for the downward deviation in the retail price so that the specified belief is intuitive.[7]

The next proposition summarizes the above discussion.

Proposition 2 *Suppose the manufacturer chooses a wholesale contract that is not observed by consumers and that* $v_L < 1 - \alpha$. *Then, there exists* $\underline{p} \in [\max\{v_L, c\}, \alpha)$ *such that for every* $p^* \in [\underline{p}, \alpha]$ *there is a pooling PBE satisfying IC-I where both types of the manufacturer offer identical two part tariff* (w^*, F^*),*where* $F^* = 0$ *and the retailer is fully squeezed.*

Proposition 2 provides a condition under which there is a continuum of pooling equilibria supported by reasonable beliefs. In the pooling PBE described above, the manufacturer is already able to fully extract the retailer's rent. It is easy to show that these pooling outcomes can also be sustained if the manufacturer uses a fixed fee $F > 0$ and set some $w^* < p^*$. In this case, the fixed F can also adjust so as to give the retailer more rent if he otherwise would not participate.

[7] One may want to consider joint deviations of both the manufacturer and the retailer to investigate whether $\mu(p) = 0$ is a reasonable out-of-equilibrium belief. It is easy to check that if the high quality manufacturer gains by reducing his wholesale price from w^* to $w \leq p < p^*$ for some belief of buyers μ' (after observing retail price p), *i.e.*, if $(w - c)d(p, \mu') \geq (p^* - c)d(p^*, \alpha)$, then the low quality manufacturer must strictly gain from this deviation, *i.e.*, $wd(p, \mu') > p^*d(p^*, \alpha)$ indicating that if some type of manufacturer would want to induce the retailer to set p it is certainly the low quality manufacturer that has an incentive to do so. Thus, it seems that any restriction on beliefs based on joint deviations, should allow for buyers to hold the belief $\mu(p) = 0$ at $p \in (v_L, p^*)$.

Note that the belief refinement has bite and restricts the set of parameters for which a class of pooling PBE outcomes where the retailer is fully squeezed exists. Without refinement, one could simply stipulate $\mu(p) = 0$ for all $p > p^*$. This would immediately imply that neither the manufacturer nor the retailer would have an incentive to deviate upwards. The requirement requires that one can find a non-empty interval of equilibrium prices where (9) holds and Lemma 3 argues that this is only the case if $\upsilon_L < 1 - \alpha$.

Next, we investigate the conditions under which some of these pooling outcomes can yield higher ex ante expected profit and expected consumer surplus than when wholesale contracts are observable. Recall that Proposition 1 establishes that when the manufacturer's upstream (two-part tariff) pricing is observable by consumers, in any perfect Bayesian equilibrium satisfying the Intuitive Criterion with coordinated beliefs, the ex ante expected industry profit (sum of manufacturer and retailer's profit) is bounded above by $\overline{\pi} = \upsilon_L(1 - \alpha c)$ and the ex ante expected consumer's surplus is bounded above by $\frac{\alpha}{2}(\upsilon_L)^2$.

Proposition 3 *Suppose the manufacturer chooses a wholesale contract that is not observed by consumers and* $\upsilon_L < 1 - \alpha$*.There exists a continuum of pooling PBE satisfying IC-I as described in Proposition 2 such that the ex ante expected consumer surplus in every such equilibrium is strictly higher than* $\frac{\alpha}{2}(\upsilon_L)^2$*, the upper bound of expected consumer surplus under observable wholesale contract. Further, the following hold:*

(i) there exists $c_0 > 0$ *such that if* $c \in (0, c_0)$*,there are pooling PBE satisfying IC-I as described in Proposition 2 where the ex ante expected profit for the manufacturer (and the industry) is strictly higher than* $\overline{\pi}$*;*

(ii) there exists $\upsilon_0 > 0$ *such that the conclusion in (i) holds if* $\upsilon_L \in (0, \upsilon_0)$*.*

For the pooling equilibria under unobservable contracts to yield higher profits than in any separating equilibrium under observable contracts, we essentially need that the signaling distortions in the latter equilibria are large enough. In the previous section we have argued that this is the case if υ_L and/or c are small enough. This is the key message of Proposition 3. The fact that consumers are also better off can be understood as follows. Under observable contracts, consumers only get a surplus from buying when the quality is high. The prices that can be sustained in a pooling equilibrium under unobservable contracts are smaller than α, which in turn are restricted to be smaller than $1 - \upsilon_L$. Thus, consumers substantially gain in case quality is high. In case quality is low, they make a negative surplus as $\upsilon_L < p^*$. It turns out that this loss is dominated by the gain when quality is high. A simple way to convey the intuition of why this may be so is to consider the (lowest) expected price at which consumer buy in the separating equilibria under observable wholesale contracts, which is equal to $\alpha \cdot 1 + (1 - \alpha)\upsilon_L$. This is clearly larger than α, the highest pooling price that can be sustained under observable contracts.

Finally, we compare the pooling equilibrium outcomes described above with the outcome under full information. One can view the full information outcome as the market outcome if the manufacturer can credibly disclose product quality at sufficiently low cost. In particular, one can see it as the outcome of a regulatory policy that imposes severe fines for false advertising. To begin, we compare the consumer surplus under full information to that in the pooling equilibrium outcomes in Proposition 2. The retail prices in the low and high quality states under full information are υ_L and $\frac{1+c+\upsilon_L}{2}$, respectively. The expected consumer surplus under full information is therefore given by $\frac{\alpha}{8}\left(1 + \upsilon_L - c\right)^2$. On the other hand, the expected consumer surplus in a pooling equilibrium outcome with retail price p^* is $\frac{1}{2\alpha}(\alpha + \upsilon_L - p^*)^2$ and this exceeds the full information consumer surplus if $(\alpha + \upsilon_L - p^*)^2 > \frac{\alpha^2}{4}\left(1 + \upsilon_L - c\right)^2$

i.e.,

$$p^* < \frac{\alpha}{2}(1+c) + \left(1 - \frac{\alpha}{2}\right) \upsilon_L. \tag{10}$$

The next proposition considers the situation where every $p^* \in [\max\{\upsilon_L, c\}, \alpha]$ can be sustained as the pooling price in an equilibrium described in Proposition 2 and shows that the inequality (10) is satisfied for all p^* that lie below the pooling price that maximizes expected industry profits on this interval. For all such pooling prices, expected consumer surplus is higher than that under full information. Further, the expected total social surplus generated in some of these pooling equilibria can be higher than under full information.

Proposition 4 *Suppose that* $\upsilon_L < \min\{1-\alpha, \alpha(3-4\alpha)\}$ *so that every* $p^* \in [\max\{\upsilon_L, c\}, \alpha]$ *can be supported as the retail price in a pooling equilibrium described in Proposition 2. Let* $\widetilde{p} \in [\max\{\upsilon_L, c\}, \alpha]$ *be the pooling price that maximizes expected industry profit across these equilibria.*

(a) The expected consumer surplus generated in a pooling outcome with $p^* \leq \widetilde{p}$ *is strictly higher than the expected consumer surplus under full information*

(b) If c *is sufficiently small, pooling equilibrium outcomes with* p^* *close to* υ_L *generate higher expected total social surplus than in the full information outcome. If, in addition,* $\upsilon_L > \frac{2\alpha}{3(1+\alpha)}$*, this holds for all* $p^* \leq \widetilde{p}$*.*

Thus, in a strong sense, consumers may benefit from not knowing quality. A regulatory policy that aims to enable credible disclosure of product quality can benefit the firms but lower consumer surplus as firms on average increase their prices and are able to achieve maximum profits under full information. The effect on consumer surplus can be so large that it more than offsets the decrease in industry profits. Thus, total surplus can also be higher in a pooling equilibrium; in the presence of market power, a vertical market with asymmetric information about product quality and unobserved vertical contracts may be more efficient than the full information market.

5 Discussion and Conclusion

In this paper we show that having upstream contracts to be unobserved by end users allows manufacturers to hide their private information, creating the possibility of higher profits. If, on the contrary, wholesale contracts are observed, manufacturers signal their private information resulting in potentially large signaling distortions. Interestingly, consumers are typically also better off not knowing the quality of the products they buy as they buy on average at lower prices. Hiding information is only possible if the manufacturer sells through a third party (a retailer) who is free to determine the parameters (retail prices) that are of interest to consumers. The outcomes of the vertical industry when wholesale contracts are observed mimic the outcomes when the manufacturer sells directly to end users.

Policies that enable credible disclosure of product information, for example by introducing severe penalties on false communication (advertising) by firms, may backfire. With such policies in place, a high quality manufacturer will always directly communicate quality and the resulting equilibrium is equivalent to the full information outcome where manufacturers exploits their market power. Even the pooling equilibrium that maximizes expected profit may yield higher consumer surplus and sometimes, higher total social surplus, than the full information outcome.

The methodology we develop on how to think of equilibrium refinement in "intermediate signaling games" where a Sender with private information sends an action to an Intermediary that is not observed by the Receiver should be of interest in a wide set of other strategic situations where the sender may potentially want to hide information by not interacting directly with the Receivers.

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Appendix

Appendix A (Proofs)

Proof of Lemma 1. We begin stating with some useful facts. Suppose that the (coordinated) belief is that quality is high with probability μ. Then for any unit wholesale price $w \leq \mu + \upsilon_L$ the optimal price set by the retailer (if he accepts the contract) is

$$\begin{aligned} p(w,\mu) &\geq \upsilon_L + \mu, \text{ if } w \geq \upsilon_L + \mu \\ &= \frac{\mu + \upsilon_L + w}{2}, \text{ if } w \in [\upsilon_L - \mu, \upsilon_L + \mu) \\ &= \upsilon_L, \text{ if } w < \upsilon_L - \mu. \end{aligned} \tag{11}$$

For $w > \mu + \upsilon_L$, the retailer sells zero at any $p \geq w$ and so $p(w,\mu)$ is any price at least as large as w. The quantity sold by the retailer is then $d(p(w,\mu),\mu) = \frac{\mu+\upsilon_L-w}{2\mu}$,if $w \in [\upsilon_L - \mu, \upsilon_L + \mu]$; further, $d(p(w,\mu),\mu) = 1$,if $w \leq \upsilon_L - \mu$, and equals zero if $w \geq \upsilon_L + \mu$.Note that given $w \geq \upsilon_L$, $d(p(w,\mu),\mu)$ is non-decreasing in μ and for $w < \upsilon_L$, $d(p(w,\mu),\mu)$ is non-increasing in μ. Consider a perfect Bayesian equilibrium satisfying the Intuitive Criterion with coordinated beliefs where the H and L of the manufacturer pool with strictly positive probability and let (w^*, F^*) be a pooling contract that can be offered in equilibrium by the manufacturer.Observe that $c \leq w^* \leq \alpha + \upsilon_L, F^* \leq (p^* - w^*)d(p^*, \alpha)$.The retailer's equilibrium strategy $p_R(w,F)$ in such an outcome must be such that $p^* = p_R(w^*, F^*)$ is given by

$$\begin{aligned} p^* &= \frac{\alpha + \upsilon_L + w^*}{2}, \text{ if } w^* \in [\upsilon_L - \alpha, \upsilon_L + \alpha] \\ &= \upsilon_L, \text{ if } w^* < \upsilon_L - \alpha. \end{aligned}$$

Note that the equilibrium profit of the high type manufacturer would then be $\pi^H =$

$\frac{1}{2\alpha}(w^*-c)(\alpha+\upsilon_L-w^*)+F^*$, if $w^* \geq \upsilon_L-\alpha$, and $\pi^H=(w^*-c)+F^*$, if $w^*<\upsilon_L-\alpha$; the equilibrium profit of the high type manufacturer is $\pi^L=\frac{1}{2\alpha}w^*(\alpha+\upsilon_L-w^*)+F^*$, if $w^* \geq \upsilon_L-\alpha$,and $\pi^L=w^*+F^*$,if $w^*<\upsilon_L-\alpha$. Consider any unit wholesale price $w \in (w^*, 1+\upsilon_L)$ and an associated fixed fee $F(w)$,where $F(w)=[p(w,1)-w]d(p(w,1),1)$.The profit earned by the low type manufacturer by deviating to such $(w, F(w))$ when buyers' belief is $\mu=1$ equals $g(w)=\frac{1}{4}(1+\upsilon_L+w)(1+\upsilon_L-w)$. Note that $g(w)$ is continuous (and strictly decreasing) in w on $[\upsilon_L, 1+\upsilon_L]$. As $w \downarrow w^*$, $g(w) \to p(w^*,1)d(p(w^*,1),1)=\frac{1}{4}(1+\upsilon_L+w^*)\left(1-(w^*-\upsilon_L)\right)>\frac{1}{4}(\alpha+\upsilon_L+w^*)\left(1-\frac{w^*-\upsilon_L}{\alpha}\right)=p^*d(p^*,\alpha)=w^*d(p^*,\alpha)+(p^*-w^*)d(p^*,\alpha) \geq \frac{1}{2\alpha}w^*(\alpha+\upsilon_L-w^*)+F^*=\pi^L$.[To see the first inequality observe that for any $w^* \geq 0$, the function $f(\mu)=\frac{1}{4\mu}[(\mu+\upsilon_L)^2-(w^*)^2]$ is strictly increasing in μ on $[\upsilon_L, 1]$ and as $\alpha \in (\upsilon_L, 1]$ under assumption (3), $f(1)>f(\alpha)$.]. On the other hand, as $w \uparrow (1+\upsilon_L)$, $g(w) \to 0$. Thus, there exists a unique $w_0 \in (w^*, 1+\upsilon_L)$ such that $g(w_0)=\pi^L$. We now claim that the low type manufacturer loses by deviating to the contract $(w_0, F(w_0))$ for any belief $\mu \in [0,1]$. As noted above, $w_0>\upsilon_L$ implies $d(p(w_0,\mu),\mu)$ is non-decreasing in μ. So, if the contract $(w_0, F(w_0))$ is feasible for belief μ, (i.e., the retailer makes non-negative profit) the low type manufacturer's deviation profit:$w_0 d(p(w_0,\mu),\mu)+F(w_0)=g(w_0)-w_0[d(p(w_0,1),1)-d(p(w_0,\mu),\mu)] \leq g(w_0)=\pi^L$. If the contract (w^*, F^*) is not feasible for belief μ, the low type manufacturer makes zero profit. Thus, regardless of the beliefs of buyers, the low type manufacturer can never gain by deviating to a contract $(w_0, F(w_0))$. Note that $g(w_0)=\pi^L$ implies $p(w_0,1)d(p(w_0,1),1)=w^*d(p^*,\alpha)+F^* \leq p^*d(p^*,\alpha)=\frac{\alpha+\upsilon_L+w^*}{2}d(p^*,\alpha)$. As $\alpha<1, w_0>w^*$, $p(w_0,1)=\frac{1+\upsilon_L+w_0}{2}>\frac{\alpha+\upsilon_L+w^*}{2}$, it follows that $d(p(w_0,1),1)<d(p^*,\alpha)$.If a high type manufacturer deviates to a contract $(w_0, F(w_0))$ and belief is $\mu=1$ his deviation profit is: $(p(w_0,1)-c)d(p(w_0,1),1)=g(w_0)-cd(p(w_0,1),1)=\pi^L-c\ d(p(w_0,1),1)=w^*d(p^*,\alpha)+F^*-c\ d(p(w_0,1),1)=\pi^H+c[d(p^*,\alpha)-d(p(w_0,1),1)]>\pi^H$, using

$d(p(w_0,1),1) < d(p^*,\alpha)$. Note that $(w_0, F(w_0))$ must be an out-of-equilibrium contract.[8] Based on the above arguments, the Intuitive Criterion with coordinated beliefs requires that the out-of-equilibrium belief satisfy $\mu(w_0, F(w_0)) = 1$,which immediately implies that the high quality manufacturer has an incentive to deviate to $(w_0, F(w_0))$, a contradiction. This completes the proof.

Proof of Lemma 2.(i) Observe that if L type deviates to $(w_H, F_H) = (1 - \upsilon_L, (\upsilon_L)^2)$, the retail price would be 1 and the quantity sold is $d(1,1) = \upsilon_L$ and his deviation profit would be $\upsilon_L(1-\upsilon_L) + (\upsilon_L)^2 = \upsilon_L$ and his incentive constraint holds with equality. If H type deviates to $(w_L, F_L) = (0, \upsilon_L)$, the quantity sold would be 1 and his deviation profit would be $\upsilon_L - c < (1-c)\upsilon_L$, his equilibrium profit. It is obvious that given the restriction on out-of-equilibrium beliefs outlined in the lemma (belief assigns probability one to low quality), neither type of the manufacturer can gain strictly by deviating to a two-part tariff (w, F) where $w \in (w_L, w_H)$. So. consider deviation to (w, F) where $w > w_H = 1-\upsilon_L$; here, the out-of-equilibrium beliefs assign probability one to high quality; the retail price following such deviation is $\frac{1+\upsilon_L+w}{2}$. If the manufacturer is of high quality, his deviation profit is bounded above by the industry profit following this deviation which is $\frac{1}{4}(1+\upsilon_L-w)(1+\upsilon_L+w-2c)$; the latter is strictly increasing in w for $w > c$; as $c < 1-\upsilon_L < w$, this industry profit is less than the equilibrium profit of H type. Same holds for L type. So, we only need to show that the out-of-equilibrium beliefs meet our refinement.

First, consider deviation two-part tariff (w, F) where $w \in (w_L, w_H)$. It is sufficient to show that if such a deviation is gainful for the H type manufacturer for some coordinated belief $\mu \in [0,1]$, then it is not equilibrium dominated for the L type

[8] $(w_0, F(w_0))$ cannot be a separating contract offered with positive probability by the H (L) type manufacturer in this equilibrium as it yields strictly higher (strictly lower) payoff to the H (L) type mamufacturer than the pooling contract (w^*, F^*) when $\mu = 1$ $(\mu = 0)$. Further, it cannot be a pooling contract offered with positive probability both types of the manufacturer as it yields strictly lower payoff to the L type manufacturer than the pooling contract (w^*, F^*) when $\mu = \alpha$.

manufacturer so that assigning probability one to L type does not violate the Intuitive Criterion. For the deviation to be gainful for the H type manufacturer for belief μ,

$$(w-c)d(p(w,\mu),\mu)+F \geq (1-c)\upsilon_L, \tag{12}$$

where $p(w,\mu)$ and $d(p(w,\mu))$ are as indicated at the beginning of the proof of Lemma 1. Note that for $w \geq \upsilon_L$, $d(p(w,\mu),\mu)$ is non-decreasing in μ and for $w < \upsilon_L$, $d(p(w,\mu),\mu)$ is non-increasing in μ. In particular, for $w \geq \upsilon_L$, (12) implies $(w-c)d(p(w,1),1)+F \geq (1-c)\upsilon_L$ so that $wd(p(w,1),1)+F-\upsilon_L \geq c[d(p(w,1),1)-\upsilon_L] = c[\frac{1+\upsilon_L-w}{2}-\upsilon_L] > 0$, as $w < w_H = 1-\upsilon_L$. This implies that the L type manufacturer strictly gains by deviating to the same contract if the coordinated belief puts probability one on high quality and thus the deviation is not equilibrium dominated for the L type. Now suppose $w < \upsilon_L$. Then, $d(p(w,\mu),\mu) \leq 1$ and (12) implies $(w-c)+F \geq (1-c)\upsilon_L$,i.e., $w+F-\upsilon_L \geq c(1-\upsilon_L) > 0$ so that the L type manufacturer strictly gains by deviating to the same contract if the coordinated belief probability zero on high quality and thus the deviation is not equilibrium dominated for the L type.

Next, consider out-of-equilibrium two-part tariff (w,F) where $w > w_H = 1-\upsilon_L$; here, our specified beliefs assign probability one to high quality and we need to show that this is consistent with our refinement. Suppose $w \geq \upsilon_L$. Then, the L type's deviation profit at any coordinated belief μ: $wd(p(w,\mu),\mu)+F \leq p(w,\mu)d(p(w,\mu),\mu) = \frac{1}{4\mu}[(\mu+\upsilon_L)^2-w^2] \leq \frac{1}{4}[(1+\upsilon_L)^2-w^2] < \frac{1}{4}[(1+\upsilon_L)^2-(1-\upsilon_L)^2] = \upsilon_L$ (the last inequality follows from $w > w_H = 1-\upsilon_L$). Thus, (w,F) is equilibrium dominated for L type manufacturer; assigning probability one to H type for such an out-of-equilibrium contract is therefore consistent with our refinement. For $w \in (1-\upsilon_L, \upsilon_L)$, $d(p(w,\mu),\mu)$ is decreasing in μ and bounded above by 1. If (w,F) is not equilibrium dominated for the L type manufacturer, there exists $\mu \in [0,1]$ such that $v_L \leq wd(p(w,\mu),\mu) + F$ so that $(w-c)d(p(w,\mu),\mu) + F \geq v_L - cd(p(w,\mu),\mu) \geq v_L - c$ and therefore, (w,F) is not equilibrium dominated for the L type manufacturer. So, assigning probability one to H type for such an out-of-equilibrium contract is consistent with our refinement. This concludes the proof of part (i) of the lemma.

(ii) Consider a fully separating perfect Bayesian equilibrium where the low and high type manufacturers set distinct two-part tariffs (w_L, F_L) and (w_H, F_H). In a separating equilibrium, consumers can infer quality from wholesale prices and the retailer can mark-up the retail price without affecting consumers' beliefs about quality. It is easy to see that in any such fully revealing equilibrium, the low type manufacturer will set $w_L \leq v_L$ and $F_L = (v_L - w_L)$; the retailer's optimal price must be $p(w_L, 0) = v_L$ and the low quality manufacturer's equilibrium profit is therefore v_L.Further, the retailer's optimal retail price after observing (w_H, F_H) is $p(w_H, 1) = (1 + v_L + w_H)/2$. The condition that the low quality manufacturer should not have an incentive to imitate the high quality type is:

$$\frac{1}{2} w_H \left(1 - w_H + v_L\right) + F_H \leq v_L. \tag{13}$$

First, suppose that $w_H < 1 - v_L$.The ex ante expected profit of the high quality manufacturer in a separating equilibrium is $\pi_H = \frac{1}{2}(w_H - c)\left(1 - w_H + v_L\right) + F_H \leq v_L - \frac{1}{2}c\left(1 - w_H + v_L\right) < v_L(1-c)$ where the first inequality uses (13) and the second inequality follows from $w_H < 1 - v_L$. Then, for $\epsilon > 0$ small enough

$$\pi_H < (v_L - \frac{\epsilon}{2})(1 - c + \frac{\epsilon}{2}). \tag{14}$$

Consider the following deviation contract $(\widehat{w}, \widehat{F})$ where $\widehat{w} = 1 - v_L + \epsilon$, $\widehat{F} = \left(v_L - \frac{\epsilon}{2}\right)^2$.

Using an identical argument as in the last part of the proof of Lemma 1, one can show that if $\epsilon > 0$ is small enough and the manufacturer offers the deviation contract $(\widehat{w}, \widehat{F})$ the retailer rejects the contract if the coordinated belief $\mu < 1$ as he cannot break even (and so the deviating manufacturer gets zero profit); further, while the retailer can break even if $\mu = 1$, a low type manufacturer earns strictly lower than his equilibrium profit from this deviation at $\mu = 1$. Thus, this deviation is equilibrium dominated for the L type manufacturer. However for belief $\mu = 1$ and when ϵ is sufficiently small, the H type manufacturer can strictly gain from deviation to $(\widehat{w}, \widehat{F})$ which yields him profit $(\upsilon_L - \frac{\epsilon}{2})(1 - c + \frac{\epsilon}{2}) > \pi_H$ (using (14)). So using the Intuitive Criterion with coordinated beliefs, buyers should believe that the deviation comes from the manufacturer of type H with probability one which then makes this deviation by the H type manufacturer strictly gainful, a contradiction. Hence, $w_H \geq 1 - \upsilon_L$,which immediately implies that the retail price faced by buyers when the manufacturer is of H type $p_H = p(w_H, 1) = \frac{1+\upsilon_L+w_H}{2} \geq 1$.Thus, the ex ante expected consumer surplus in the industry is bounded above by $\frac{\alpha}{2}(\upsilon_L)^2$.

Finally, as $(p-c)(1+\upsilon_L-p)$ is strictly decreasing in p for $p \geq 1$ (using assumption (2)), $(p_H - c)(1 + \upsilon_L - p_H) \leq \upsilon_L(1-c)$ so that the sum of manufacturer and retailer's equilibrium profit, i.e., the industry profit when manufacturer is of H type is $(p_H - c)(1 + \upsilon_L - p_H) \leq \upsilon_L(1 - c)$.The ex ante expected industry profit is then bounded above by $\alpha\upsilon_L(1 - c) + (1 - \alpha)\upsilon_L = \upsilon_L(1 - \alpha c) = \overline{\pi}$.

Proof of Lemma 3. At $p^* = \alpha$ and at $p^* = \upsilon_L$,the LHS of (9) is equal to υ_L and the inequality in (9) is satisfied if, and only if, $\upsilon_L < 1 - \alpha$. As $\left(1 - \frac{p^*-\upsilon_L}{\alpha}\right) p^*$ is concave and maximized at $p^* = \frac{\alpha+\upsilon_L}{2}$, (9) holds for all $p^* \in [\upsilon_L, \alpha]$) (and therefore, for all $p^* \in [\max\{\upsilon_L, c\}, \alpha]$) if, and only if, $\frac{1}{4\alpha}(\alpha + \upsilon_L)^2 < (1 - \alpha)(\alpha + \upsilon_L)$,i.e., if, and only if, $\upsilon_L < \alpha(3 - 4\alpha)$. This establishes the second part of the lemma. If $\upsilon_L \geq \alpha(3 - 4\alpha)$, then there exist z_1, z_2,where $\upsilon_L < z_1 \leq z_2 < \alpha$ such that (9) is satisfied for $p^* \in [\upsilon_L, z_1] \cup [z_2, \alpha]$. Setting $\underline{p} = \max\{c, z_2\}$, we have the first part of the lemma. It follows that (9) is not satisfied for any $p^* \in [\upsilon_L, \alpha]$ if $\upsilon_L > 1 - \alpha$.

Proof of Proposition 3. Note that in any pooling PBE satisfying IC-I described in Proposition 2 the expected consumer surplus generated is given by

$$\frac{1}{2\alpha}(\alpha + \upsilon_L - p^*)^2 \geq \frac{1}{2\alpha}(\upsilon_L)^2 > \frac{\alpha}{2}(\upsilon_L)^2$$

where the first inequality uses the fact that $p^* \leq \alpha$. Next, we investigate whether if there are $p^* \in [\underline{p}, \alpha]$ (where $\underline{p}$ is defined in Proposition 2) such that

$$\left(1 - \frac{p^* - \upsilon_L}{\alpha}\right)(p^* - \alpha c) > \upsilon_L(1 - \alpha c) \tag{15}$$

First, consider c small and in particular, $c \leq \upsilon_L$. Then, $\underline{p}$ as defined in Proposition 2 does not depend on c. Note that the function $f(p) = \left(\frac{\alpha+\upsilon_L-p}{\alpha}\right)p$ is strictly decreasing in p for $p \in (\frac{\alpha+\upsilon_L}{2}, \alpha]$.Further, $f(\alpha) = \upsilon_L$. Fix $p^* \in (\max\{\underline{p}, \frac{\alpha+\upsilon_L}{2}\}, \alpha)$. Then, $f(p^*) > f(\alpha) = \upsilon_L$. As $c \to 0$, the right hand side of (15) converges to υ_L while the left hand side converges to $f(p^*) > \upsilon_L$. It follows that (15) holds for c sufficiently small. This establishes (i). Next, suppose that υ_L is small and in particular, $\upsilon_L < \alpha(3 - 4\alpha)$ in which case $\underline{p} = \max\{\upsilon_L, c\}$. It is easy to check that on the interval $[\max\{\upsilon_L, c\}, \alpha]$ the left hand side of (15) attains a maximum at $p^* = \max\left\{\frac{\alpha(1+c)+\upsilon_L}{2}, c\right\}$. Inequality (15) holds for some $p^* \in [\max\{\upsilon_L, c\}, \alpha]$ if and only if,

$$\begin{aligned} \frac{1}{4\alpha}(\alpha(1-c) + \upsilon_L)^2 &> \upsilon_L(1 - \alpha c), \text{ if } c \leq \frac{\alpha(1+c)+\upsilon_L}{2} \\ \frac{1}{\alpha}(\alpha + \upsilon_L - c)(1-\alpha)c &> \upsilon_L(1 - \alpha c), \text{if } c > \frac{\alpha(1+c)+\upsilon_L}{2} \end{aligned}$$

which is always satisfied for υ_L sufficiently small as $\frac{1}{4\alpha}(\alpha(1-c))^2 > 0$ and $\frac{1}{\alpha}(\alpha -$

$c)(1-\alpha)c > 0$. This establishes (ii). This concludes the proof.

Proof of Proposition 4. From Lemma 3 and Proposition 2, $\upsilon_L < \min\{1-\alpha, \alpha(3-4\alpha)\}$ implies every $p^* \in [\max\{\upsilon_L, c\}, \alpha]$ can be supported as the retail price in a pooling equilibrium as described in Proposition 2. Let $\widetilde{p}$ be the pooling price that maximizes expected industry profit over $p^* \in [\max\{\upsilon_L, c\}, \alpha]$. If $\widetilde{p} \in (\max\{\upsilon_L, c\}, \alpha)$, then $\widetilde{p} = \frac{\alpha(1+c)+\upsilon_L}{2} < \frac{\alpha}{2}(1+c) + \upsilon_L\left(1-\frac{\alpha}{2}\right)$ so that (10) holds at $p^* = \widetilde{p}$ and therefore at $p^* \leq \widetilde{p}$. If $\widetilde{p} = \alpha$, then we must have $\frac{\alpha(1+c)+\upsilon_L}{2} \geq \alpha$ and using this one can check that $\alpha < \frac{\alpha}{2}(1+c) + \upsilon_L\left(1-\frac{\alpha}{2}\right)$ i.e., (10) holds at $p^* = \alpha$ and therefore at $p^* \leq \alpha$. Finally, consider the situation where $\widetilde{p} = \max\{\upsilon_L, c\}$ which requires $\frac{\alpha(1+c)+\upsilon_L}{2} \leq \max\{\upsilon_L, c\}$. As $\alpha > \upsilon_L$, this can only happen if $\max\{\upsilon_L, c\} = c$. It is easy to see that (10) holds at $p^* = c$ if $\frac{\alpha}{2} + \left(1-\frac{\alpha}{2}\right)(\upsilon_L - c) > 0$ which is true. This establishes part (a). The expected total surplus under full information equals $(1-\alpha)\upsilon_L + \frac{3\alpha}{8}(1+\upsilon_L-c)^2$. The expected total surplus in a pooling equilibrium with $p^* \in [\max\{\upsilon_L, c\}, \alpha]$ is

$$\frac{1}{2\alpha}(\alpha+\upsilon_L-p^*)^2 + \frac{p^*-\alpha c}{\alpha}(\alpha+\upsilon_L-p^*) = \frac{1}{2\alpha}\left[(\alpha+\upsilon_L)^2 - p^{*2}\right] - c(\alpha+\upsilon_L-p^*)$$

which is strictly larger than $(1-\alpha)\upsilon_L + \frac{3\alpha}{8}(1+\upsilon_L-c)^2$ if, and only if,

$$\begin{aligned} p^{*2} - 2\alpha c p^* &< (\alpha+\upsilon_L)^2 - 2\alpha(1-\alpha)\upsilon_L - \frac{3\alpha^2}{4}(1+\upsilon_L-c)^2 - 2\alpha c(\alpha+\upsilon_L) \\ &= \frac{\alpha^2}{4} + \upsilon_L^2 + \frac{\alpha^2}{2}\upsilon_L - \frac{3\alpha^2}{4}(\upsilon_L-c)^2 - \frac{\alpha^2}{2}c - 2\alpha c\upsilon_L. \end{aligned}$$

It is easy to verify that this holds at $p^* = \upsilon_L$ as $c \to 0$ and is therefore satisfied for c small enough and p^* close to υ_L. Further, note that as $c \to 0$, $\widetilde{p} \to \frac{\alpha+\upsilon_L}{2}$. If $\upsilon_L > \frac{2\alpha}{3(1+\alpha)}$, the above inequality holds at $p^* < \frac{\alpha+\upsilon_L}{2}$ (and $c = 0$) and therefore, $p^* \leq \widetilde{p}$ when c is small enough. This establishes (b).

Appendix B: Belief Refinement for Intermediated Signaling Games (used in Section 4)

In this appendix B, we develop a notion of equilibrium refinement that can be applied to our game of selling through a retailer with unobserved vertical pricing. It is based on considerations similar to the Intuitive Criterion in standard signaling games. The refinement we introduce is potentially useful in other contexts and can be applied to a general class of *intermediated signaling games.* These are dynamic games of incomplete information with three players: a Sender (S), an Intermediary (I) and a Receiver (R), with the following extensive form:

1. Nature draws a type t for the sender from a finite set of types T according to a probability distribution $\beta(t), t \in T$, where $\beta(t) > 0, \sum_{t \in T} \beta(t) = 1$.

2. The Sender observes t and then chooses a message m from a set M of messages.

3. The Intermediary observes m (but not t) and then chooses an action a from a set of feasible actions A.

4. The Receiver observes a (but not m or t) and then chooses a response r from a set of feasible responses ρ.

5. Payoffs for the Sender, the Intermediary and the Receiver are given by $U_S(t, m, r), U_I(m, a, r)$ and $U_R(t, a, r)$ respectively.

The standard signaling game analyzed widely in the existing literature has two players - a Sender and a Receiver. The Sender has private information about its type and chooses a message which is observed directly by the Receiver who then chooses a response. The payoffs of both players may depend on the type of the Sender, the message and the response of the Receiver.

The intermediated signaling game outlined above differs from this standard signaling game in several respects. First, there are three players including an Intermediary who moves after the Sender and before the Receiver. Like the Receiver, the Intermediary does not observe the type of the Sender. Second, the message sent by the Sender is observed only by the Intermediary, while the Receiver only observes the action chosen by the Intermediary. Third, the Receiver's payoff does not depend directly on the message sent by the Sender to the Intermediary, but only depends on the action chosen by the Intermediary. Finally, the Sender's payoff does not depend directly on the action chosen by the intermediary (though the latter may influence the Sender's payoff through the Receiver's response).

Focusing on pure strategy Perfect Bayesian Equilibrium (PBE), the equilibrium strategies are as follows. (a) The Sender's equilibrium strategy is a function $m^* : T \to M$ with a Sender of type t choosing message $m^*(t), t \in T$, (b) The Intermediary's equilibrium strategy is a function $a^* : M \to A$, following any message m from the Sender the Intermediary chooses action $a^*(m)$. (c) The Receiver's strategy is a function $r^*(a) : A \to R$, following any action a by the Intermediary, the Receiver chooses response $r^*(a)$.

On the equilibrium path, the message sent by the Sender lies in the set M^* where

$$M^* = \{m^*(t) : t \in T\}$$

and the action chosen by the intermediary lies in the set A^*

$$A^* = \{a^*(m^*(t)) : t \in T\}.$$

Let $U_s^*(t)$ denote the equilibrium payoff of type t sender. For $m^* \in M^*$, let $U_I^*(m^*)$ denote the equilibrium payoff of the Intermediary in the continuation game after the manufacturer chooses m^*. For $a \in A/A^*$, let the probability distribution $\mu_a(t) \geq 0, t \in T$, $\sum_{t\in T} \mu_a(t) = 1$, be the out-of-equilibrium belief of the Receiver when it observes action a of the intermediary.

Consider any out-of-equilibrium action $a \in A/A^*$of the Intermediary. Let Σ_T be the set of all probability distributions on T. Further, for any $\widehat{\mu} \in \Sigma_T$ let

$$BR(\widehat{\mu}, a) = \arg\max_{r \in \rho} \sum_{t \in T} U_R(t, a, r)\widehat{\mu}(t)$$

be the set of best responses of the Receiver after observing a when it has belief $\widehat{\mu}$; let $BR(T, a)$ be the set of all such best responses for all possible beliefs of the Receiver i.e.,

$$BR(T, a) = \cup_{\widehat{\mu} \in \Sigma_T} BR(\widehat{\mu}, a).$$

In line with the Intuitive Criterion (Cho and Kreps, 1987) for the standard signaling game and in accordance with the principle of focusing on unilateral deviations when accounting for an observed out-of-equilibrium action, we introduce two definitions of equilibrium domination, one for the Sender and one for the Intermediary:

Definition 1 *Given the equilibrium strategy $m^* : T \to M$ of the Sender, an out-of-equilibrium action $a \in A/A^*$ is said to be equilibrium dominated for the Intermediary who has observed message $\widetilde{m} \in M^*$if*

$$\max_{r \in BR(T,a)} U_I(\widetilde{m}, a, r) < U_I^*(\widetilde{m}).$$

Definition 2 *Given the equilibrium strategy $a^* : M \to A$ of the Intermediary, for $t \in T$, a message $\overline{m} \in M/M^*$ is said to be equilibrium dominated for the Sender of type t if*

$$\max_{r \in BR(T, a^*(\overline{m}))} U_S(t, \overline{m}, r) < U_S^*(t).$$

The belief formation process by the Receiver after observing an out-of-equilibrium action $a \in A/A^*$ has two components. First, the Receiver assigns a probability $x_a^I \in [0,1]$ that the observed out-of-equilibrium action a results from a unilateral deviation by the Intermediary (given the equilibrium strategy of the Sender) and a probability $x_a^S = 1 - x_a^I$ that the out-of-equilibrium action a results from a unilateral deviation by the Sender (given the equilibrium strategy of the Intermediary). Second, conditional on a unilateral deviation by the Intermediary, the Receiver assigns a probability $y_a^I(t)$ to the Sender being of type $t, y_a^I(t) \geq 0, \sum_{t\in T} y_a^I(t) = 1$, while conditional on a unilateral deviation by the Sender, the Receiver assigns a probability $y_a^S(t)$ to the Sender being of type $t, y_a^S(t) \geq 0, \sum_{t\in T} y_a^S(t) = 1$. Note that even when considering a unilateral deviation by the Intermediary, one assigns probabilities $y_a^I(t)$ to the Sender being of type t as only the Sender and not the Intermediary can be of different types. Nevertheless, as explained in more detail below, depending on the type of equilibrium (pooling, semi-separating or separating), a deviation by the Intermediary can reveal some information to the Receiver.

Next, we use the above definitions of equilibrium domination to define when an out-of-equilibrium action can be attributed to a unilateral deviation by the Intermediary and/or the Sender.

Definition 3 *An out-of-equilibrium action $a \in A/A^*$ of the Intermediary can be attributed to a* unilateral *deviation by the Intermediary if given the equilibrium strategy $m^* : T \to M$ of the Sender, a is* not *equilibrium dominated for the Intermediary for some equilibrium message $\widetilde{m} \in M^*$ from the Sender.*

Definition 4 *An out-of-equilibrium action $a \in A/A^*$ of the Intermediary can be attributed to a* unilateral *deviation by the Sender if $a = a^*(\overline{m})$ for some $\overline{m} \in M$ and further, given the equilibrium strategy of the Intermediary, there exists $t \in T$ such that $\overline{m}$ is* not *equilibrium dominated for the Sender of type t.*

Note that the above two definitions incorporate an asymmetry in that any out-of-equilibrium action a can, in principle, be attributed to a unilateral deviation of the Intermediary, whereas this is not the case for the Sender. In particular, if a is not in the reach of the equilibrium strategy of the Intermediary, then no unilateral deviation by the Sender can account for a.

We now outline a set of restrictions on the out-of-equilibrium belief $\mu_a(t), t \in T$, that adapt the Intuitive Criterion for standard signaling games to our game of intermediated signaling. The criterion follows the principle that if observations can be rationalized by unilateral deviations, they should get priority (see, e.g., Bagwell and Ramey 1991). This implies, among other things, that if a is not in the reach of the equilibrium strategy of the Intermediary and a is not equilibrium dominated for the Intermediary, then the Receiver should blame the Intermediary for the deviation from equilibrium play.

Condition 1 ***Intuitive Criterion for Intermediated Signaling Games (IC-I)****: Consider an out-of-equilibrium action $a \in A/A^*$ of the intermediary that can be attributed to a unilateral deviation by either the Sender or the Intermediary (or both). Then, the out-of-equilibrium belief $\mu_a(t), t \in T$ satisfies the Intuitive Criteria for Intermediated Signaling Games (IC-I) if the following restrictions are satisfied:*

(i)

$$\mu_a(t) = x_a^I y_a^I(t) + (1 - x_a^I) y_a^S(t), t \in T$$

and further, if action a cannot be attributed to a unilateral deviation by the Sender (Intermediary), then $x_a^I = 1$ ($x_a^I = 0$).

(ii) Suppose that action a can be attributed to a unilateral deviation by the Intermediary. Let $T_0(a) = \{t \in T : a$ is equilibrium dominated for the Intermediary when the Sender is of type t and sends equilibrium message $m^*(t)\}$. Then, $y_a^I(t) = 0$ for all

$t \in T_0(a)$. Further, for any $\widehat{t} \in T/T_0(a)$, $\tau(\widehat{t}) = \{t \in T/T_0(a) : m^*(t) = m^*(\widehat{t})\}$, the following holds:

$$\frac{y_a^I(\widehat{t})}{\sum_{t\in\tau(\widehat{t})} y_a^I(t)} = \frac{\beta(\widehat{t})}{\sum_{t\in\tau(\widehat{t})} \beta(t)}.$$

(iii) Suppose that the action a can be attributed to a unilateral deviation by the Sender. If $a = a^*(\overline{m})$ and further, given the equilibrium strategy of the Intermediary, the message $\overline{m}$ is equilibrium dominated for the Sender of type t, then $y_a^S(t) = 0$.

The first part of the definition considers when to attribute an out-of-equilibrium action $a \in A/A^*$ exclusively to the retailer or the manufacturer or whether unilateral deviations by both players can account for the deviation (given the equilibrium strategy of the other). For any such attribution to a unilateral deviation by a player, the second and third requirements impose restrictions on assignment of beliefs to different types of the Sender. The third requirement simply adjusts the implications of the original Intuitive Criterion to the setting of intermediated signaling where the Receiver does not observe the Sender's message and can only infer which Sender could have deviated given the Intermediary's equilibrium strategy.

The second requirement is more involved and a few examples may clarify. If, for example, one considers a candidate pooling equilibrium and an out-of-equilibrium action $a \in A/A^*$ that is not equilibrium dominated for the retailer, then $y_a^I(\widehat{t}) = \beta(\widehat{t})$, i.e., conditional on a unilateral deviation by Intermediary, as the Receiver cannot infer any information from the incentive of the Intermediary to deviate (given the deviation is not based on any learning of the type of the Sender), the Receiver should assign the beliefs to be identical to the prior for all types. In other words, as the Intermediary does not acquire any additional information after observing the Sender's message, to the extent that the Receiver blames out-of-equilibrium action on a unilateral deviation by the Intermediary it should assign the same beliefs as the Intermediary has at that stage. A similar logic applies if a subset of Sender types pool on a message and one looks at the event of unilateral deviation by the intermediary conditional on this pooled message: the relative likelihood of each type that pools should be as in the prior belief. Finally, if an out-of-equilibrium action $a \in A/A^*$ is equilibrium dominated for the Intermediary given the Sender's equilibrium strategy $m^*(t)$ for only a strict subset of Sender types, then conditional on attributing attributing a to a unilateral deviation by the Intermediary, the Receiver should assign probability zero to this subset and probability one to the complement of this subset. Overall, the second requirement is a conservative way to implement the considerations underlying the Intuitive Criterion: the Receiver attributes deviations in such a way that they are consistent with the information the Intermediary may have had when deviating.

The criterion outlined above confines attention to incentives for unilateral deviations. If an out-of-equilibrium action cannot be accounted for by unilateral deviations, then our criterion does not impose any restriction on the out-of-equilibrium belief.